Lecture 15

Synchrotron Radiation & Damping

Steve Peggs
February 12, 2021

At GE [in 1947], Pollack got permission to assemble a team to build a 70-MeV electron synchrotron to test the idea [of the phase-stability principle]. Fortunately for the future of synchrotron radiation, the machine was not fully shielded and the coating on the doughnut-shaped electron tube was transparent, which allowed a technician to look around the shielding with a large mirror to check for sparking in the tube. Instead, he saw a bright arc of light, which the GE group quickly realised was actually coming from the electron beam."

A.L.Robinson, ".. History of Synchrotron Radiation".

Lecture 15: Synchrotron Radiation & Damping **Feb 12, 2021**

Introduction: classical E&M, relativity ... but h = 0 !

Radiated power
- local total - 1 particle
- spectrum
- angular distribution - headlight effect
- energy loss-per-turn - 1 particle
  - isomagnetic ring

Longitudinal damping
- continual energy replenishment
- energy loss-per-turn - dependence on delta
- Time out - acceleration without radiation
- Time in - strange attractors

Vertical damping

Partition numbers
- combined or separated function magnets

$\hbar = 0$

- LARMOR (19-th century) knew how charged particles radiated BEFORE ① special relativity ② quantum mechanics
- POLLACK (1947) first saw synch. radiation

- ELECTRON STORAGE RINGS

0) Radiate copiously : protons "not" at all
1) H, V, S motion is damped, stabilized CLASSICAL
2) H, S motion is driven by QUANTUM excitation ($\hbar \neq 0$)
3) Dynamical equilibrium defines "natural" emittances

BUT HERE $\boxed{\hbar = 0}$

LOCAL POWER ("local")

SEE Fig 11.1

Larmor total power $P = \frac{1}{6\pi\epsilon_0} \cdot \frac{q^2 a_c^2}{c^3}$

boosted, becomes $P = \frac{1}{6\pi\epsilon_0} \cdot \frac{q^2 c}{\rho^2} \cdot \beta^4 \gamma^4$

dipole bend radius

more conveniently: $\boxed{P = \frac{1}{6\pi\epsilon_0} \cdot \frac{e^4}{m^4 c^5} \cdot B^2 E^2}$

energy

$B \lesssim 1.0$ T

Protons "don't" radiate because $\left(\frac{m_p}{m_e}\right)^4 \simeq 10^{13}$

Q: WHY are muon colliders attractive?

$m_\mu = 105.7 \frac{\text{MeV}}{c^2}$

SPECTRUM universal shape ($\omega = 2\pi f$)

$$\boxed{\mathcal{P}(\omega) = \frac{P}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)}$$

characteristic frequency

Fig 11.2

$$\boxed{\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}}$$

$\frac{1}{2}$ power radiated $\omega > \omega_c$, $\frac{1}{2}$ with $\omega < \omega_c$

$$\boxed{S(\xi) = \frac{9\sqrt{3}}{8\pi}\,\xi \int_{\xi}^{\infty} K_{5/3}(\bar{\xi})\, d\bar{\xi}}$$

Modified Bessel fn.

WAVELENGTH

$$\boxed{\lambda_c\,[m] = \frac{1.86 \times 10^{-9}}{B[T]\, E^2\,[GeV^2]}}$$

Q: TYPICAL VALUES

$B = 0.5$ T

$E = 3$ GeV

$\lambda_c = 4 \times 10^{-10}$ m $= 4$ Å

ANGULAR DISTBTN.

in plane of synchrotron at angle θ

$$\frac{dP}{d\Omega} = \frac{P_0}{(1-\beta\cos(\theta))^3} \cdot \left[1 - \frac{\sin^2(\theta)}{\gamma^2(1-\beta\cos(\theta))^2}\right]$$

RELATIVISTIC LIMIT $\gamma \gg 1$

$$\boxed{\frac{dP}{d\Omega} \approx \frac{P_1}{(1+\gamma^2\theta^2)^3} \cdot \left[1 - \frac{4\gamma^2\theta^2}{(1+\gamma^2\theta^2)^2}\right]}$$

fn. of $(\gamma\theta)$

HEADLIGHT EFFECT Fig 11.3

TYPICAL: $\gamma = 6{,}000 \rightarrow \pm 0.1$ mrad

ENERGY LOSS PER TURN

1 particle, 1 turn: $U = \oint P(s) \frac{ds}{c}$

ideally:
$$\boxed{U_0 = C_\gamma E_0^4 \cdot \frac{C}{2\pi} \langle G^2 \rangle}$$

circumference

$G = \frac{1}{\rho}$

where angle brackets $\langle \rangle$ denote a ring average

and constant
$$\boxed{C_\gamma = \frac{4\pi}{3} \frac{r_0}{(mc^2)^3}}$$

classical radius

$= 8.85 \times 10^{-5}$ $[\text{m GeV}^{-3}]$ ELECTRONS

$= 7.78 \times 10^{-18}$ $[\text{m GeV}^{-3}]$ PROTONS

MUONS?

