USPAS Accelerator Physics 2021
(Virtually) Texas A&M University

Ch 12: Quantum Excitation and Synchrotron Light Facility Lattices

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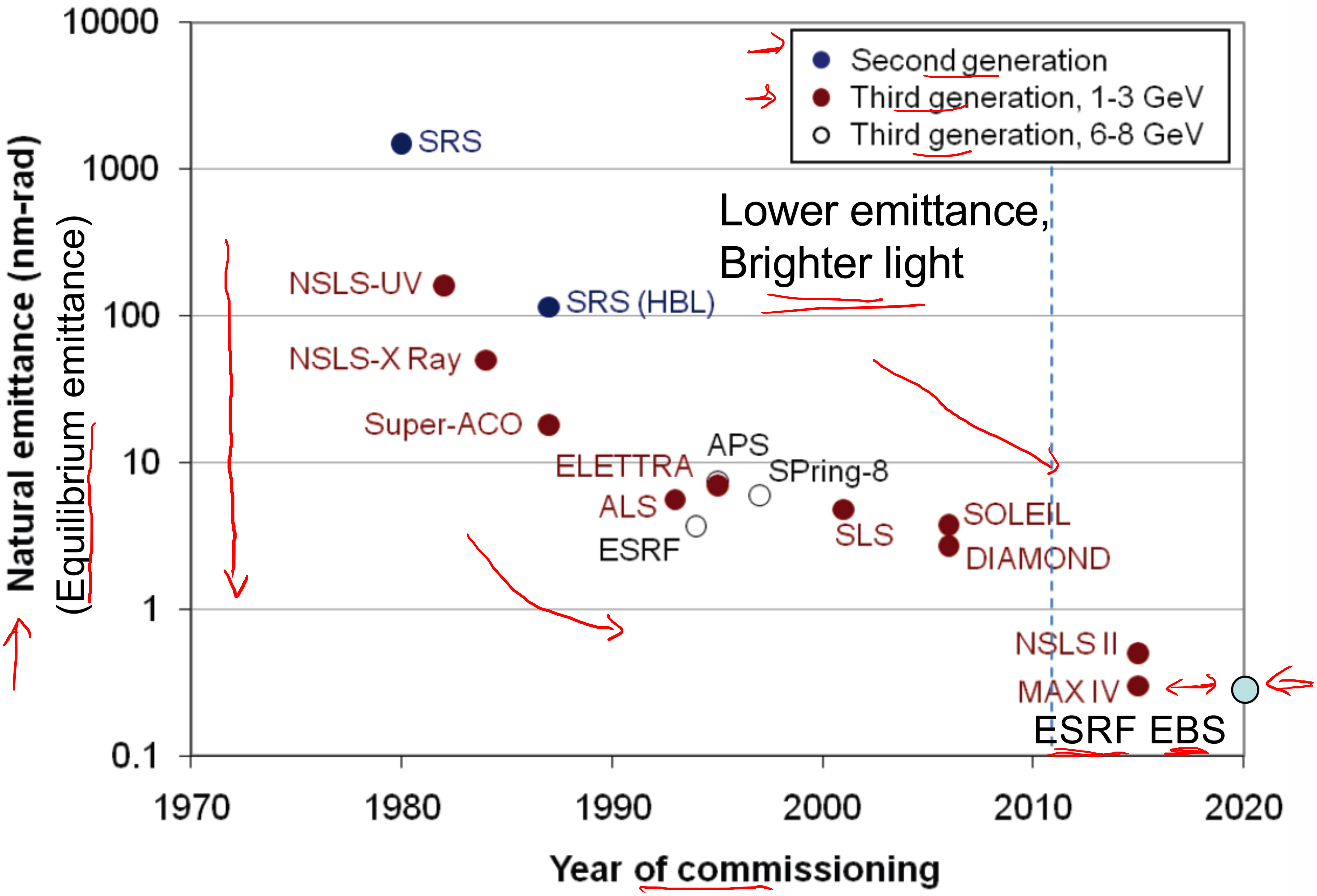
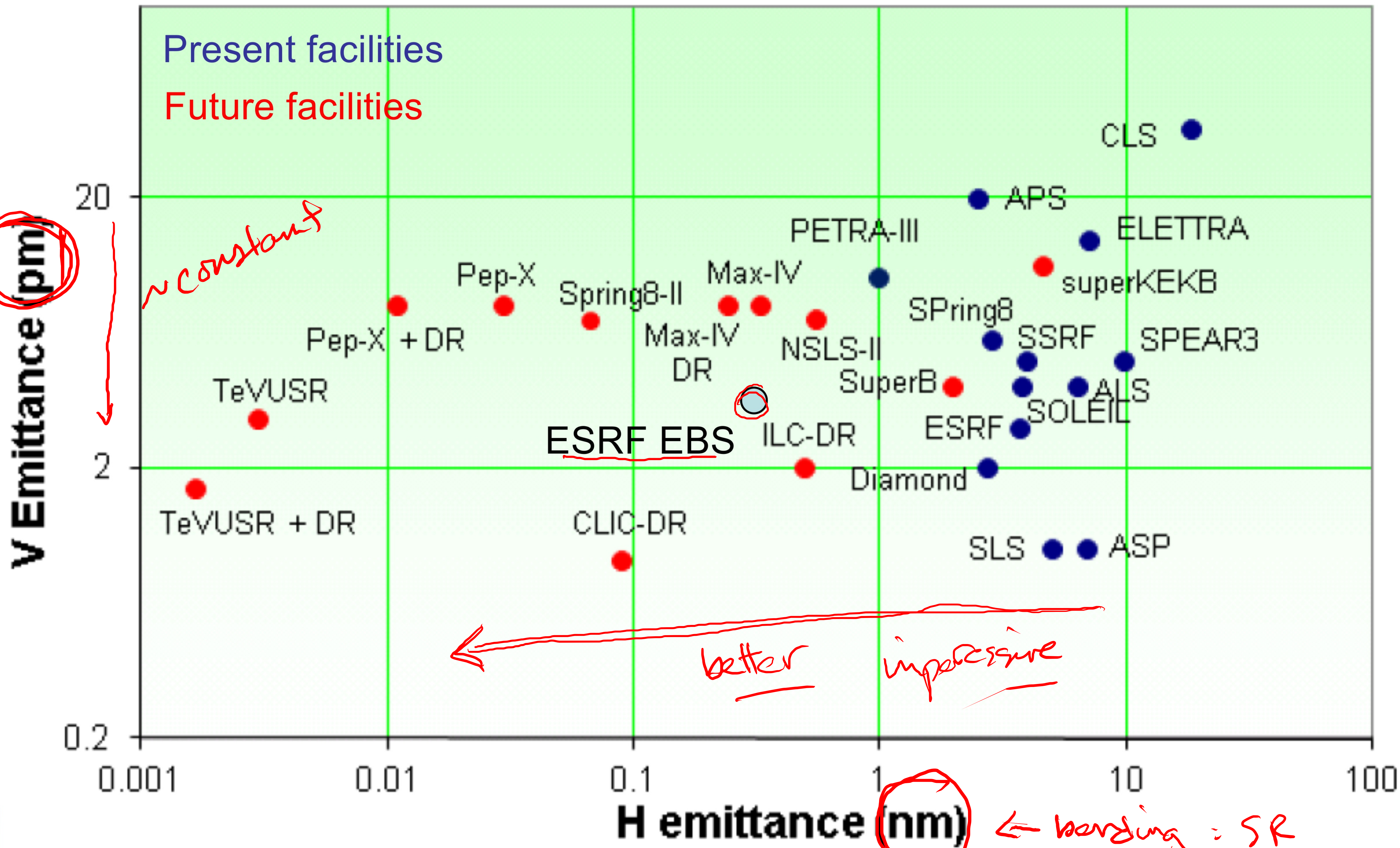
Synchrotron Light Source Emittance Evolution



A. Wolski, 2011 CERN Accelerator School Lectures, Greece

Another View of Light Source (unequal) Emittances



R. Bartolini, ICFA Beam Dynamics Newsletter 57 (2012)

Chapter 11: Synchrotron Radiation Review

$$\frac{p\,[\mathrm{GeV/c}]}{q\,[e^-]} \approx 0.3\, B[\mathrm{T}]\, \rho[\mathrm{m}]$$

- Synchrotron light **photon** characteristic energy $\boxed{u_c = \frac{3}{2}\frac{\hbar c \gamma^3}{\rho}}$

$$\hbar = 6.582 \times 10^{-16}\ [\mathrm{eV\ s}] \qquad \boxed{u_c\ [\mathrm{keV}] = 0.665\ E^2\ [\mathrm{GeV}^2]\ B\,[\mathrm{T}]}$$

e^-

3 GeV electrons (γ=5871) in B=0.4T, ρ=25m ring => u_c = 2.4 keV photons

- Energy loss **per turn** given by eqns (11.19-20)

single particle

$$U_0 = \frac{C_g E_0^4}{\rho} \qquad U_0\ [\mathrm{keV}] = 88.5 \frac{E_0^4\ [\mathrm{GeV}^4]}{\rho\ [\mathrm{m}]}$$

- Average number of **photons per turn** for electrons is then

$$\frac{U_0}{u_0} \approx 39.9\ E\ [\mathrm{GeV}]$$

3 GeV electrons (γ=5871) in B=0.4T,
ρ=25m ring => 120 photons/turn
or 0.76 photons/m/turn

SR kicks are discrete quanta => random walk

Looking ahead

- Why don't electrons just damp down to zero emittance?
- Number of photons N emitted per second is related to n(u), classical radiated power distribution

$$N = \int_0^\infty n(u)\, du$$

- We will see the equilibrium of quantum excitation from random walks in longitudinal phase space from **photon emission** is

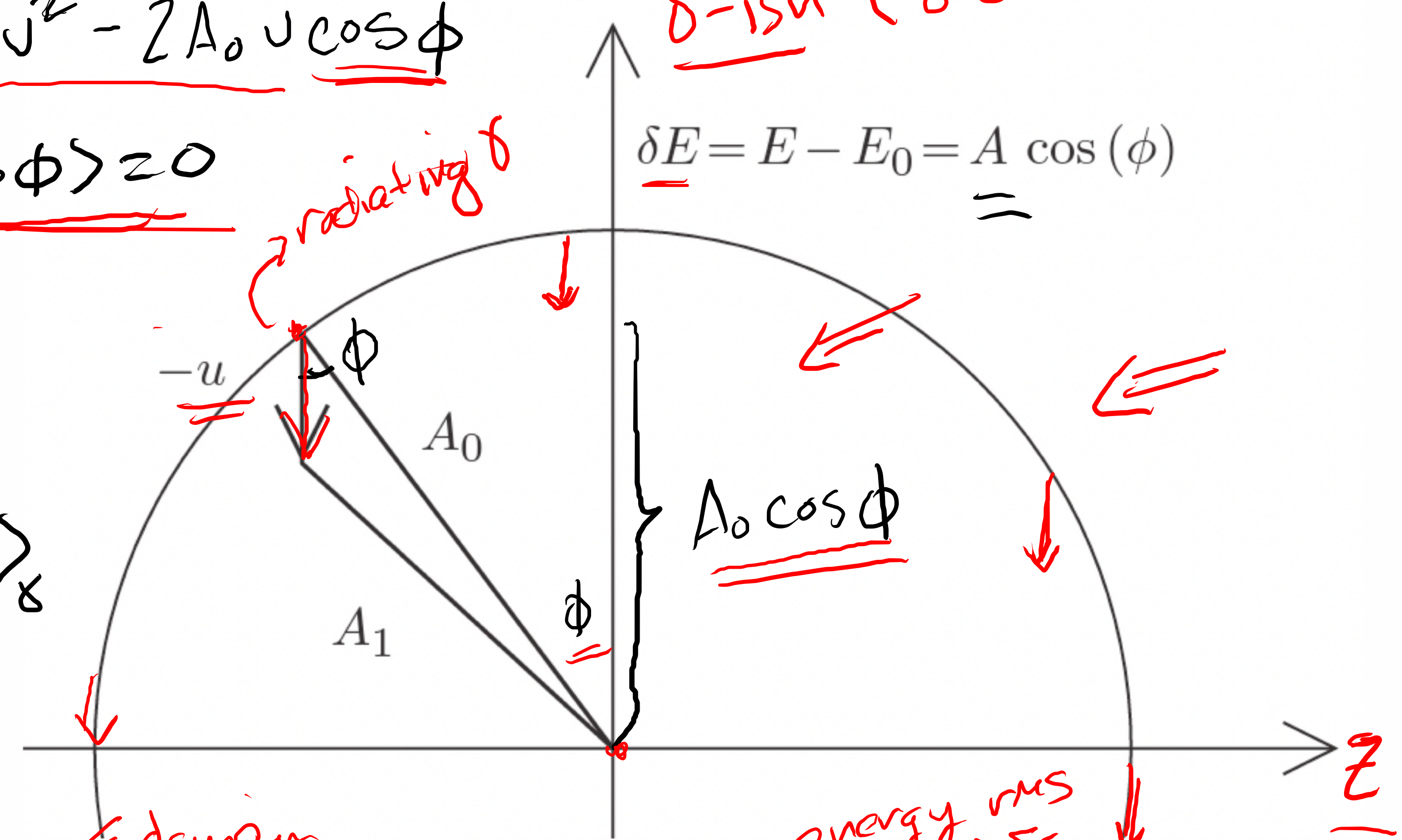
$$N\langle u^2\rangle = \int_0^\infty n(u)\, u^2\, du$$

$$= \frac{55}{24\sqrt{3}} r_0 \hbar m c^4 \frac{\gamma^7}{\rho^3}$$

12.1: Energy Spread

Law of cosines: $A_1^2 = A_0^2 + u^2 - 2A_0 u \cos\phi$

average over ϕ: $\langle \Delta A^2 \rangle = \langle u^2 \rangle$ $\quad \langle \cos\phi \rangle = 0$



