Lecture 2A:
Linear Motion & Stability

Steve Peggs
January 25, 2021

210125

LECTURE 2A: LINEAR MOTION & STABILITY

ANALYTIC TOOLS (pre-computer) make dynamic problems look like DIFFERENTIAL equs.

NUMERICAL TOOLS make problems look like like DIFFERENCE equns

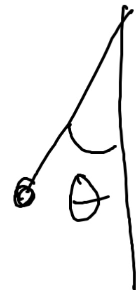
Q1: Which is right?

Q2: IS TIME CONTINUOUS?

A: IT DEPENDS !!

EXAMPLE: PENDULUM ...

PENDULUM



$$\theta'' = -g \cdot \sin(\theta) \quad \left[= \frac{d^2\theta}{dt^2}\right]$$

HOW TO SIMULATE ?

until finished {

$$\theta = \theta + \theta' \cdot \Delta t$$

$$\theta' = \theta' - g \cdot \sin(\theta) \cdot \Delta t \longrightarrow = 0 \,?$$

large)

}

This is more like an accelerator

Circular accelerators are inherently DISCRETE

: gravity pulsed on once per turn:

$$\theta'' = -\sum_{n=1}^{\infty} \delta(t - n\Delta t) \cdot g \cdot \sin(\theta) \cdot \Delta t$$

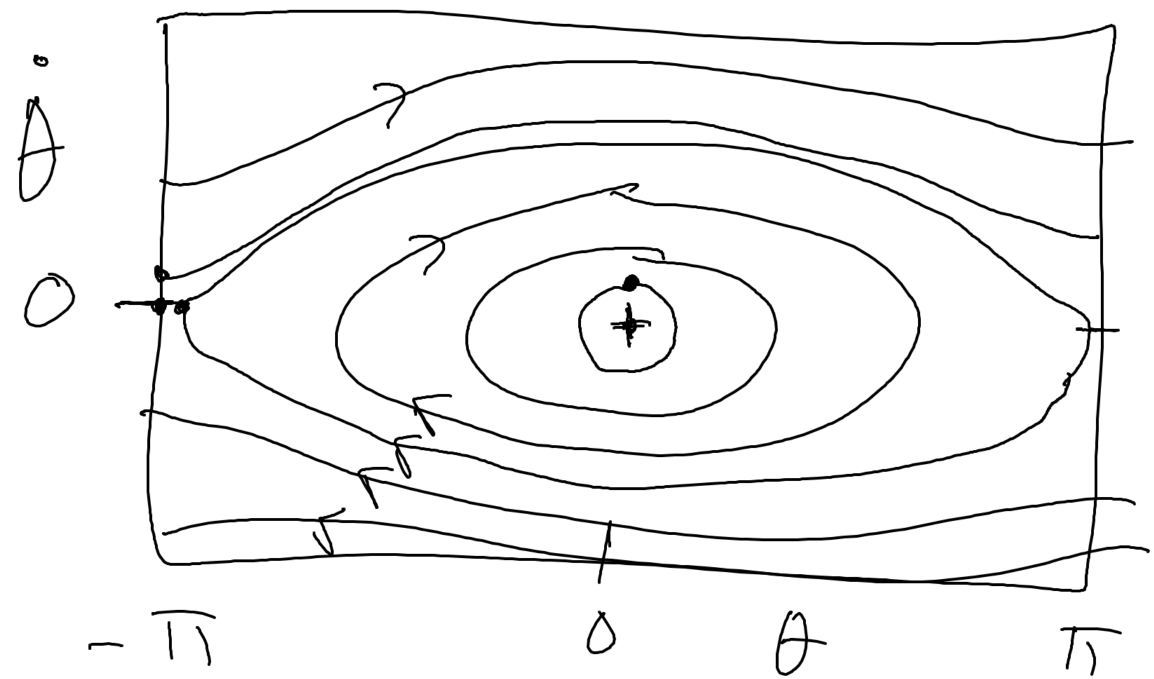
Why bother to force into differential form?

DIFFERENCE MAPS ARE LEGIT

WELL-MATCHED TO ACCELS.

