Lecture 7

Radio Frequency Cavities

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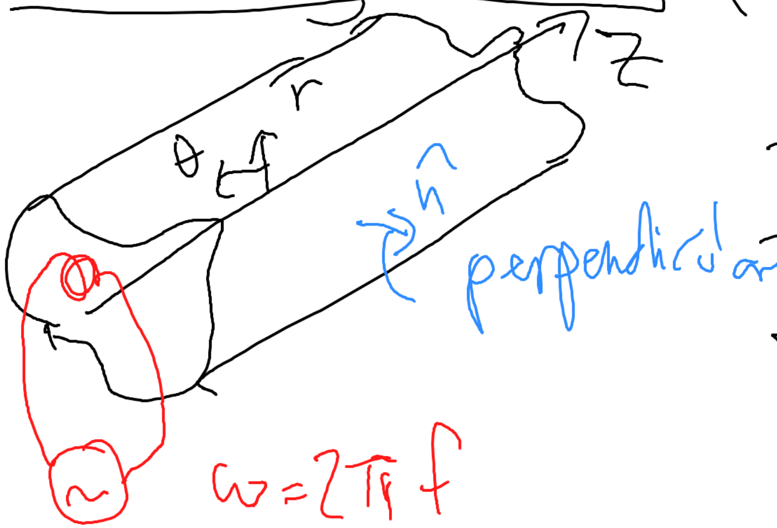
A: Waveguides
B: Transverse modes
C: Pill boxes
D: Performance limits
- Transit time factor
- Kilpatrick criterion
- Superconducing cavities (1 slide)

A: Waveguides



Consider a waveguide

- open ends
- constant cross-section
- perfectly conducting walls

Transmitting (?) fields oscillating at $\omega = 2\pi f$

Boundary conditions at walls

$$\hat{n} \times \bar{E} = 0$$ no parallel E-field

$$\hat{n} \cdot \bar{B} = 0$$ no perpendicular B-field

Real parts of complex $\bar{E}$ + $\bar{B}$ are physical

Consider a mode labeled by WAVENUMBER k

(A)
$$\boxed{\begin{aligned} \bar{E} &= \bar{E}(r,\theta)\, e^{i(kz-\omega t)} \\ \bar{B} &= \bar{B}(r,\theta)\, e^{i(kz-\omega t)} \end{aligned}}$$

where phase velocity in z-direction is $V = \dfrac{\omega}{k}$

If k is complex, the wave is damped NO PROPAGATION

Q1: HOW does k vary with ω ??

Q2: WHAT happens when walls are added at waveguide ends? CAVITY

B: TRANSVERSE MODES

3 categories of modes solve (A)

1 TRANSVERSE MAGNETIC (TM)

$B_z = 0$ everywhere, with $E_z = 0$ @ walls

2 TRANSVERSE ELECTRIC (TE)

$E_z = 0$ everywhere, with $\frac{\partial B_z}{\partial n} = 0$ @ walls

TM modes are most useful. E_z accelerates, confines

~~Crab~~ Crab cavity? Exception

3 TRANSVERSE ELECTROMAGNETIC (TEM)

No longitudinal fields:

FREE SPACE WAVES

$$\boxed{k = \sqrt{\mu_R \epsilon_R}\,\frac{\omega}{c}}$$

speed of light.

From here on $\mu_R = \epsilon_R = 1$ VACUUM

TEM modes have $\lambda = \frac{2\pi}{k}$ MUCH smaller than waveguide width.

SOLVE FOR TE & TM MODES

A particular cross-section geometry has a family of modes, each with a CUTOFF-FREQUENCIES ω_n with $n = 0, 1 \ldots \infty$

where $$\boxed{k_n = \frac{1}{c}\sqrt{\omega^2 - \omega_n^2}}$$

① Clearly, mode n does not propagate if $\omega < \omega_n$

② $\pm k$ modes propagate forwards or backwards

Many modes can co-exist if ω is large enough.