Random walk: average over many photons:

$$\frac{d\langle A^2\rangle}{dt} = \left\langle \frac{dA^2}{dt} \right\rangle_\gamma = N\langle u^2\rangle_\gamma$$

At equilibrium, growth and damping match on average around the ring:

$$\underbrace{\frac{\langle N\langle u^2\rangle_\gamma\rangle_s}{2}}_{\text{growth rate}} = \frac{\langle A^2\rangle}{\underbrace{\tau_s}_{\text{damping rate}}} \Rightarrow \text{equilibrium}$$

damping

SR damping rate

energy rms spread δE

$$\sigma_E^2 = \frac{1}{2}\langle A^2\rangle = \frac{\tau_s}{4}\langle N\langle u^2\rangle_\gamma\rangle_s$$

or

$$\boxed{\left(\frac{\sigma_E}{E_0}\right)^2 = \frac{C_q}{J_s}\gamma^2\frac{\langle G^3\rangle}{\langle G^2\rangle}}$$

long partition #

$$C_q = \frac{55}{32\sqrt{3}}\frac{\hbar}{mc}$$
$$= 3.84 \times 10^{-13}\ [\mathrm{m}] \quad \text{electrons}$$

nicely dimensionless

$$\left(\frac{\sigma_E}{E_0}\right)^2 = \frac{C_q}{J_s}\gamma^2\frac{\langle G^3\rangle}{\langle G^2\rangle}$$

J_s: longitudinal partition #

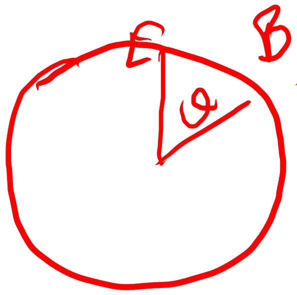
quantum constant

$$C_q = \frac{55}{32\sqrt{3}}\frac{\hbar}{mc}$$

$$= 3.84\times 10^{-13}\ [\mathrm{m}] \qquad \text{electrons}$$

$$G^{(s)} \equiv \frac{1}{\rho(s)}$$

For isomagnetic ring: $J_s = 2$, $\langle G^3\rangle = \frac{1}{\rho^3}$, $\langle G^2\rangle = \frac{1}{\rho^2}$



$\vec{B} = B_y$

for all θ

$$\boxed{\left(\frac{\sigma_E}{E_0}\right)^2 = \frac{C_q}{2}\frac{\gamma^2}{\rho\,[\mathrm{m}]}}$$

$$\sigma_E \propto \gamma/\rho^{1/2}$$

For most rings e^- ~ constant

12.1

What about horizontal?

12.2: Horizontal equilibrium emittance

Recall dispersion discussion
(motivated by δ changing in SR)
$\uparrow$ kicks

$$x_{TOT} = x_\beta + \eta \frac{\delta E}{E_0} \quad \left(\sim \frac{\delta p}{p_0}\right)$$

$\uparrow$ betatron

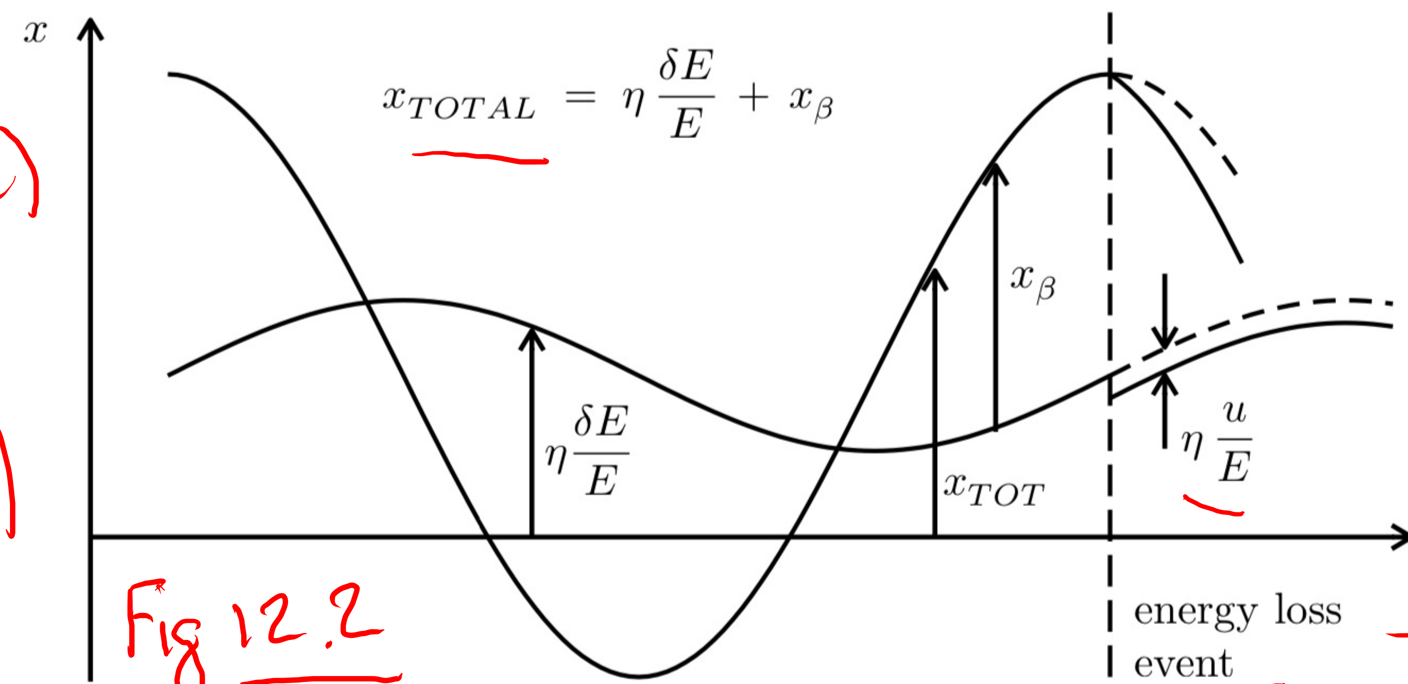


Fig 12.2

$\rightarrow \gamma$

$$\begin{pmatrix} \Delta x_\beta \\ \Delta x_\beta' \end{pmatrix} = \begin{pmatrix} \eta \\ \eta' \end{pmatrix} \frac{u}{E_0}$$

moving onto different betatron orbit
changing betatron amplitude $\Rightarrow$ via $\eta, \frac{u}{E}$

Betatron amplitude (conserved quantity, ~ action)

$$a_x^2 = \gamma_x x_\beta^2 + 2\alpha_x x_\beta x_\beta' + \beta_x x_\beta'^2 \Rightarrow \Delta a_x^2 = \mathcal{H}\left(\frac{u}{E_0}\right)^2$$

"Courant-Snyder Invariant" $\rightarrow$ No longer invariant!

$\mathcal{H}$: what is it? $\Rightarrow$ related to η, η' $\Rightarrow$ dominate discussion of eq. Horz ϵ.

RESULTS

$$\boxed{\mathcal{H}^{(s)} = \gamma_x y^2 + 2\alpha_x y y' + \beta_x y'^2}$$ ⟸ everything depends on $\underline{\underline{s}}$

(Just like C-S invariant with $x_\beta \to y,\ x'_\beta \to y'$)

NOT INVARIANT → depends on s

But you promised results on ϵ_x . . . $\langle\ \rangle_s$ ← $\mathcal{H}(s)$ ↓ $\mathcal{H}(s)$

book (12.2) ~~p...~~

Horz damping from SR

$$\text{equilibrium } \epsilon_x = \frac{1}{2}\langle a_x^2\rangle = \frac{\tau_x}{4E_0^2}\langle N \langle u^2\rangle \mathcal{H}\rangle_s$$

average over emitted γ

average over 1 turn in ring

$$\boxed{\epsilon_x \propto \langle \mathcal{H}(s)\rangle_s}$$

only contributes where there is SR

Onwards to Synchrotron Light Sources

- The equilibrium emittance, balanced between synchrotron radiation damping and quantum excitation effects, is

Nomenclature

$$\epsilon_{\text{rms}} = \frac{\tau_x}{4LU_s^2} \oint \langle u_\gamma^2 \rangle \, \mathcal{H}(s) N_\gamma \, ds$$

$$L: \ \text{circumference}$$

$$U_s: \ \text{Energy of synchronous particle}$$ E_0

$$\tau_x: \text{Horizontal damping time}$$

$$\langle u_\gamma^2 \rangle N_\gamma: \ \text{photon energy integral terms}$$

$$\boxed{\mathcal{H}(s) = \beta_x(s)\eta_x'^2 + 2\alpha_x\eta_x\eta_x' + \gamma_x\eta_x^2}$$

$$(\text{“Curly} - \text{H function”})$$

Review

$$G \equiv \frac{1}{\rho}$$

- Energy loss per turn

$$U_\gamma \approx \frac{C_\gamma U^4}{2\pi} \boxed{\oint \frac{ds}{\rho^2}} \quad \underline{\langle G^2 \rangle_s}$$

Integral only in dipoles
Property of lattice

$$\text{constant} \quad C_\gamma = \frac{4\pi}{3} \frac{r_3}{(mc^2)^3} = 8.85 \times 10^{-5} \frac{\text{m}}{(\text{GeV})^3}$$

- The integral above is sometimes called the second synchrotron radiation integral (e.g. Wolski, Handbook):

$$\boxed{I_2 \equiv \oint \frac{ds}{\rho^2(s)}} \qquad U_\gamma \approx \frac{C_\gamma U^4}{2\pi} I_2$$

A. Wolski, Joint US-CERN-Japan-Russia school on particle accelerators, April 2011
http://cas.web.cern.ch/cas/JAS/Erice-2011/Lectures/StorageRingDesign2-Handout.pdf

Reference

Radiation Integrals

- There are several other radiation integrals that come into play in evaluation of effects of radiation on dynamics of ultra-relativistic particles in a storage ring or beamline, including one that depends on curly-H.

$$I_1 \equiv \oint \frac{\eta_x(s)}{\rho(s)} ds \rightarrow \quad \text{momentum compaction } \alpha_p = \frac{I_1}{L}$$

don't worry!

$$I_2 \equiv \oint \frac{ds}{\rho^2(s)} \leftarrow \sigma_E^2 \qquad I_4 \equiv \oint \frac{\eta_x(s)}{\rho(s)} \left(\frac{1}{\rho^2(s)} + 2k_1(s) \right) ds$$

$$I_3 \equiv \oint \frac{ds}{|\rho(s)|^3} \leftarrow \sigma_E^2 \qquad I_5 \equiv \oint \frac{\mathcal{H}_x}{|\rho^3(s)|} ds \leftarrow \epsilon_x$$

These integrals only $\boxed{\text{depend on the lattice design}}$

Equilibrium Horizontal Emittance

- The evolution of horizontal emittance, including both damping and quantum excitation, is

$$\frac{d\epsilon_x}{dt} = -\frac{2}{\tau_x}\epsilon_x + \frac{2}{J_x \tau_x} C_q \gamma^2 \frac{I_5}{I_2}$$

Partition #

$$J_x = 1 - \frac{I_4}{I_2}$$

damping   Quantum excitation

"quantum constant" $C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc} \approx 3.83 \times 10^{-13}$ m

- This is at an equilibrium for the "natural" emittance

$$\boxed{\epsilon_{0x} = C_q \frac{\gamma^2}{J_x} \frac{I_5}{I_2}}$$

  - This only depends on beam energy and radiation integrals!