$\boxed{\text{PHILOSOPHY OF BOOK}}$

GRAVITY PENDULUM



CH 2 : LINEAR MOTION & STABILITY

DESIGN & CLOSED ORBITS

DESIGN

CLOSED ORBIT

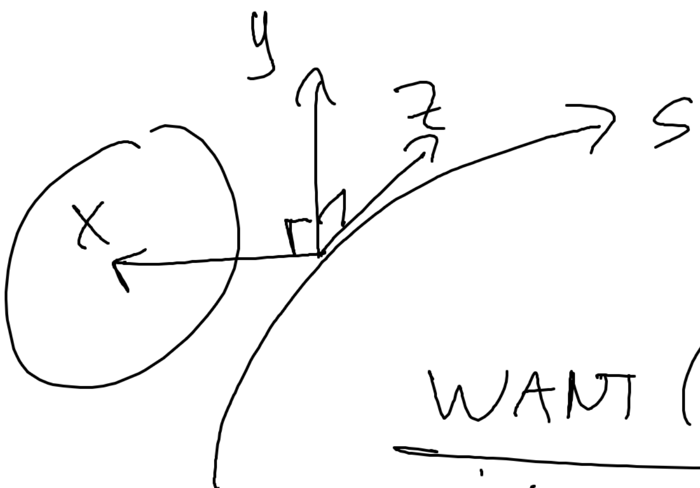
It can be shown that if fields are static

$$\frac{dB}{dt} = \frac{dE}{dt} = 0$$

then there is 1 orbit that repeats itself $\rightarrow$ CLOSED ORBIT

MEASURE TEST PARTICLE OFFSETS RELATIVE TO C. ORBIT

BETATRON OSCILLATIONS



RIGHT HANDED CO-ORD SYSTEM

(x, y, z) rotates, tangential to orbit at that location.

WANT (LINEAR) STABILITY for small oscillation about closed orbit:

$$\boxed{x(n) = a \cdot \cos(2\pi \cdot Q_x \cdot n + \phi_0)}$$

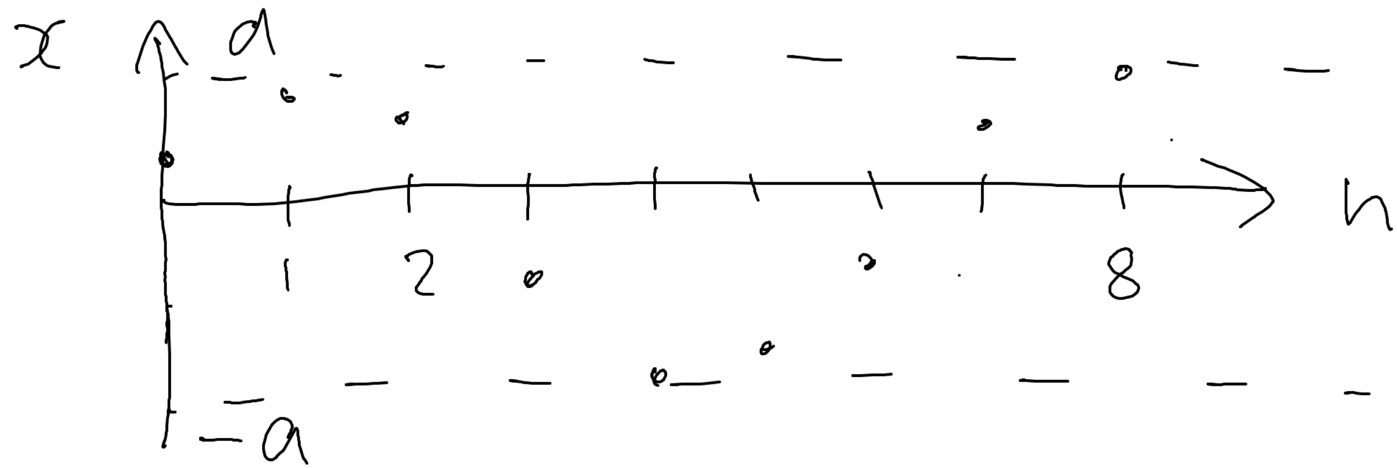
amplitude ~1 mm

BETATRON TUNE

integer turn number

initial phase

turn number $0 < n < \infty$



$Q_x \gg 1$

ALSO WANT LONG STABILITY (Relative to bunch center)

$$\boxed{z_n = a_z \cos(2\pi Q_s \cdot n)}$$

SYNCHROTRON TUNE ~ 0.003 ?