- It is advantageous if:

  1) transmit at $\omega_0 < \omega < \omega_1$. ONE MODE

  2) use a geometry s.t. $\omega_0 \ll \omega_1$ DYNAMIC RANGE

- The beam ITSELF can drive many unwanted modes HIGHER ORDER MODES (HOM) which sometimes need explicit damping .....

$$\boxed{\omega_{max} = 2\pi \frac{c}{\sigma_z}}$$

RMS bunch length

$$\lambda = \frac{c}{f}$$

C: PILLBOXES — CYLINDRICAL RESONANT CAVITIES

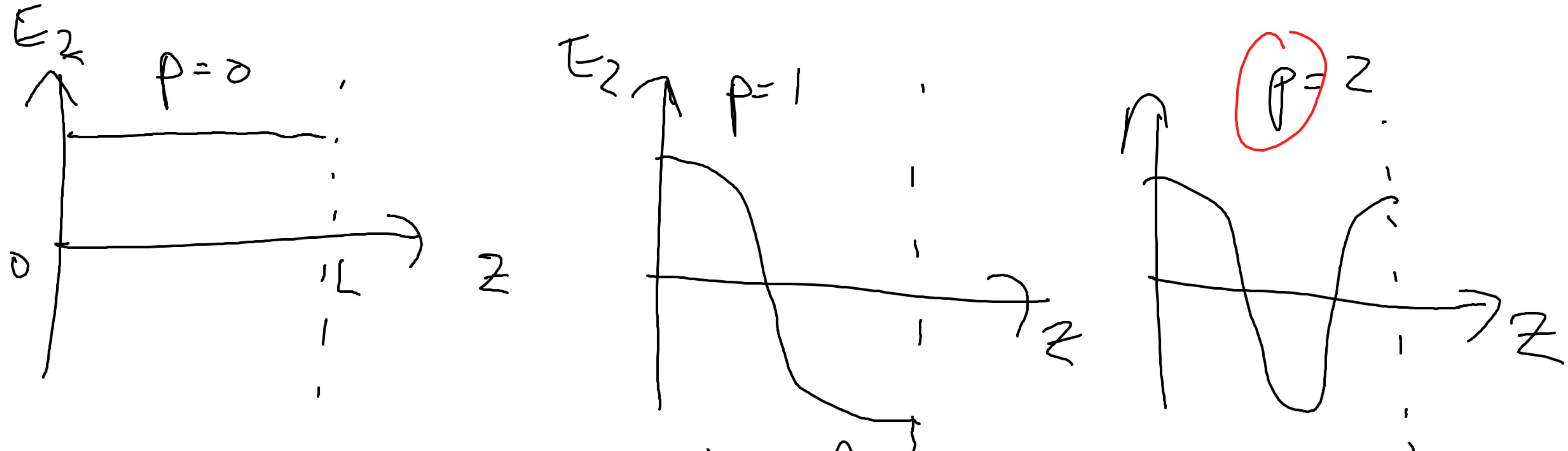
Add flat ends at $z = 0$ & $z = L$

Take a pair of waveguide modes with

$$k = \pm k_p = \pm p \frac{\pi}{L}, \quad p = 0, 1 \ldots \infty$$

add together ($\sim e^{\pm ikz}$) to get a resonant TM mode

$$\boxed{E_z = \psi(r,\theta) \cdot \cos\left(p\pi \frac{z}{L}\right) \cdot e^{i\omega_{RES} t}}$$



NEXT, SOLVE $\psi(r,\theta)$ for a circle

This adds 2 more integer indices m, n where the resonant frequency is

$$\boxed{\omega_{mnp,RES} = c\sqrt{\left(\frac{u_{mn}}{R}\right)^2 + \left(\frac{p\pi}{L}\right)^2}}$$

B

where $\upsilon_{mn} \geq 1$ is n'th root of Bessel function J_m

$$\boxed{J_m(\upsilon_{mn}) = 0}$$

- Lowest resonant frequency ω_{mnp} when $p=0$
- Here, simplify further with $m=0$ (no azimuthal structure)!

. . . .

TM_{0n0} mode has ONLY 2 non-zero field components:

$$\boxed{\begin{aligned} E_z &= E_0 * J_0\left(u_{0n}\frac{r}{R}\right)e^{-i\omega_{0n0}t} \\ B_\theta &= -B_0\frac{\omega_{0n0}R}{u_{0n}c} * J_1\left(u_{0n}\frac{r}{R}\right)\cdot e^{-i\omega_{0n0}t} \end{aligned}}$$

See Figure 7.4 for TM_{020} .....