Equilibrium Energy Spread

- We can average the quantum excitation effects on beam momentum offset to find the evolution of energy spread:

$$\frac{d\sigma_\delta^2}{dt} = C_q \gamma^2 \frac{2}{J_u \tau_u} \frac{I_3}{I_2} - \frac{2}{\tau_u} \sigma_\delta^2 \qquad J_u = 2 + \frac{I_4}{I_2}$$

Quantum excitation　　damping

- We can also find the equilibrium energy spread and bunch length

$$\boxed{\sigma_{\delta 0}^2 = C_q \gamma^2 \frac{I_3}{J_u I_2}}$$

2

$$\text{bunch length} \quad \boxed{\sigma_{z0} = \frac{\alpha_p c}{\Omega_s} \sigma_{\delta 0}}$$

  - Note the lack of RF parameters! This equilibrium distribution is again determined only by the lattice (and collective effects). We can shorten bunch length by raising RF voltage, Ω_s

Evaluating Radiation Integrals

- If bends have no quadrupole component (a modern separated function synchrotron), $J_x \approx 1$ and $J_u \approx 2$
- To find the equilibrium emittance, we then need to evaluate two synchrotron radiation integrals

- I_2 depends on only detailed knowledge of dipole magnets
  - e.g. for all dipole magnets being the same, total bend 2π

$$I_2 = \oint \frac{ds}{\rho^2(s)} = \frac{2\pi B}{(B\rho)} \approx \frac{2\pi c B}{U/e}$$

- Evaluating I_5 depends on detailed knowledge of optics

$$I_5 \equiv \oint \frac{\mathcal{H}_x}{|\rho^3(s)|} ds \qquad \mathcal{H}(s) = \beta_x \eta_x'^2 + 2\alpha_x \eta_x \eta_x' + \gamma_x \eta_x^2$$

FODO Lattice I_5 $\propto \int \mathcal{H} \to \eta, \eta'$

- Just like our excursions into the FODO lattice before, we had calculated our optical functions in terms of
  - Thin quadrupole focal length f
  - Dipole bending radius ρ (for dispersion contributions)
  - Dipole lengths $L = \rho\theta$ (full space between quadrupoles)

- These calculations are usually done with computer programs that find the optical functions and integrate $\mathcal{H}$ for us.
  - But Wolski (see below) writes out some of the logic to progress through a FODO lattice and evaluate some reasonably realistic approximations

thin quads

$$\underline{\theta \ll 1} \quad \Rightarrow \quad \rho \gg 2f \quad \Rightarrow \quad \underline{4f \gg L}$$

small θs

A. Wolski, 2011 CERN Accelerator School Lectures, Greece

FODO Lattice I_5

- Similar to the dogleg, the analysis is most easily done in an expansion of small dipole bend angle θ

math ☹

$$\frac{I_5}{I_2} = \left(4 + \frac{\rho^2}{f^2}\right)^{-\frac{3}{2}} \left[8 - \frac{\rho^2}{2f^2}\theta^2 + O(\theta^4)\right]$$

$$\approx \left(1 - \frac{\rho^2}{16f^2}\theta^2\right)\left(1 + \frac{\rho^2}{4f^2}\right)^{-\frac{3}{2}} \qquad \sin\frac{\mu}{2} = \frac{\rho\theta}{2f}$$

$$\rho >> 2f \quad \Rightarrow \quad \frac{I_5}{I_2} \approx \left(1 - \frac{L^2}{16f^2}\right)\frac{8f^3}{\rho^3}$$

$$4f >> L \quad \Rightarrow \quad \frac{I_5}{I_2} \approx \frac{8f^3}{\rho^3}$$

Approximate Natural Emittance of FODO Lattice

Horz

- We can then write the approximate natural horizontal emittance of the FODO lattice, again with $J_x \approx 1$

Lower is better

$$\epsilon_0 = C_q \frac{\gamma^2}{J_x} \frac{I_5}{I_2} \approx \boxed{C_q \gamma^2 \left(\frac{2f}{L}\right)^3 \theta^3 = \epsilon_{0x}}$$

$\left(\frac{f}{L}\right)^3$ θ^3

  - Proportional to square of beam energy γ
  - Proportional to cube of bending angle per dipole
    - Increase number of cells to reduce bending angle per dipole and thus reduce FODO emittance.
  - Proportional to cube of quadrupole focal length
    - Stronger quads gives stronger focusing, lower natural emittance
  - Inversely proportional to cube of the cell (or dipole) length
    - Longer cells also reduce overall natural emittance

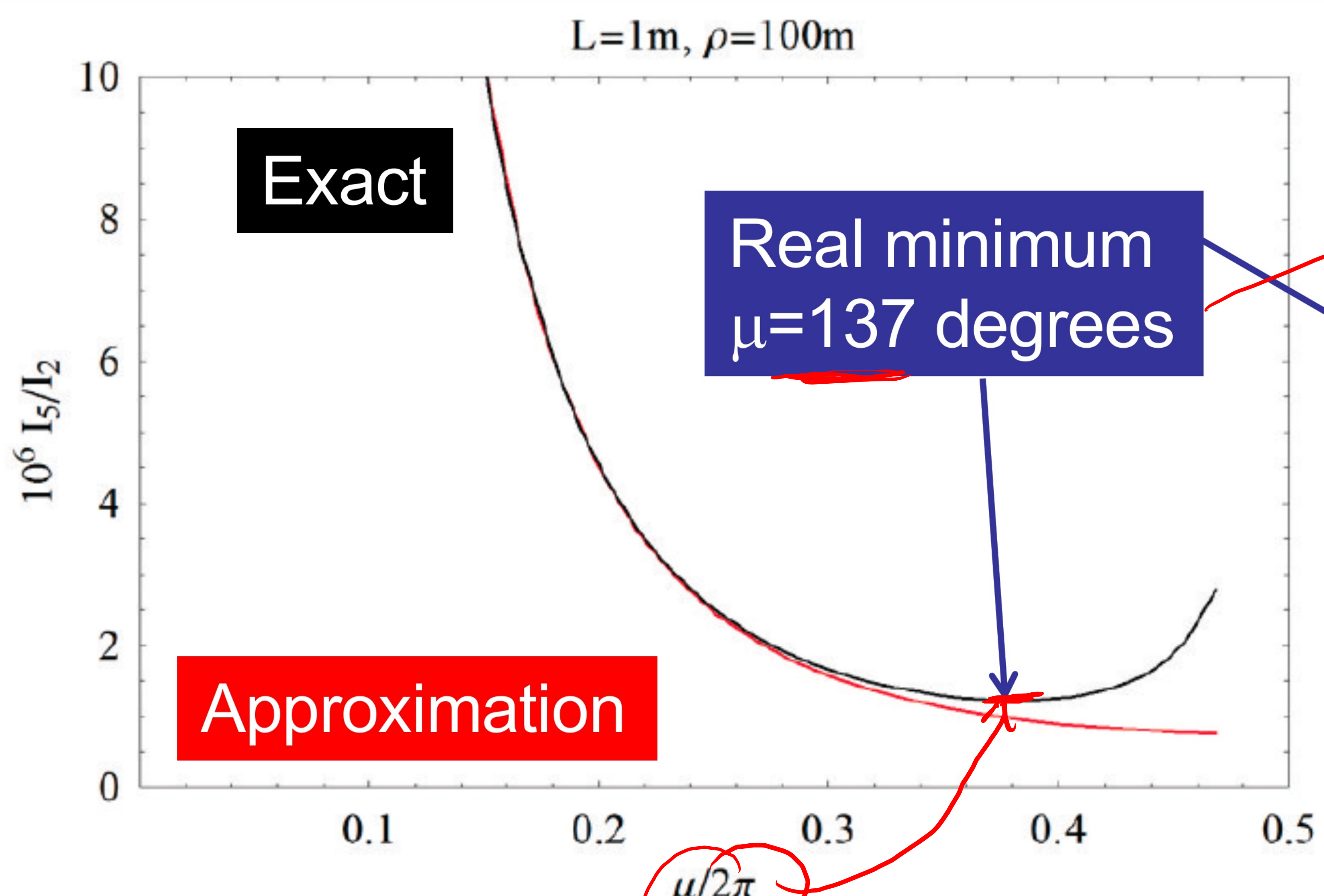
A. Wolski, 2011 CERN Accelerator School Lectures, Greece

Minimum Emittance of FODO Lattice?

- The stability criterion for FODO lattices with these parameters is $f \geq L/2$ with a minimum of $f/L = 1/2$
  - Estimated FODO lattice minimum emittance

$$\epsilon_0 \approx C_q \gamma^2 \theta^3$$

  - But approximations start to break down for large f



phase adv/cell ~137°

$$\boxed{\epsilon_0 \approx 1.2\, C_q \gamma^2 \theta^3}$$

"ultimate FODO"

We Can Do Better!

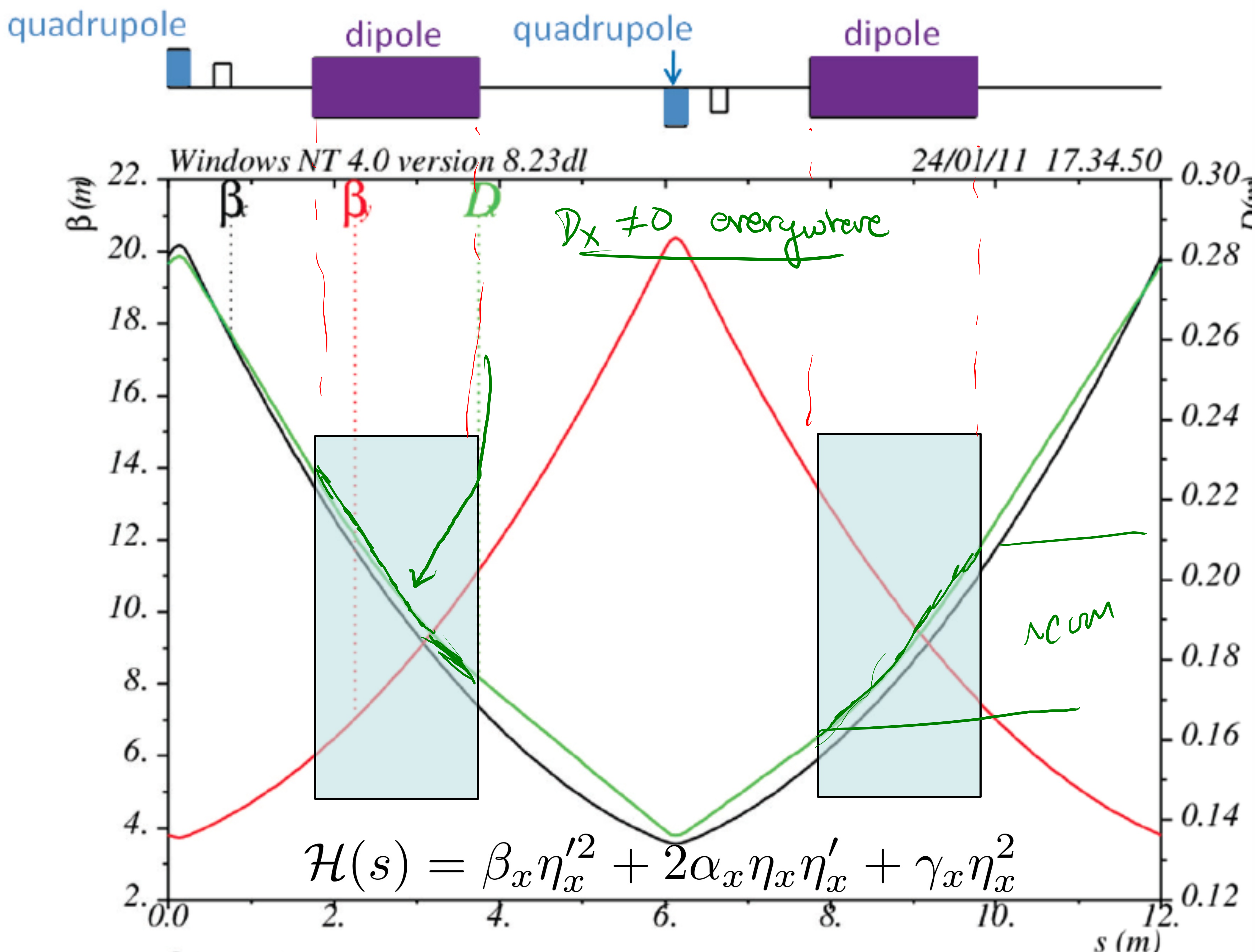
- It turns out that this emittance isn't usually good enough for modern third-generation light source requirements
  - 1-2 orders of magnitude too big
- How do we fix this?
  - Beam energy determines some properties of the sync light
  - So the remaining handle we have is the optics

$$I_5 \equiv \oint \frac{\mathcal{H}_x}{|\rho^3(s)|} ds \qquad \mathcal{H}(s) = \beta_x \eta_x'^2 + 2\alpha_x \eta_x \eta_x' + \gamma_x \eta_x^2$$

