Lecture 9

Linear Errors & Their Correction

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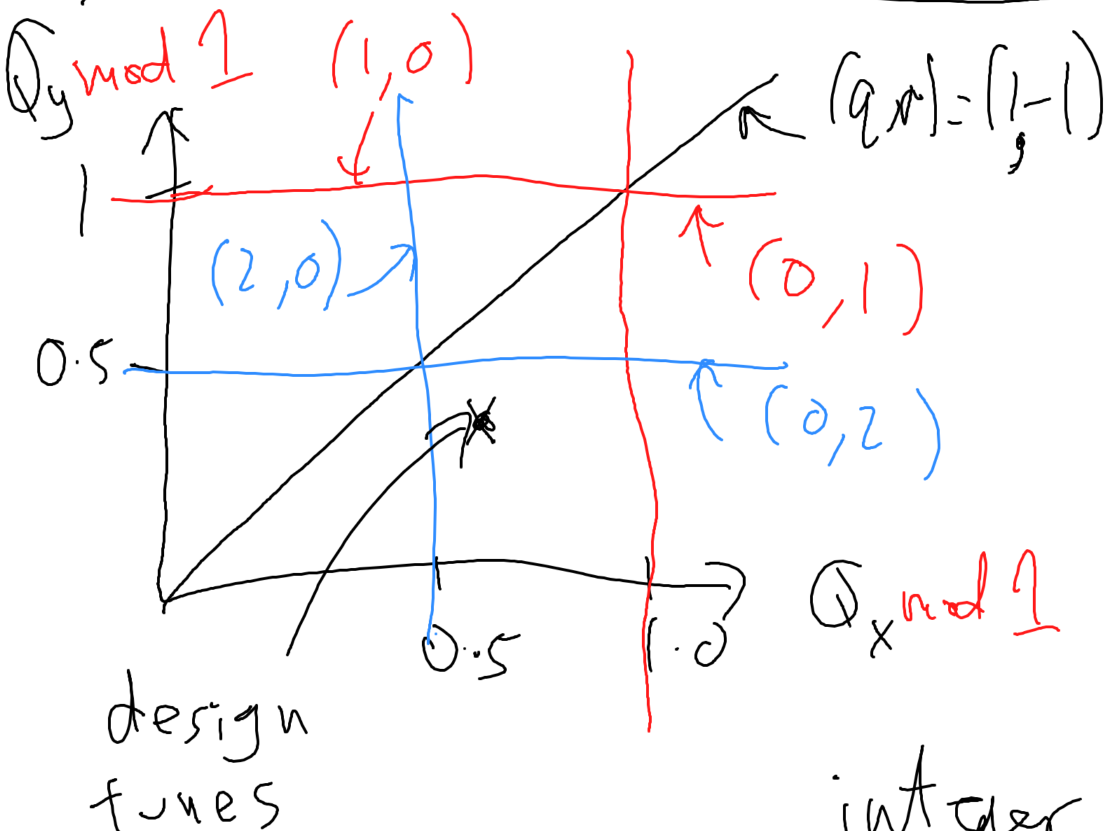
Lecture 9: Linear errors & their correction **Feb 4, 2021**

- Tune plane - introduction
- Trajectory & closed orbit
  - quadrupole displacement
  - dipole roll
  - free-wave & Periodic Boundary Conditions
  - closed orbit correction
- Horizontal-vertical linear coupling
- Quadrupole strength errors
  - tune shifts
  - beta-waves
    - free-wave
    - PBC

Hidden agenda: introduction to resonances

INTRODUCTION TO RESONANCE



$$\boxed{p = q\,Q_x + r\,Q_y}$$

p, q, r integers

- p can be v. large ....
  but (often) we
  just ignore the
  integer parts of Q_x + Q_y

TRAJECTORY + CLOSED ORBIT : MISALIGNMENTS

Take a multipole magnet

$$B_y + iB_x = C_n (x+iy)^n$$

consider $y=0$

real $\begin{cases} C_1 & \text{quad} \\ C_2 & \text{sextupole} \\ C_3 & \end{cases}$

Displace by Δx, Δy

$$x \to x + \Delta x \,, \quad y \to y + \Delta y$$

$$\boxed{B_y + iB_x = C_n \left[(x+iy)^n - n(x+iy)^{n-1}(\Delta x + i\Delta y) + \cdots \right]}$$

feed-down!