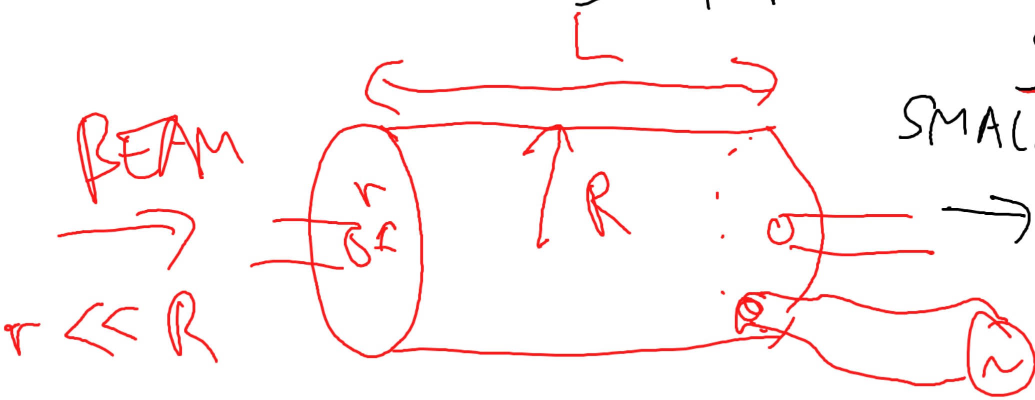
"n" counts the E_z zero-crossings from $r=0$ to R

B_ϕ is non-zero at $r = R$

$$\boxed{\omega_{0n0} = \frac{u_{0n}\, c}{R}}$$

$$\begin{pmatrix} u_{01} \\ u_{02} \\ u_{03} \end{pmatrix} = \begin{pmatrix} 2.41 \\ 5.52 \\ 8.65 \end{pmatrix}$$

ADD end holes for beam



SMALL HOLES HAVE SMALL CUT-OFF FREQS.

→ usually no propagation outside, little perturbation inside

$\omega_{DRIVE} \approx \omega_{0n0,RES}$

- Field "leaks" a little way only into beam pipe:

$\Rightarrow$ a good place to add drive attendance OR HOM dampers.

$\boxed{\text{D: PERFORMANCE}}$

1) Adjust R to get frequency of choice

HOW LONG SHOULD L BE?

- A particle going through that pill box with speed βc passes cavity center $(z = \frac{L}{2})$ at $t = 0$

It aquires a voltage of

$$V_A = \int_{-L/2}^{+L/2} E_z \, dz = \beta c . E_0 \int_{-L/2\beta c}^{+L/2\beta c} e^{i\omega t} . dt$$

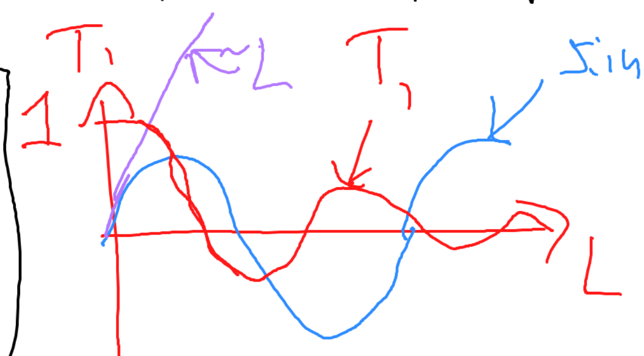
For a TM_{010} mode NOTE

$$\boxed{V_A = E_0 L . T_1(L)}$$

TRANSIT TIME FACTOR

where

$$\boxed{T_1(L) = \frac{\sin(\omega L/2\beta c)}{\omega L/2\beta c}}$$



so V_A has a maximum WHEN $T_1(L)$

$T_1 = 2/\pi$ when $\boxed{L_{OPT} = \pi \frac{\beta c}{\omega}}$

E.g. in the TM_{010} mode

$$\boxed{L_{OPT} = \frac{\pi}{2.41} \beta R}$$

Pill-boxes are very inefficient when $\beta \ll 1$

e.g. protons & ions

eg protons and ions

NON RELATIVISTIC particles ($\beta \ll 1$)

need different cavity shapes. Resonant zoo:

SPOKE

SPLIT RING

inter-digital

1/4 wave resonators

1/2 wave

⋮

(crab cavities)