  - Minimizing η and η' in the dipoles will minimize the overall integral of $\mathcal{H}$ and thus I_5

- How do the dispersion functions look though FODO dipoles?

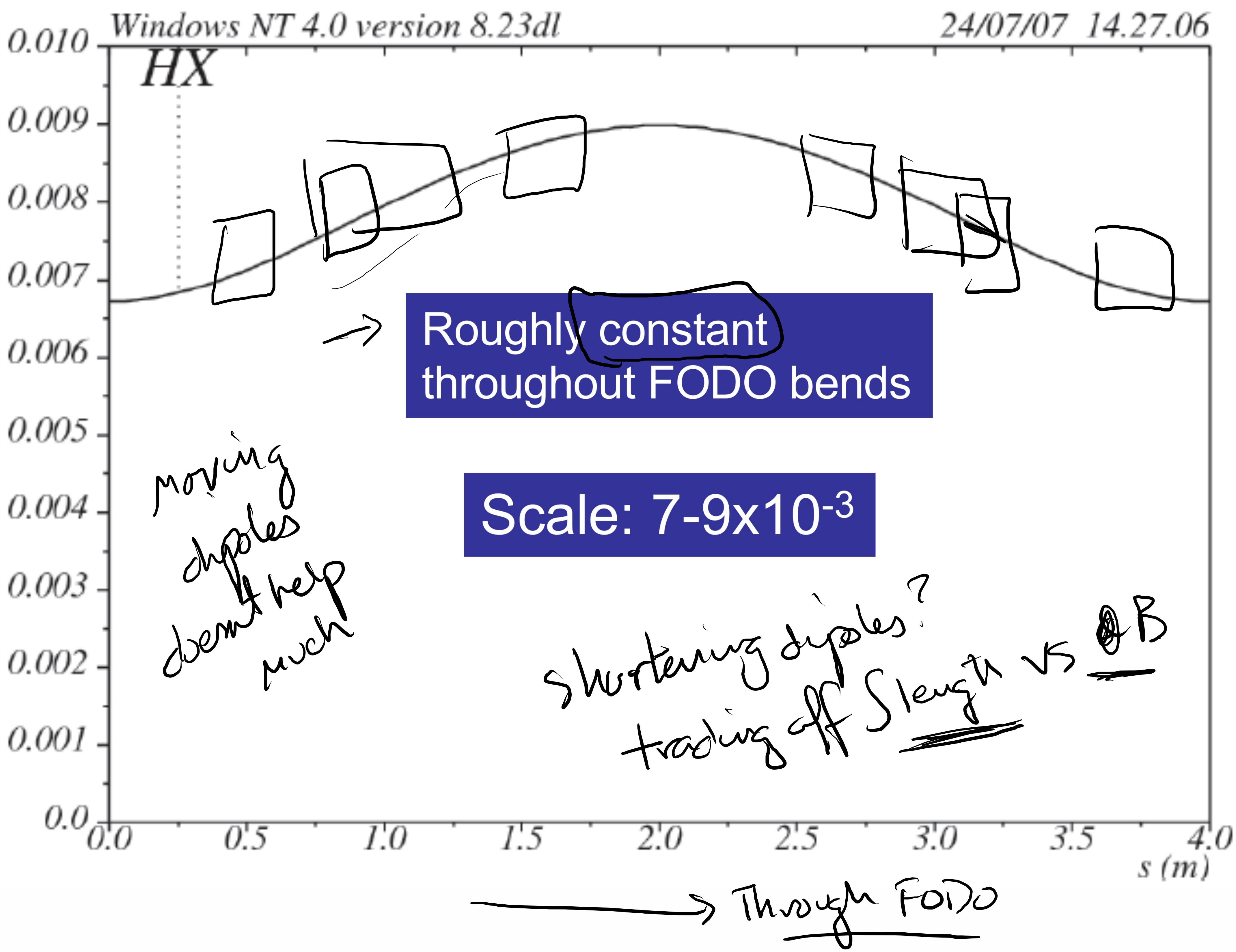
FODO Cell Optics



$$\mathcal{H}(s) = \beta_x \eta_x'^2 + 2\alpha_x \eta_x \eta_x' + \gamma_x \eta_x^2$$

A. Wolski, 2011 CERN Accelerator School Lectures, Greece

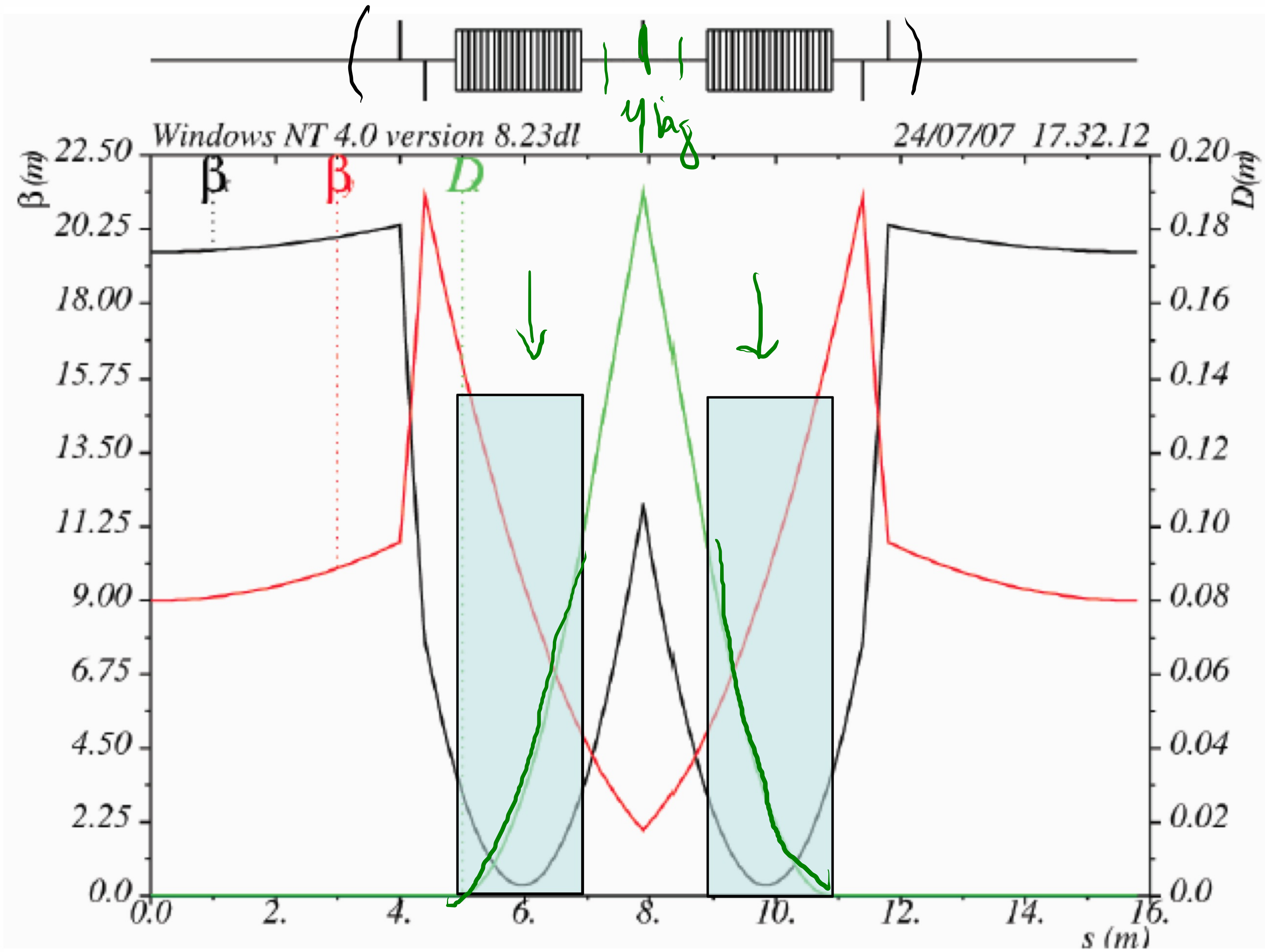
FODO Dipole $\mathcal{H}(s)$



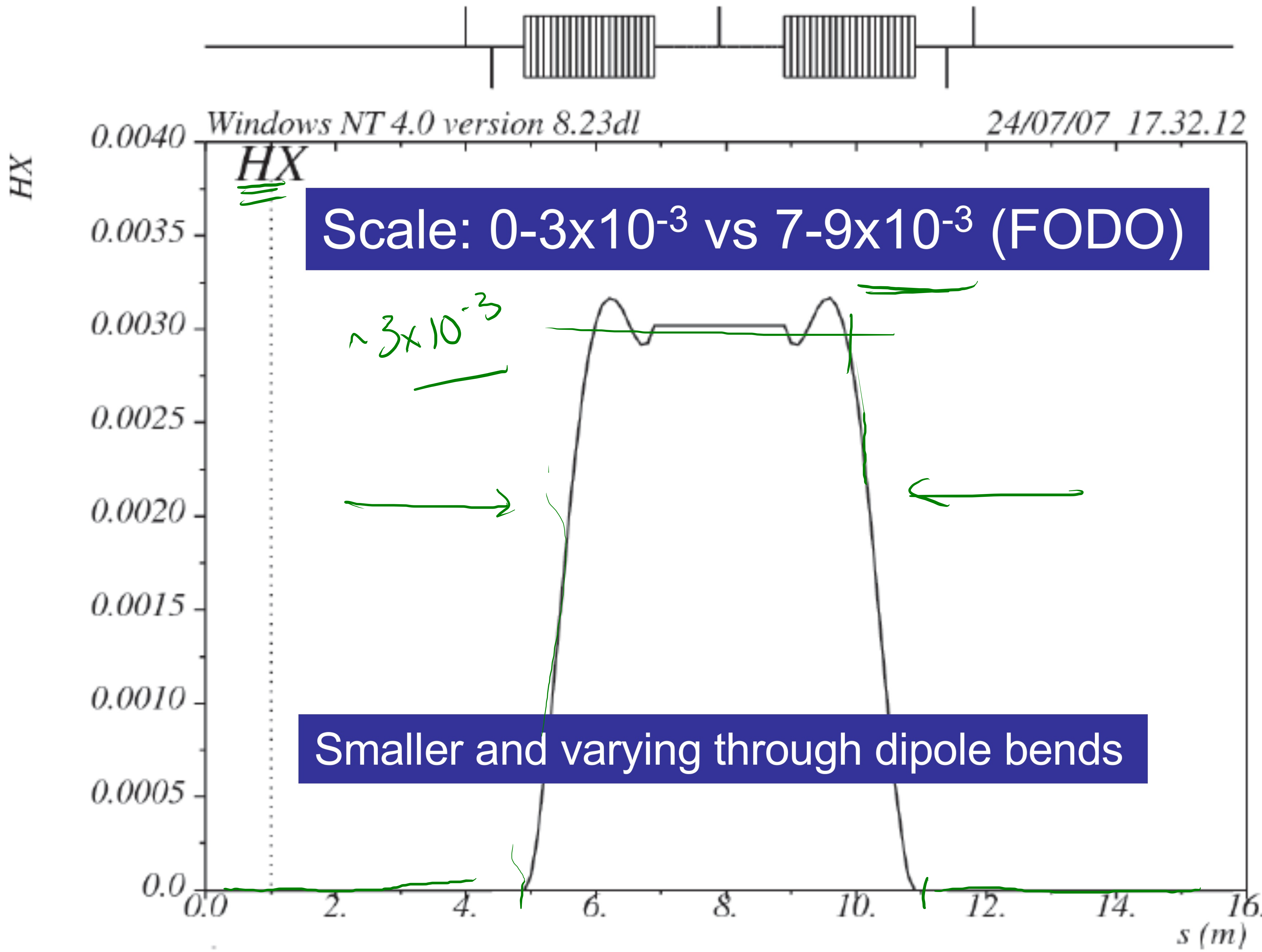
Double Bend Achromats : Chasman-Green

NSLS

- If only we had a lattice that had dipoles that had zero η and η' somewhere near their ends
- We do, the double bend achromat!
  - Add extra focusing at ends for periodic matching with $\alpha_{x,y} \approx 0$