QUADRUPOLE DISPLACEMENT

$$\Delta x' = -\frac{x}{f} + \underbrace{\frac{\Delta x}{f}}$$

CONSTANT TERM: DIPOLE

EG, $f = 10$ m, $\Delta x = 1$ mm

$$\frac{\Delta x}{f} = \frac{10^{-3}}{10} = 100 \text{ } \mu\text{radians}$$

DIPOLE ROLL (about longitudinal axis)

Roll a dipole (of BEND angle θ) by α

$\boxed{\Delta y' = \alpha\theta}$ EG $\theta = 40$ mrad, $\alpha = 1$ mrad, $\Rightarrow \Delta y' = 40$ μrad

So a SINGLE translated quad or rolled dipole delivers angular kicks of order 10^{-4} radians, V+H

Q1 : How bad is it?

Q2 : Free wave or closed orbit

Q3 : How to correct "it"?

FREE WAVE

In a transfer line or linac

$$\boxed{\frac{x}{\sqrt{\beta_x}} = \left(\Delta x' \underbrace{\sqrt{\beta_0}}_{\text{SENSITIVITY}}\right) \cdot \sin\left(\psi_s - \psi_0\right)} \qquad s > s_0$$

E.G., $\Delta x' = 100\ \mu\mathrm{rad}$, $\beta = 50\ \mathrm{m} = \beta_0$

$$\left.\frac{\Delta x}{\sqrt{50}}\right|_{MAX} = 10^{-4}\cdot\sqrt{50}\ ,\qquad \underline{\Delta x_{max} = 5\ \mathrm{mm}\ !!}$$

Can't take many such errors!

Q: WHAT happens in a ring? Periodic Bound. Condx (PBC)

CLOSED ORBIT RESPONSE TO ERRORS

Apply PBC to 1-turn 2x2 matrix M

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{co} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{co} + \begin{pmatrix} 0 \\ \Delta x' \end{pmatrix}$$

$$\boxed{\begin{pmatrix} x \\ x' \end{pmatrix}_{co} = (I-M)^{-1} \begin{pmatrix} 0 \\ \Delta x' \end{pmatrix}}$$

I : 2x2 identity matrix

Watch out for det(I-M) → 0 !

$$\boxed{(I-M)^{-1} = \frac{1}{2(1-C)} \cdot \begin{pmatrix} 1-C & \beta S \\ -\frac{S}{\beta} & 1-C \end{pmatrix}}$$

$C = \cos(2\pi Q_x)$
$S = \sin(2\pi Q_x)$

watch out for $C \approx 1$

$Q_x = 0, 1, \ldots$ integer ... !

AT KICK LOCATION

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{co} = \frac{\Delta x'}{2\sin(\pi Q_x)} \cdot \begin{pmatrix} \beta_0 \cos(\pi Q_x) \\ \sin(\pi Q) - \alpha_0 \cos(\pi Q_x) \end{pmatrix}.$$

AT ANY LOCATION s

$$\boxed{\frac{x_{co}}{\sqrt{\beta}} = \frac{\Delta x' \sqrt{\beta_0}}{2\sin(\pi Q_x)} \cdot \cos\left(|\psi - \psi_0| - \pi Q_x\right)}$$

$\boxed{\text{RESONANCE EXAMPLE 1}}$ (LINEAR)

Tunes with $Q_x \approx p$ are dangerous !!

OR, Beware $(q, r) = (1, 0)$ or $(0, 1)$!!

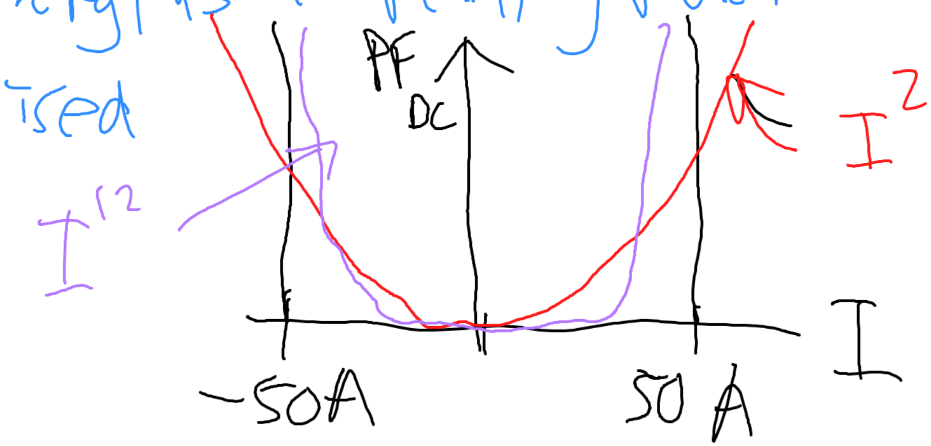
CLOSED ORBIT CORRECTION

1. Measure orbit with Beam Position Mons (BPM)
2. Apply algorithm
3. Adjust dipole CORRECTOR strengths

e.g.

$$PF = \sum_{BPM} x_{co,BPM}^2$$

INCLUDE corrector strengths in Penalty Function (PF) that is to be minimised



Q1 : What PENALTY FUNCTION? Quadratic? WHY?