$0 < Q_s \ll 1$

electrons: ~ 1 mm

hadrons: ~ 100 mm

MOTION THROUGH A MAGNET

Much longer than its bore : 2-D fields

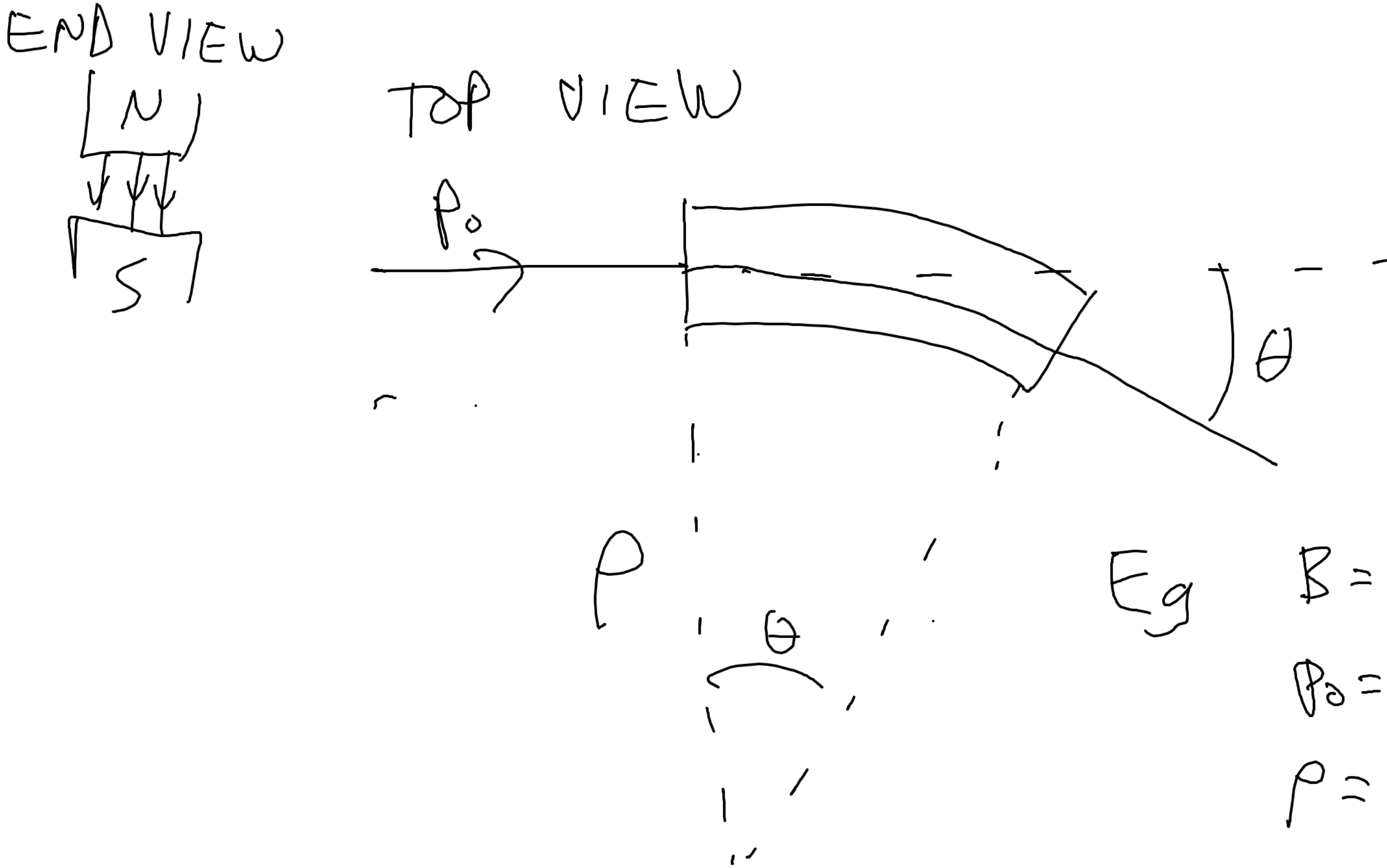
$$B_x = B_x(x,y) \;,\; B_y = B_y(x,y) \;,\; B_s = 0$$