DBA Dipole $\mathcal{H}$



Scale: 0-3x10^{-3} vs 7-9x10^{-3} (FODO)

Smaller and varying through dipole bends

DBA Radiation Integrals

- We can optimize the beta functions and matching vs dipole length to produce a best (minimum) integral of I_5

$$I_{5,\mathrm{min}} = \frac{1}{4\sqrt{15}} \frac{\theta^4}{\rho} + o(\theta^6)$$

dipole ends

$$\beta_x \approx L\sqrt{12/5}$$

$$\alpha_x \approx \sqrt{15}$$

$$I_2 = \int \frac{ds}{\rho^2} = \frac{\theta}{\rho}$$

FODO 1.2 → ~1/16!

$$\epsilon_{0,\mathrm{DBA,min}} = C_q \gamma^2 \frac{I_{5,\mathrm{min}}}{J_x I_2} \approx \frac{1}{4\sqrt{15}} C_q \gamma^2 \theta^3$$

- This is about 13 times smaller (!) than the FODO lattice minimum emittance!

$$\epsilon_0 \approx 1.2\, C_q \gamma^2 \theta^3$$

But We Can Still Do Better

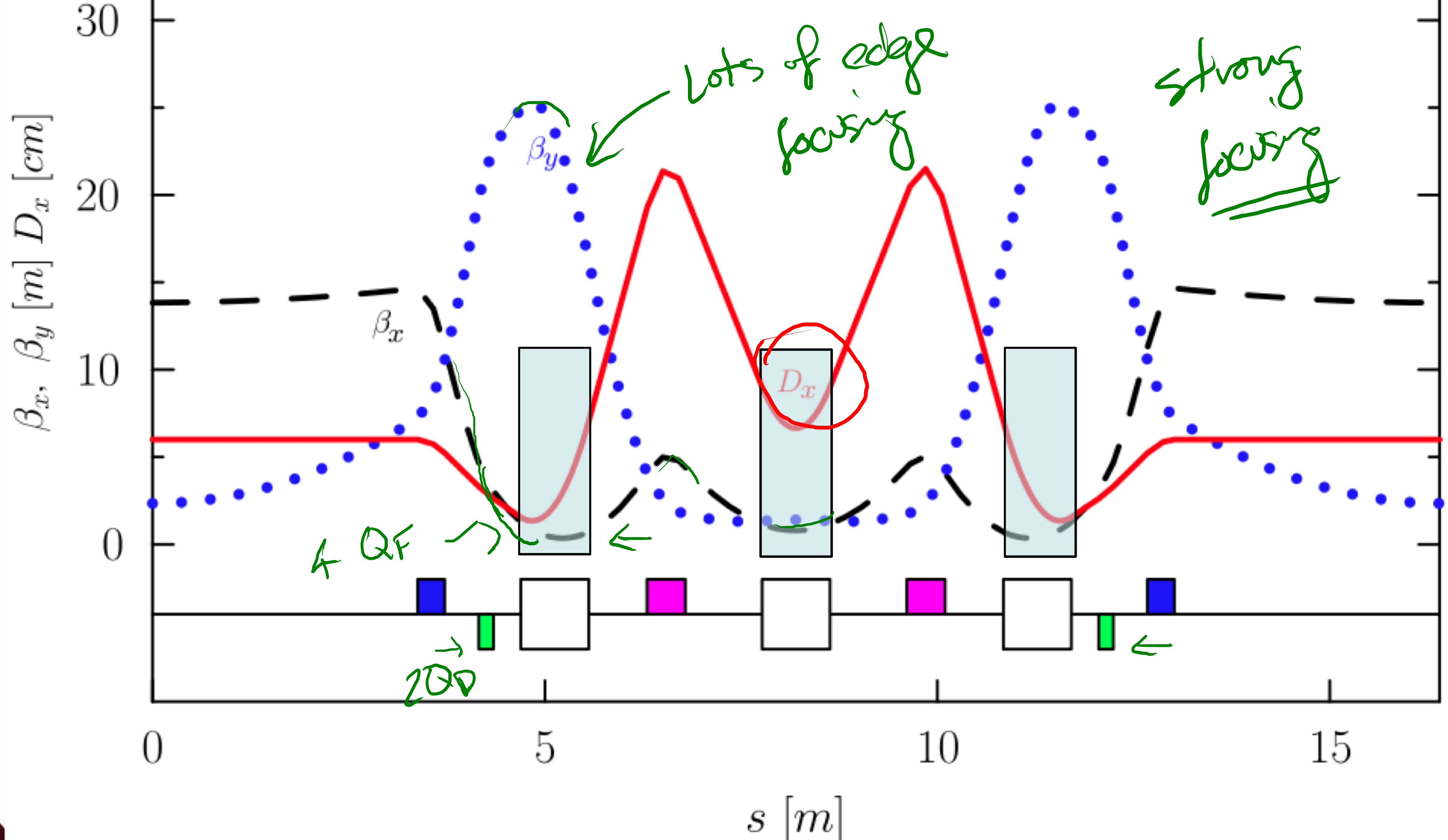
- The double bend achromat was a huge step forward
  - Made NSLS into a very successful light source
  - But we can still further optimize I_5
- One way to do this is the triple bend achromat shown earlier
  - e.g. the ALS, BESSY-II, SLS (Swiss Light Source, PSI)
  - This can place local minima at the dipoles
  - One tradeoff: more complicated lattice, more expensive…
  - More focusing also provides stronger chromatic effects
    - Correction with sextupoles requires nonlinear optimization
- Another solution: minimize I_5 wrt all lattice parameters
  - So-called TME (theoretical minimum emittance) lattices
  - Tend to not be very locally robust solutions
  - But they sure get close to minimizing the natural emittance

vs DBA

Triple Bend Achromat Cell (ALS at LBL)

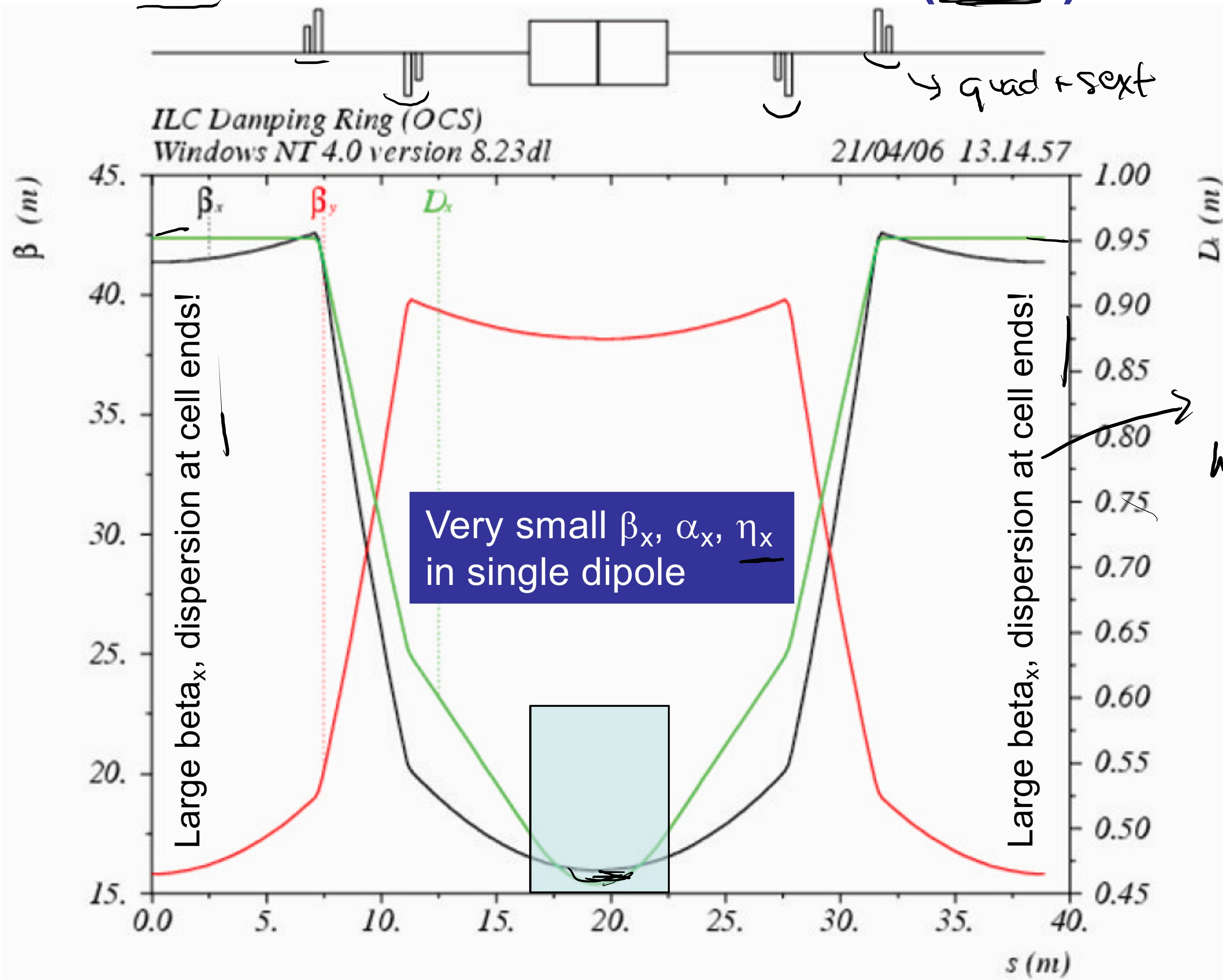
Good reasons to minimize integral of dispersion (this talk)

Extra quadrupoles help with matching at ends of cell, $\alpha_{x,y}=0$



L. Yang et al, Global Optimization of an Accelerator Lattice Using Multiobjective Genetic Algorithms, 2009

“Theoretical Minimum Emittance” (TME) Lattice



→ quad + sext

insertions here η

Summary of Some Minimum Emittance Lattices

Lattice style	Minimum emittance	Conditions/comments
90° FODO	$\varepsilon_0 \approx 2\sqrt{2} C_q \gamma^2 \theta^3$	$\frac{f}{L} = \frac{1}{\sqrt{2}}$
137° FODO	/2.36 $\varepsilon_0 \approx 1.2 C_q \gamma^2 \theta^3$	minimum emittance FODO
DBA	/18.6 $\varepsilon_0 \approx \frac{1}{4\sqrt{15}} C_q \gamma^2 \theta^3$	$\eta_{x,0} = \eta_{px,0} = 0$ $\beta_{x,0} \approx \sqrt{12/5} L \quad \alpha_{x,0} \approx \sqrt{15}$
TME	/3 $\varepsilon_0 \approx \frac{1}{12\sqrt{15}} C_q \gamma^2 \theta^3$	$\eta_{x,\min} \approx \frac{L\theta}{24} \quad \beta_{x,\min} \approx \frac{L}{2\sqrt{15}}$