Q2 : What about DIPOLE CORRECTOR limits?

Q3 : 19th century maths can be improved upon.

THERE ARE MANY ALGORITHMS TO CORRECT C.O., but "SLIDING THREE BUMP" allows ANY P.F.

Downstream of 3 dipole correctors, 1, 2, 3

$$\boxed{\frac{x(\psi)}{\sqrt{\beta}} = \sum_{i=1}^{3} \left(\Delta x'_i \sqrt{\beta_i}\right) \cdot \sin(\psi - \psi_i)}$$

IF this sum is zero, NO NEED to APPLY PBC!!

ONLY LOCALLY (between $i=1$ & 3) C.O. DISPLACEMENT.

3-BUMP LOCALISATION IS GUARANTEED IF

$$\boxed{\frac{\Delta x'_i \sqrt{\beta_i}}{\sin(\psi_j - \psi_k)} = K}$$

for (i,j,k) & cyclic combination of $(1,2,3)$

$(2,3,4)$
$(4,5,6)$
$(N,1,2)$

"SLIDING 3-BUMP ALGORITHM": $\boxed{\text{Adjust}}$ strength K for $(1,2,3)$, $(2,3,4)$, $(3,4,5)$ ... minimize local PF

$\boxed{\text{Repeat}}$ until global PF is minimized

LINEAR COUPLING (OF H&V) DUE TO ROLLED QUADS

Roll any magnet about it's longitudinal axis by angle α

4x4 $$\boxed{M_{ROLLED} = R(-\alpha)\, M\, R(\alpha)}$$

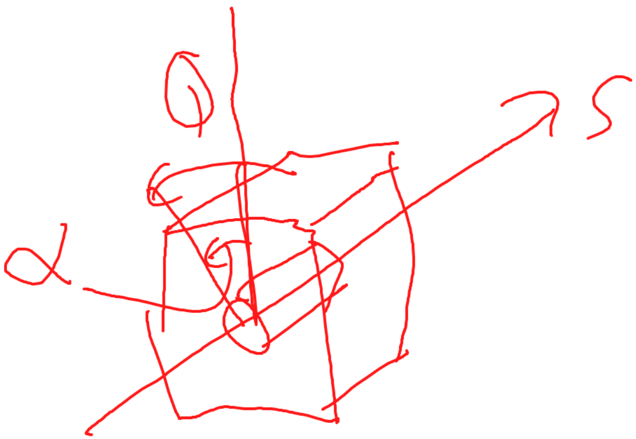
where $$R(\alpha) = \begin{pmatrix} C & 0 & S & 0 \\ 0 & C & 0 & S \\ -S & 0 & C & 0 \\ 0 & -S & 0 & C \end{pmatrix}$$ where $C = \cos(\alpha) \simeq 1$

$S = \sin(\alpha) \simeq \alpha$

So a Rolled Thin Quad (RTQ) is ...

$$M_{RTQ} \simeq M_{TQ} + \alpha \left(\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 0 & -2q & 0 \\ \hline 0 & 0 & 0 & 0 \\ -2q & 0 & 0 & 0 \end{array}\right) + O(\alpha^2)$$

where $q = \frac{1}{f}$

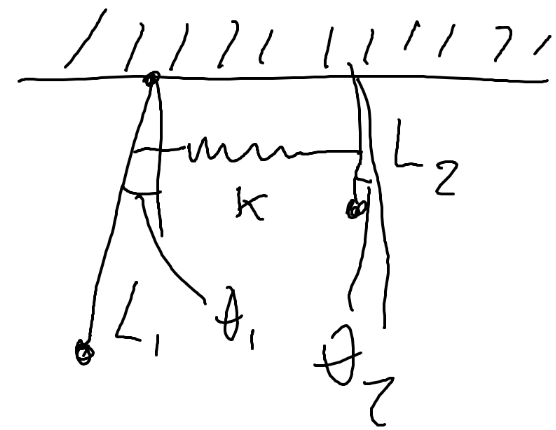


coupling H + V motion !!