Maxwell's 3-D equs. become 2-D

$$\nabla \cdot \bar{B} = 0 \;,\quad \nabla \times \bar{B} = \mu_0 \bar{J} = 0$$

$$\boxed{\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0} \;,\quad \boxed{\frac{\partial B_x}{\partial B_y} - \frac{\partial B_y}{\partial x} = 0}$$

SIMPLEST SOLN: $B_y = \text{constant}$, $B_x = 0$ $\boxed{\text{DIPOLE}}$



Eg $B = 1$ T

$p_0 = 3.4$ GeV/c

$\rho = 10$ m

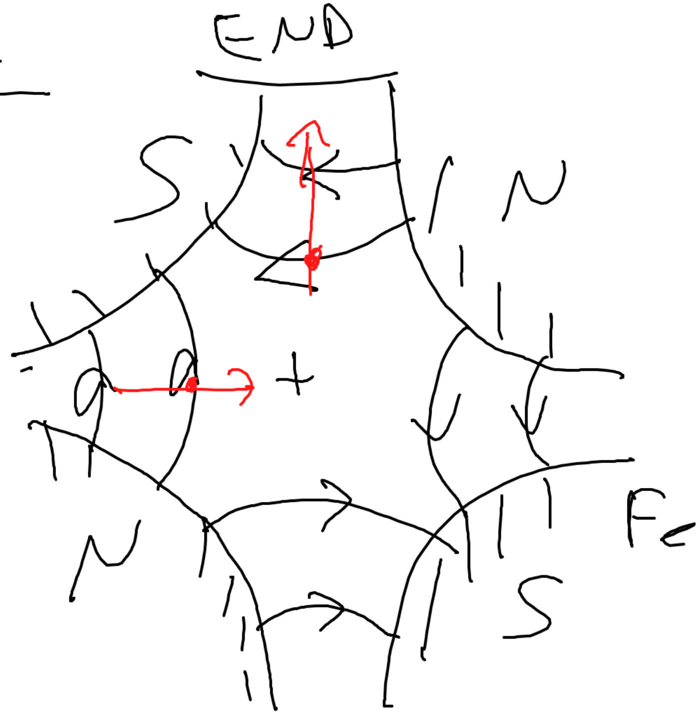
RIGIDITY $(B\rho)\,[\mathrm{Tm}] = 3.34\, p_0 \left[\frac{\mathrm{GeV}}{c}\right]$

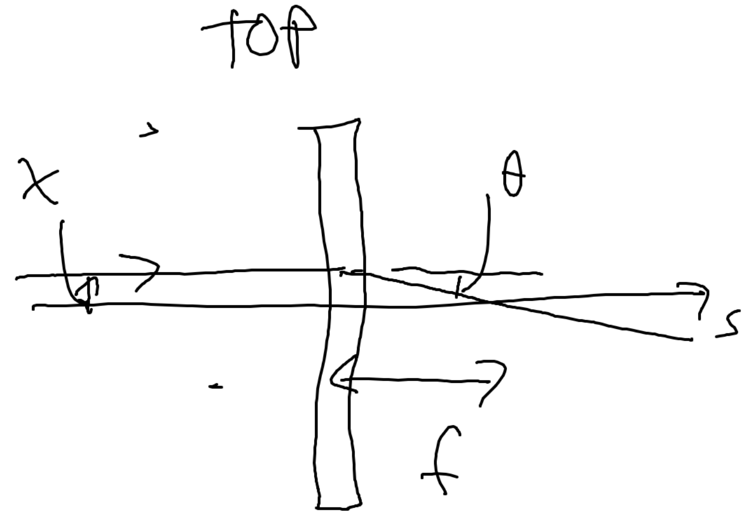
QUADRUPOLE

$B_x = B' y$

$B_y = B' x$

CONSTANT





$$\boxed{\theta = \frac{q B_y}{p_0} L = \frac{B'}{(B\rho)} L x}$$

A QUAD THAT FOCUSES IN x DE FOCUSES IN y.

WHAT TO DO??