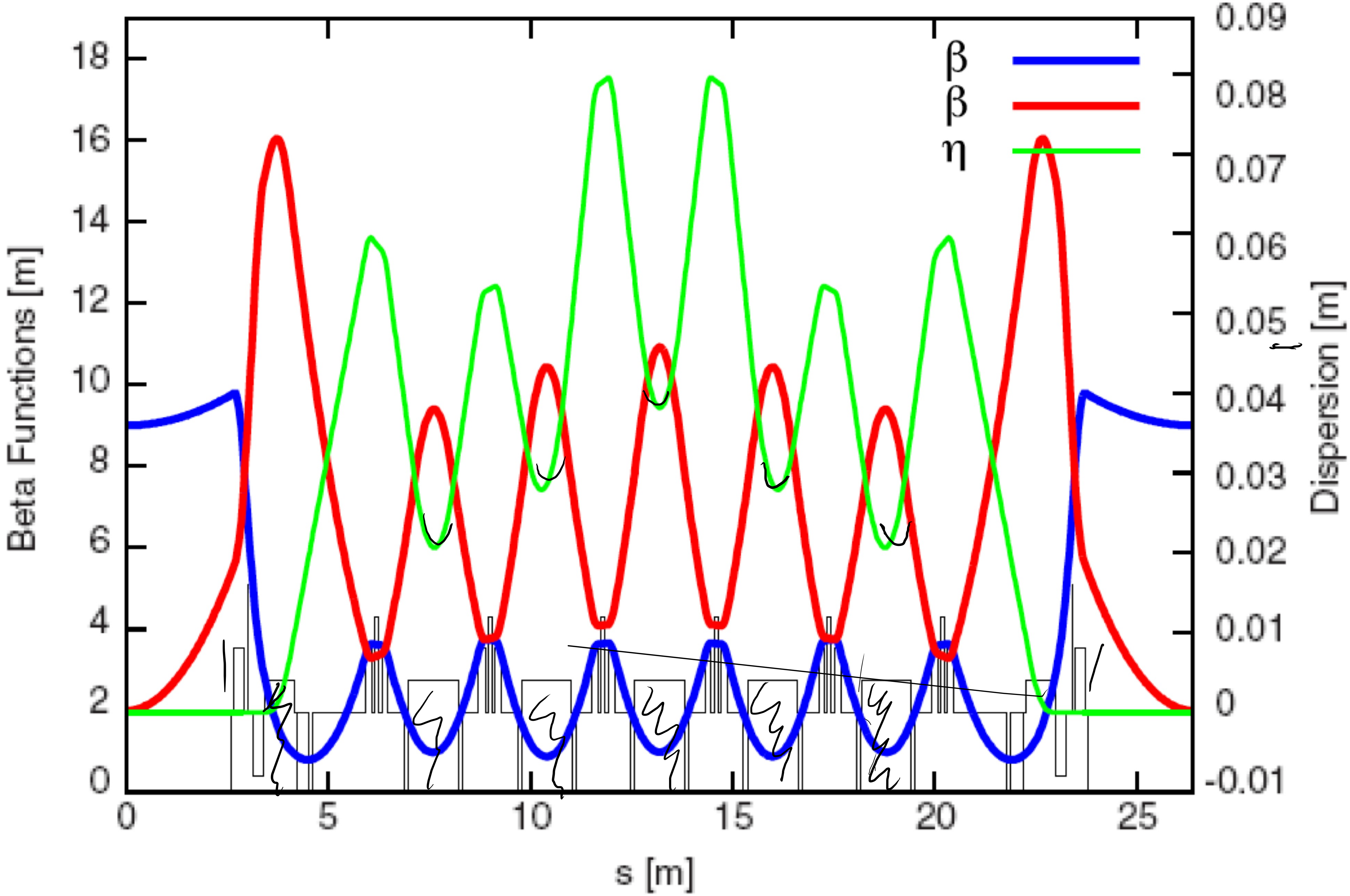
A. Wolski, 2011 CERN Accelerator School Lectures, Greece

Why Not TME All The Time?

- Optimizing one parameter (beam emittance) does not necessarily optimize the facility performance!
  - TME lattices are considered by many to be over-optimized
  - High chromaticities give very sensitive sextupole distributions
    - These in turn give very sensitive nonlinear beam dynamics
    - Momentum aperture, dynamic aperture, …
    - More tomorrow and Thursday

- Usually best to back off TME to work on other optimization
  - Another alternative is to move towards machines with many dipoles
    - Reduces bending angle per dipole and brings emittance down
    - MAX-IV: 7-bend achromat; SPRING-8 6- and 10-bend achromats

MAX-IV 7-Bend Achromat



Operating in Lund, Sweden

MAX-IV Nonlinear Optics

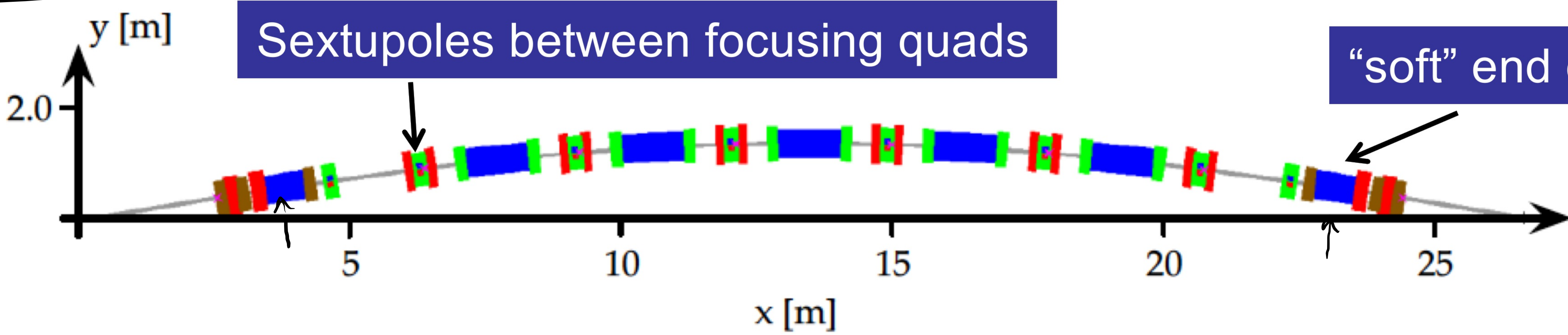


Figure 1: Schematic of one of the 20 achromats of the MAX IV 3 GeV storage ring. Magnets indicated are gradient dipoles (blue), focusing quadrupoles (red), sextupoles (green), and octupoles (brown).

- MAX-IV represents an interesting case in optics design
  - Soft end dipoles minimize synchrotron radiation on SC IDs
  - All dipoles have vertical gradient
  - Strong focusing -> large chromaticities
  - Low dispersion -> very strong chromaticity sextupoles
  - Three sextupole families optimize higher-order chromaticity and driving terms
  - Additional octupoles also correct tune change vs amplitude

S. Leeman, ICFA Beam Dynamics Newsletter 57 (2012)

MAX-IV Parameters

Parameter	***Unit***	***Value***
Energy	GeV	3.0
Main radio frequency	MHz	99.931
Circulating current	mA	$\boxed{500}$
Circumference	m	528
Number of achromats	…	20
Number of long straights available for IDs	…	19
Betatron tunes (H/V)	…	$\boxed{42.20 / 16.28}$
Natural chromaticities (H/V)	…	-50.0 / -50.2 !
Corrected chromaticities (H/V)	…	+1.0 / +1.0
Momentum compaction factor	…	3.07×10^{-4}
Horizontal damping partition	…	1.85
Horizontal emittance (bare lattice)	nm·rad	0.326
Radiation losses per turn (bare lattice)	keV	360.0
Natural energy spread	…	0.077%
Required momentum acceptance	…	$\boxed{4.5\%}$

(vs RHIC $Q_x \simeq 28.19$ $Q_y \simeq 29.18$)

4.5×10^{-2} (vs $\sim 1\times10^{-2}$)