ANALOGY: 2 COUPLED PENDULA

IF $k=0$ (NO COUPLING)

$$f_1 \sim \frac{1}{\sqrt{L_1}} \, , \quad f_2 \sim \frac{1}{\sqrt{L_2}}$$



WITH COUPLING ($k \neq 0$)

$$\boxed{\begin{aligned} \theta_1 &= a_{1A}\cos(f_A t + \phi_{1A}) + a_{1B}\cos(f_B t + \phi_{1B}) \\ \theta_2 &= a_{2A}\cos(f_A t + \phi_{2A}) + a_{2B}\cos(f_B t + \phi_{2B}) \end{aligned}}$$

TWO EIGENFREQUENCIES $f_1, f_2 \rightarrow f_A, f_B$

Q1: What happens when L_1 + L_2 are very different?

Q2: WHAT happens as $\overset{f_1}{\wedge} \sim \frac{1}{\sqrt{L_1}}$ is scanned across $\overset{f_2}{\wedge} \sim \frac{1}{\sqrt{L_2}}$?

- $f_A > f_B$: association with f_1 + f_2 swaps!
- There is a closest approach of f_A + f_B, depending on coupling strength

SIMILARLY, H + V betatron motion is COUPLED + CONFUSED

if $\boxed{|p + Q_x - Q_y| \lesssim Q_{MIN}}$

RESONANCE EXAMPLE 2 $(q, r) = (1, -1)$

≈ 0.001 WELL corrected
0.1 BAD!

QUAD STRENGTH ERRORS : TUNE SHIFTS & β-WAVES

A quad at s_0 has an error $\Delta q = \Delta\left(\frac{1}{f}\right)$

A) Is the linear motion still stable? If yes....

B) How much do Q_x & Q_y change?

C) What happens to β-functions everywhere?

One turn matrix becomes

$$\tilde{M} = M \begin{pmatrix} 1 & 0 \\ -\Delta q & 1 \end{pmatrix}$$

Perturbed tune $\tilde{Q}$ is found by solving

$$\mathrm{Tr}(\tilde{M}) = 2\cos(2\pi\tilde{Q})$$

OR

$$\boxed{\cos(2\pi\tilde{Q}) = \cos(2\pi Q) - \left(\frac{\beta_0 \Delta q}{2}\right)\cdot \sin(2\pi Q)}$$

EXACT !!

sensitivity $\sim \beta_0$

A) Motion is still stable if $|\mathrm{Tr}(\tilde{M})| < 2$

B) If Δq is small, then

$$\boxed{\Delta Q = \tilde{Q} - Q = \frac{\beta_0 \Delta q}{4\pi}}$$

first order in Δq

- Δq has different signs in H + V
- EG, 1% error in a quad with $f = 10$ m at a location with $(\beta_x, \beta_y) = (50, 10)$ m gives $(\Delta Q_x, \Delta Q_y) = (0.0040, -0.0008)$
- Tune shifts EASILY CORRECTED if (as common) F + D quad families in arc FODO cells can be tuned

→ β-function correction is trickier !!

$\boxed{\text{FREE } \beta\text{-waves}}$ just DOWNSTREAM of error quad

$$\begin{pmatrix} \tilde\beta \\ \tilde\alpha \\ \tilde\gamma \end{pmatrix}_{0+\epsilon} = \begin{pmatrix} 1 & 0 & 0 \\ \Delta q & 1 & 0 \\ (\Delta q)^2 & 2\Delta q & 1 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_0$$

launching a FREE WAVE

$$\boxed{\frac{\Delta\beta}{\beta} = -[\Delta q \beta_0] \cdot \sin\left(2(\psi - \psi_\epsilon)\right)}$$

advances at TWICE the speed of a betatron oscillation !

E.G. $\Delta q = 10^{-2}\left(\frac{1}{10}\right)$, $\beta_0 = 50$ m $\Rightarrow$ 5% error wave!!

Dangerous if many such random errors!! Control quad strengths at the 10^{-4} level is possible.

β-waves with PBC

$$\frac{\Delta\beta}{\beta} = \frac{-\Delta q \beta_0}{2\sin(\boxed{2}\pi Q)} \cdot \cos(2|\psi - \psi_0| - 2\pi Q)$$

RESONANCE DENOMINATOR:

twice as fast C.O. error

RESONANCE EXAMPLE 3

Optics are especially vulnerable to small errors IF

$$p \simeq 2Q_x \qquad r \simeq 2Q_y$$

That is, if $(q,r) = (2,0)$ or $(0,2)$

Q: WHY also include nonlinear magnets?

Q: WHERE do higher order (non linear) resonances come from?