E.G. ISOMAGNETIC RING : All dipoles have the same radius ρ

$$\boxed{\langle G^n \rangle = \frac{\oint G^n ds}{\oint ds} = \frac{1}{R \rho^{n-1}}}$$

where

$$C = 2\pi R$$

and

$$U_0 = \frac{C_\gamma E_0^4}{\rho}$$

ELECTRONS

$$\boxed{U_0 [\mathrm{keV}] = 88.5 \frac{E_0^4 [\mathrm{GeV}^4]}{\rho\,[\mathrm{m}]}}$$

EG 1: $B = 0.4$ T, $E = 3$ GeV $\rightarrow U_0 = 0.29$ MeV/turn NSLS-II injector

2: 50 TeV protons, $\rho = 14$ km $\rightarrow U_0 = 3.5$ MeV/turn FCC/VLHC

LONGITUDINAL DAMPING

Previously: SLIP FACTOR $\eta_s = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$

Now ($\gamma \gg 1$) use COMPACTION FACTOR

$$\boxed{\alpha = \langle \eta G \rangle} = \frac{1}{\gamma_T^2}$$

Longitudinal displacement evolves turn-to-turn like

$$\boxed{z_{n+1} = z_n - \alpha C_0 \delta_n} \quad A$$

On every turn must replenish (average) energy loss:

$$\boxed{U_0 = eV\sin(\phi_s)}$$

TOTAL RF voltage

SYNCHRONOUS PHASE

FIG 11-4

TBT evolution of δ

B

$$\boxed{\delta_{n+1} = \delta_n + \frac{1}{E_\gamma}\left[eV\left(\sin(\phi_n) - \sin(\phi_s)\right) - \left(U(\delta_n) - U_0\right)\right]}$$

$$\left(\gamma \gg 1 \Rightarrow \delta = \frac{\Delta p}{p} \approx \frac{\Delta E}{E_0}\right)$$

Variation of U with δ is VITAL!!

If longitudinal motion is slow (Q_s is small)

THEN combine A + B to give

$$\boxed{\frac{d^2\delta}{dt^2} + \frac{2}{\tau_s}\frac{d\delta}{dt} + \left(\frac{2\pi Q_s}{T}\right)^2 \delta = 0}$$

t is time (seconds)

$T = \frac{C}{c}$ is period (seconds)

SOLUTION

$$\boxed{\delta = \delta_0 \, e^{-\frac{t}{\tau_s}} \cdot \cos\left(2\pi Q_s \frac{t}{T}\right)}$$

with damping time

Fig 11.5

$$\boxed{\tau_s = \frac{2T}{\left.\frac{dU}{dE}\right|_0}}$$

slope matters!!

"zero" for protons!!

VARIATION OF U with δ?

$$U(\delta) \sim \oint P(\delta).ds \sim \oint B^2 E^2 . ds$$

where

$$B(\delta) \sim G + K(\eta\delta + X_{co})$$

from misalignment errors

$$E(\delta) \simeq E_0(1+\delta)$$

So USUALLY $\left.\frac{dU}{dE}\right|_0$ is +ve, motion is damped!!

TIME OUT - Brief return to CONSERVATIVE MOTION

$U(\delta) = 0$

$\phi_s \neq 0 \Rightarrow$ ~~Particles are accelerating~~ !!

Hamiltonian

$$\boxed{H(\phi, \delta) = \frac{1}{2}\alpha\omega_{RF}\delta^2 - \left(\frac{eV}{TE_0}\right)\left(\cos(\phi) + \phi\sin(\phi_s)\right)}$$

REPRODUCES (A) + (B) (with $U=0$) through canonical eqns.

$$\frac{d\delta}{dt} = \frac{\partial H}{\partial \phi}, \quad \frac{d\phi}{dt} = -\frac{\partial H}{\partial \delta}$$

Fig 11.7

Fig 11.6

VISUALIZE THE MOTION:

1. Particles follow contours of H
2. Speed is proportional to steepness!!

VERTICAL DAMPING

V is easier – dispersion is zero!!

- Consider a particle losing ΔU energy in a single dipole
  ... and (in effect) recovering ΔU from RF in that dipole (!)

Fig 11.8

$$\underline{y'_{NEW} = \left(1 - \frac{\Delta U}{E_0}\right)\frac{P_\perp}{P_0} = \left(1 - \frac{\Delta U}{E_0}\right) y'_{OLD}}$$

V. OSCILLATIONS THEREFORE DAMP

$$\boxed{y = a_y \cdot e^{-\frac{t}{\tau_y}} \cdot \cos\left(2\pi Q_y \cdot \frac{t}{T}\right)}$$

damping time $\boxed{\tau_y = 2T\frac{E_0}{U_0}}$ $= \tau_0$

PARTITION NUMBERS

The H, V, + S damping times are connected (e.g. horizontal dispersion) represented by partition numbers J_x, J_y, J_s

$$\boxed{\tau_{x,y,s} = \frac{\tau_0}{J_{x,y,s}}}$$

CHARACTERISTIC TIME

"it can be shown"

$$\boxed{J_x + J_y + J_s = 4}$$

$J_y = 1$ in a flat machine

E.G. NSLS-II injector, $E_0 = 3$ GeV, $U_0 = 0.29$ MeV / turn

$\Rightarrow \boxed{\tau_0 \simeq 2{,}000}$ turns

3-D STABILITY Both J_x + J_s must be positive

$$J_x = 1 - \mathcal{D}$$

$$J_s = 2 + \mathcal{D}$$

DIPOLE STRENGTH QUAD STRENGTH

where $\boxed{\mathcal{D} = \dfrac{\langle \eta G^3 \rangle + 2\langle \eta G.K \rangle}{\langle G^2 \rangle}}$

EG: SEPARATED FUNCTION : $GK = 0$

$$\mathcal{D} \sim \frac{\eta}{\rho} \sim \frac{2}{100} \ll 1, \quad J_x \simeq 1, \quad J_s \simeq 2$$

3D stability "guaranteed". (Beware errors in large machines)

EG COMBINED FUNCTION

D can easily be of order 1

DOESN'T MATTER if

① No radiation in hadron rings e.g. CERN PS, AGS

② if electron storage time is short

e.g. CESR injector $\tau_{STORE} \lesssim \tau_{x,s}$

Q: WHAT prevents electron sizes going to zero ??

$\boxed{\hbar \neq 0}$!! Quantum excitation

Chapter 12