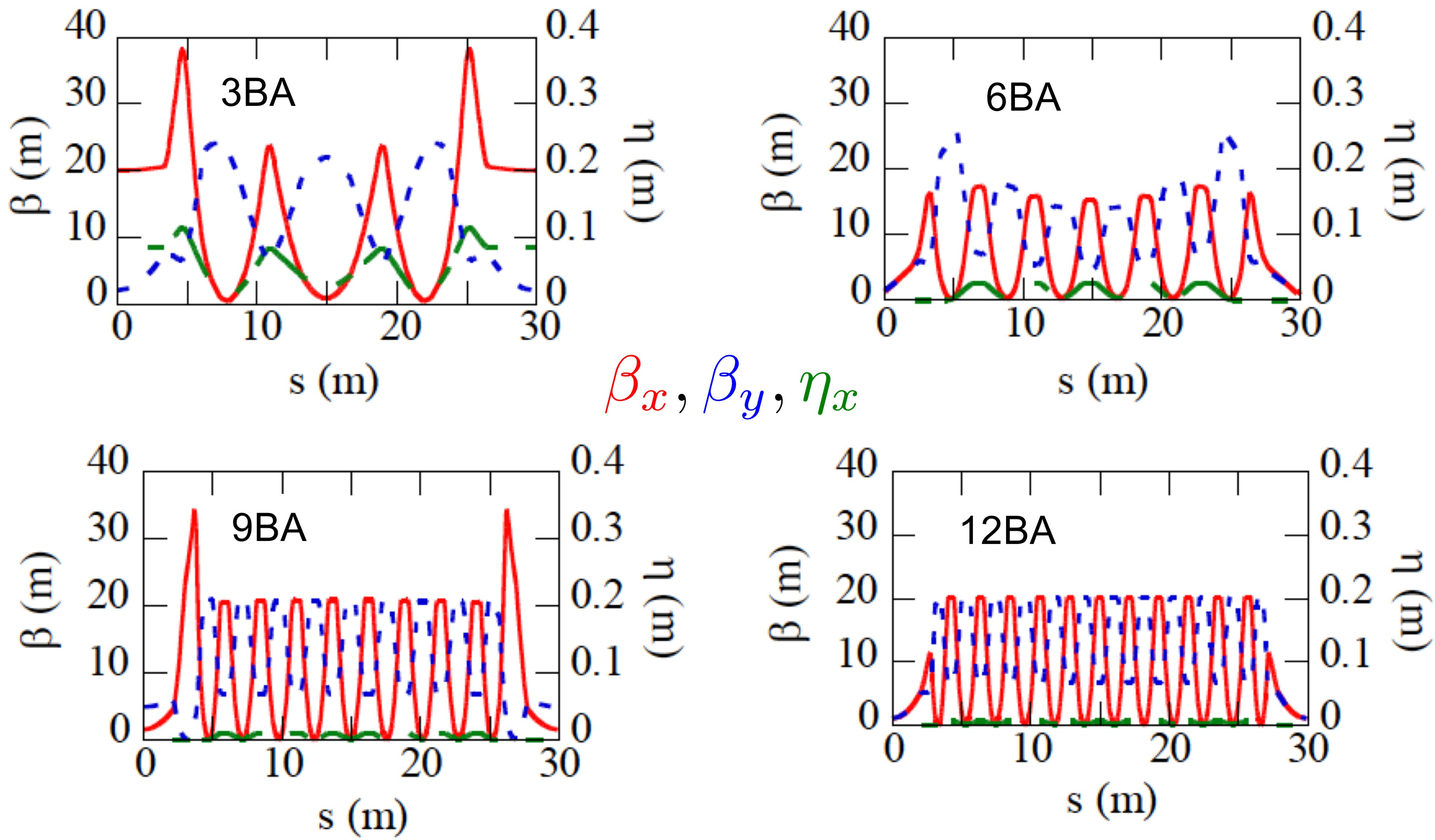
→ Touschek

S. Leeman, ICFA Beam Dynamics Newsletter 57 (2012)

Lattice	Type	E [GeV]	ε_x [nm·rad]	ε^*_x [nm·rad]	J_x	$<\mathcal{H}_x>$ [$\times10^{-3}$]	F_{rel}	ξ_x/ν_x	S
SPring-8	11×DB-4	8	3.4	3.7	1.0	1.42	4.6	2.2	58
ESRF	DB-32	6	3.8		1.0	1.68	3.5	3.6	89
APS	DB-40	7	2.5	3.1	1.0	1.35	3.3	2.5	69
PETRA III	Mod. FODO	6	1		1.0	3.62	39.8	1.2	20
SPEAR3	DB-18	3	11.2		1.0	5.73	7.4	5.5	73
ALS	TB-12	1.9	6.3	6.4	1.0	4.99	10.4	1.7	24
BESSY II	TBA-10	1.9	6.1		1.0	4.83	2.9	2.8	40
SLS	TBA-12	2.4	5		1.0	3.38	2.6	3.2	56
DIAMOND	DB-24	3	2.7		1.0	1.46	4.2	2.9	76
ASP	DB-14	3	7		1.4	5.60	3.0	2.1	28
ALBA	DB-16	3	4.3		1.3	2.96	2.6	2.1	39
SOLEIL	DB-16	2.75	3.7	5.5	1.0	1.79	2.0	2.8	67
CLS	DBA-12	2.9	18.3		1.6	16.79	2.0	1.3	10
ELETTRA	DBA-12	2	7.4		1.3	9.12	1.4	3.0	31
TPS	DB-24	3	1.7		1.0	1.08	2.7	2.9	87
NSLS-II	DBA-30	3	2		1.0	3.78	2.0	3.1	50
MAX-IV	7BA-20	3	0.33		1.9	0.40	18.1	1.2	59
PEP-X (TME)	4×8TME-6	4.5	0.095		1.0	0.34	3.3	1.7	90
PEP-X (USR)	8×7BA-6	4.5	0.029		1.0	0.10	5.3	1.4	145
TeVUSR	30×7BA-6	11	0.0031		2.4	0.02	12.0	1.4	360
TeVUSR	30×7BA-6	9	0.0029		2.7	0.02	18.4	1.4	281

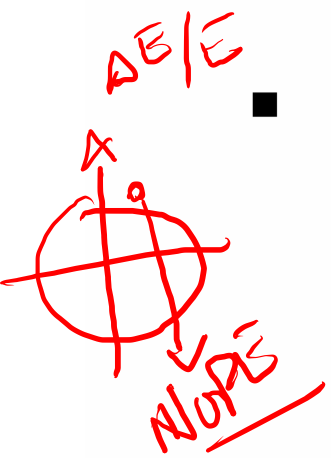
J. Bengtsson, 2012, Nonlinear Dynamics Optimization in Low Emittance Rings, ICFA Beam Dynamics Newsletter 57, April 2012

Multiple Bend Achromats (BAs)



Y. Shimosaki, 2012, Nonlinear Dynamics Optimization in Low Emittance Rings, ICFA Beam Dynamics Newsletter 57, April 2012

Small Emittance Drawbacks: Touschek Scattering

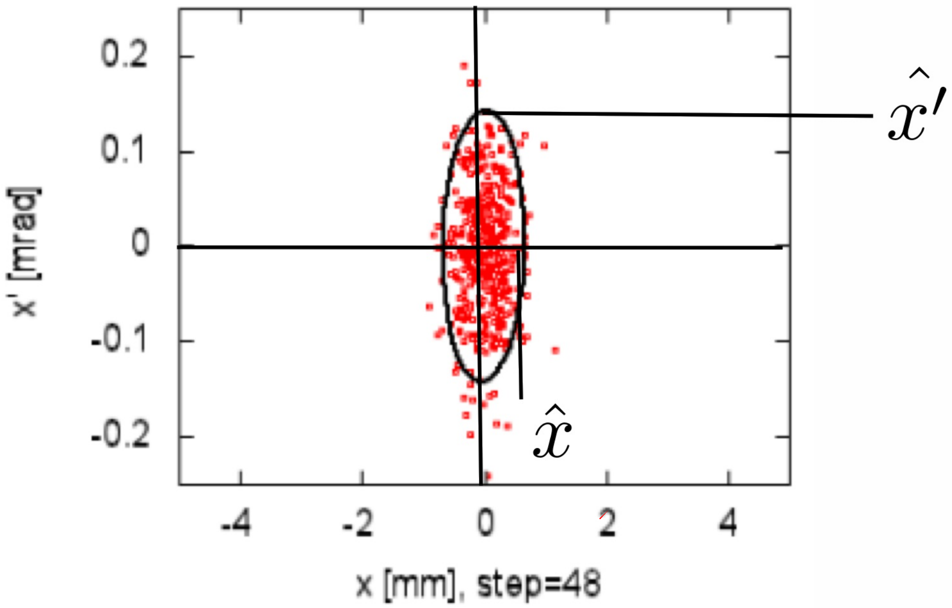


- Electrons within the electron bunches in a synchrotron light storage ring do sometimes interact with each other
  - They're all charged particles, after all

→ scatter, ΔE like SR but bigger

- Fortunately most of these interactions are negligible for high energy, ultrarelativistic electron beams
  - $\gamma \gg 1$ so, e.g., time dilation reduces effect of space charge $\propto \gamma^{-2}$
  - But these are long-distance Coulomb repulsions
  - High angle scattering can lead to sudden large momentum changes for individual electrons
  - Low emittance and high brilliance enhances this effect
    - Tighter distributions of particles => more likelihood of interactions
  - Large momentum changes can move electrons out of the stable RF bucket => particle loss

Rough Order of Magnitude



$$\hat{x} = \sqrt{\mathcal{W}\beta(s)}$$

$$\hat{x}' = \sqrt{\mathcal{W}/\beta(s)}$$

$$\epsilon = \pi\mathcal{W}$$

$$\sigma_{\text{RMS}} = \sqrt{\epsilon\beta}$$

$$\sigma'_{\text{RMS}} = \sqrt{\epsilon/\beta}$$

- For a given particle, $\hat{x} = \beta\hat{x}' = \dfrac{\beta\hat{p_x}}{p_0} \qquad \hat{p_x} = \dfrac{p_0\hat{x}}{\beta}$
- If **all** transverse momentum is transferred into δ then

$$\Delta p = \gamma p_x = \gamma\frac{p_0\hat{x}}{\beta}$$

- For realistic numbers of 2 GeV beam (γ~4000), β_x=10m, and $100\mu m$ beam displacement, we find

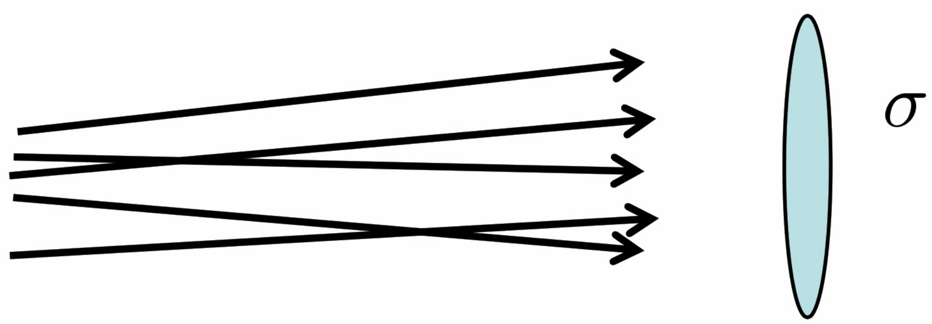
$$\Delta p \approx 80\ \text{MeV/c} \approx 0.04\, p_0$$

$\Leftarrow$ 4% ~ MAX-W

- This scattering mechanism can create electron loss
  - Even worse for particles out in Gaussian tails

Cross Section

- **Cross section** is used in high energy physics to express the probability of scattering processes: units of area



$$1 \text{ barn} \equiv 10^{-28}\text{ m}^2 = 10^{-24}\text{ cm}^2$$

- Often expressed as a **differential cross section**, probably of interaction in a given set of conditions (like interaction angle or momentum transfer): $d\sigma/d\Omega$
- In particle colliders, **luminosity** is defined as the rate of observed interactions of a particular type divided by the cross section $\mathcal{L} \equiv \dfrac{\text{event rate}}{\sigma}$ units $[\text{s}^{-1}\text{ cm}^{-2}]$

Integrating this over time gives an expected number of events in a given time period to calculate experiment statistics

Touschek Scattering Calculations

- Touschek Scattering calculations use the Moller electron elastic interaction cross section in the rest frame of the electrons
  - Then relativistically boost back into the lab frame
  - This is all too involved for this lecture!
    - Really 2nd year graduate level scattering theory calculation
  - See Carlo Bocchetta's talk at CERN Accelerator School
    - http://cas.web.cern.ch/cas/BRUNNEN/Presentations/PDF/Bocchetta/Touschek.pdf
  - As usual we'll just quote the result
  - Touschek loss exponential decay lifetime

$$V_{\text{bunch}} = 8\pi\sigma_x\sigma_y\sigma_z$$

$$\boxed{\tau = \frac{\gamma^3 V_{\text{bunch}} \sigma'_{\text{x,RMS}} \delta^2_{\text{acceptance}}}{c r_0^2 N_{\text{bunch}} (\ln(2)\sqrt{\pi})} \frac{1}{C(\epsilon)}}$$

$$C(\epsilon) \approx -[\ln(1.732\epsilon) + 1.5]$$

$$\epsilon \equiv \left(\frac{\delta_{\text{acceptance}}}{\gamma \sigma'_{\text{x,RMS}}} \right)^2$$

$\delta_{\text{acceptance}}$: $\dfrac{\Delta p}{p_0}$ at which particles are lost

$$r_0 \approx 2.818 \times 10^{-13}\ \text{cm}$$

Touschek Scaling

$$\tau = \frac{\gamma^3 V_{\mathrm{bunch}} \sigma'_{\mathrm{x,RMS}} \delta^2_{\mathrm{acceptance}}}{c r_0^2 N_{\mathrm{bunch}} (\ln(2)\sqrt{\pi})} \frac{1}{C(\epsilon)}$$

$$V_{\mathrm{bunch}} = 8\pi\sigma_x\sigma_y\sigma_z$$

$$C(\epsilon) \approx -[\ln(1.732\epsilon) + 1.5]$$

$$\epsilon \equiv \left(\frac{\delta_{\mathrm{acceptance}}}{\gamma\sigma'_{\mathrm{x,RMS}}}\right)^2$$

$\delta_{\mathrm{acceptance}}$: $\dfrac{\Delta p}{p_0}$ at which particles are lost

$$r_0 \approx 2.818 \times 10^{-13}\ \mathrm{cm}$$

- High lifetime is good, low lifetime is bad
  - Higher particle phase space density $N_{\mathrm{bunch}}/V_{\mathrm{bunch}}$ makes loss faster
    - But we want this for higher brilliance!
  - Smaller momentum acceptance makes loss faster
    - But tighter focusing requires sextupoles to correct chromaticity
    - Sextupoles and other nonlinearities reduce $\delta_{\mathrm{acceptance}}$
  - Higher beam energy γ_r makes loss slower
    - Well at least we win somewhere!

Touschek Lifetime Calculations

- Generally one must do some simulation of Touschek losses

Touschek Lifetime Calculations for NSLS-II

B. Nash, S. Kramer, Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

The Touschek effect limits the lifetime for NSLS-II. The basic mechanism is Coulomb scattering resulting in a longitudinal momentum outside the momentum aperture. The momentum aperture results from a combination of the initial betatron oscillations after the scatter and the non-linear properties determining the resultant stability. We find that higher order multipole errors may reduce the momentum aperture, particularly for scattered particles with energy loss. The resultant drop in Touschek lifetime is minimized, however, due to less scattering in the dispersive regions. We describe these mechanisms, and present calculations for NSLS-II using a realistic lattice model including damping wigglers and engineering tolerances.[1]

INTRODUCTION

LINEAR AND NON-LINEAR DYNAMICS MODELING

NSLS-II has a 15-fold periodic DBA lattice. The lattice functions for NSLS-II are shown in Figure 1. The linear lattice results in the equilibrium beam sizes around the ring that enter into Eqn. (1). Non-linear dynamics enter through the parameter $\delta_{acc}(s)$. This is the maximum momentum change that a scattered particle can endure before it is lost. There are two elements to this stability question. The first is the amplitude of the initial orbit which comes from the off-momentum closed orbit (dispersion) and beta functions. These are shown in Figures 2 and 3. The amplitude of the induced betatron oscillation following a scatter with relative energy change $\delta = \frac{\Delta E}{E_0}$ is given by

$$x_2 = (\eta^{(1)}(s_2) + \sqrt{\mathcal{H}(s_1)\beta_x(s_2)})\delta + \eta^{(2)}(s_2)\delta^2 \quad (3)$$

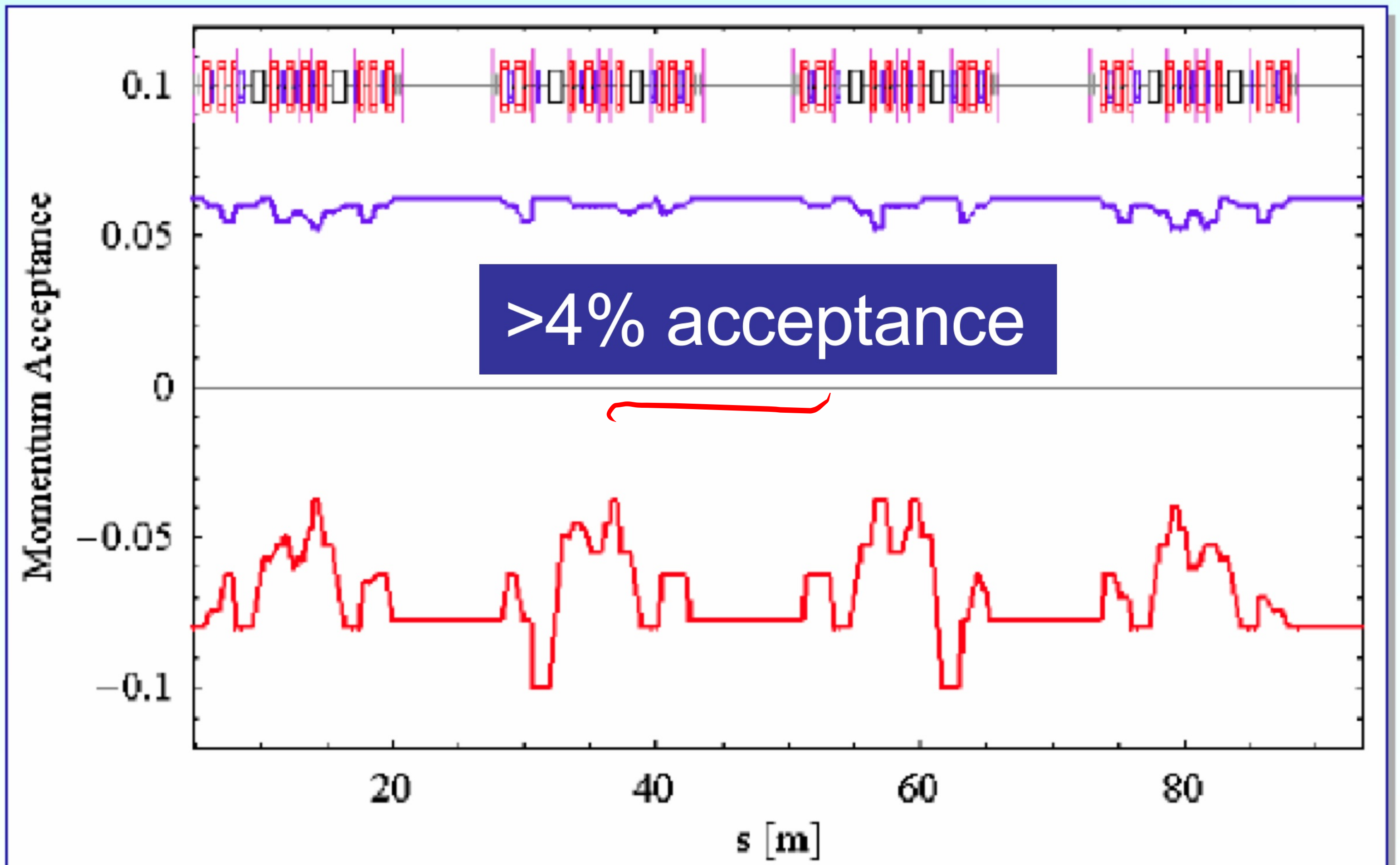
where $\mathcal{H} = \gamma_x\eta_x^2 + 2\alpha_x\eta_x\eta_x' + \beta_x\eta_x'^2$ is the dispersion in-

PAC'09 Conference: http://www.bnl.gov/isd/documents/70446.pdf

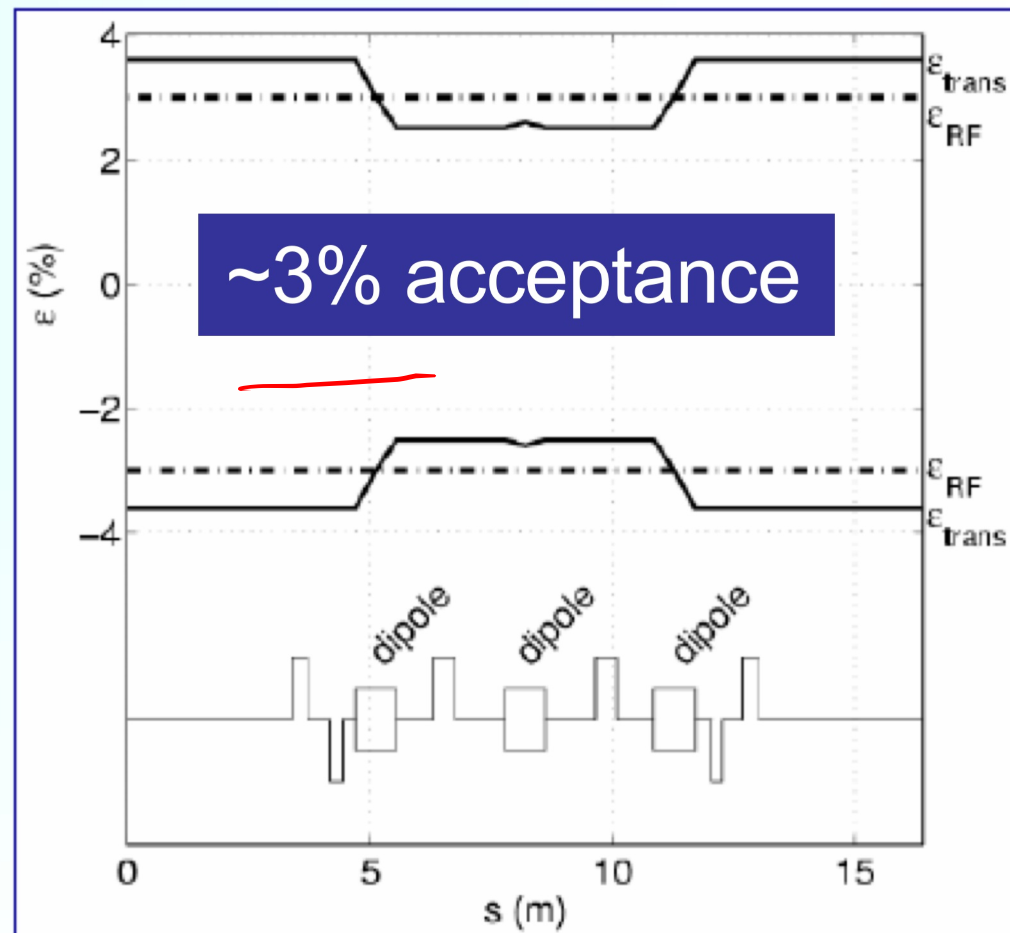
Momentum Aperture and Touschek

- Most third generation storage rings have limiting transverse acceptance
  - Much work to optimize transverse momentum aperture
  - Particularly modern machines (e.g. DIAMOND, SOLEIL)
  - Detailed nonlinear dynamics measurements required

DIAMOND - Transverse

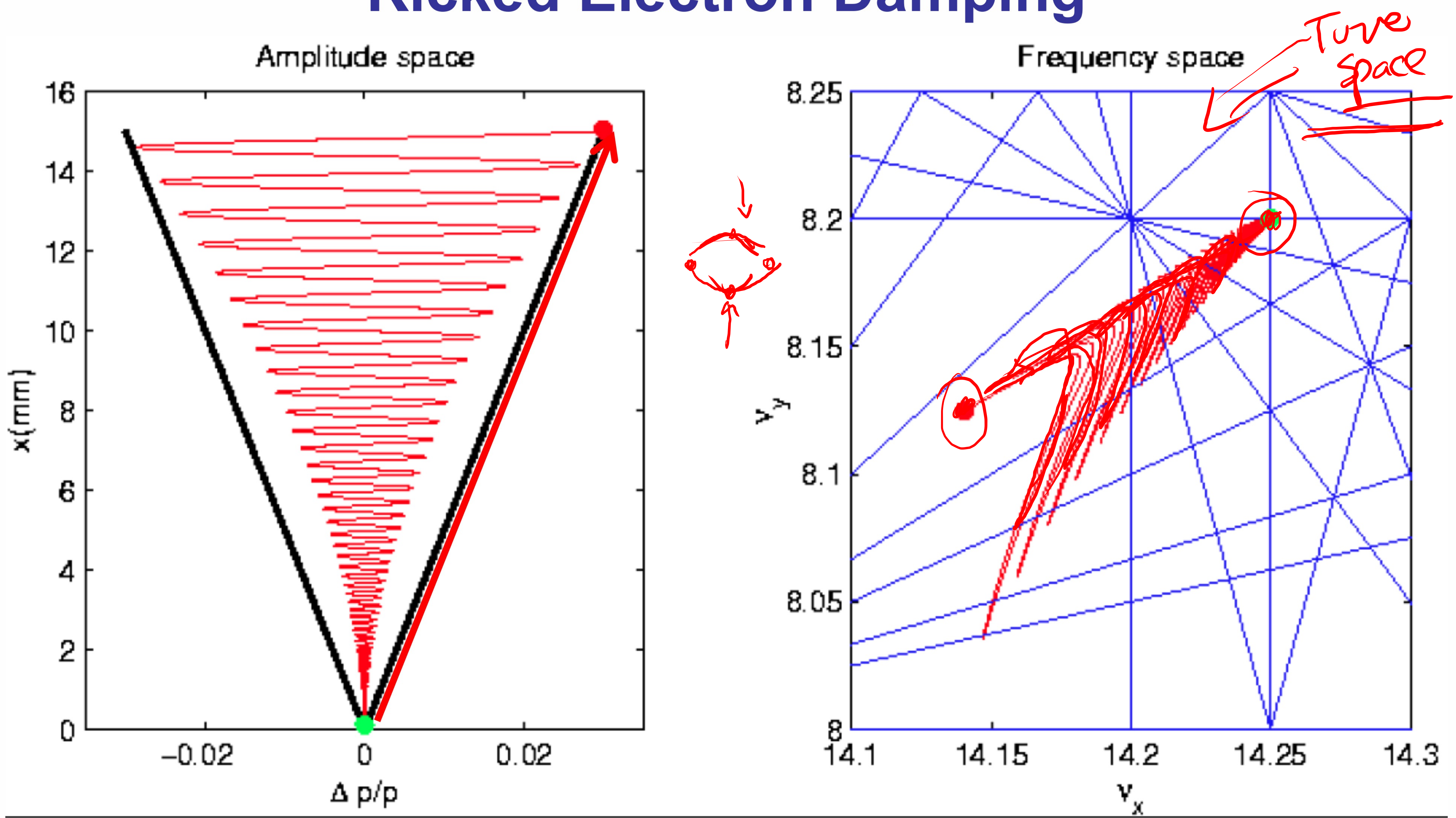


ALS - Transverse & RF



Carlo Bocchetta's talk at CERN Accelerator School

Kicked Electron Damping



- After a Touschek kick, electrons damp again
  - But they move through tunes and amplitudes in complicated way

D. Robin, ALS