University Physics 226N/231N
Old Domination University

Exam Review, Friction and Work

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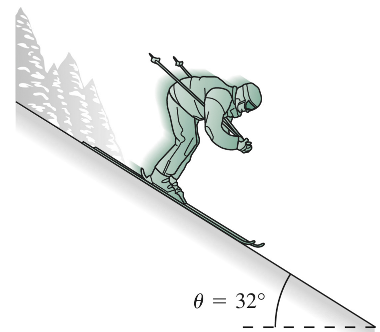


Wednesday, September 28, 2016

Reminder: The Second Midterm will be Weds Oct 19 2016

You should have received email from me late last night re exam 1.

Happy Birthday Hilary Duff, St. Vincent, Naomi Watts,
Confucius, and Henri Moissan (1906 Nobel)!
Happy National Women's Fitness Day and National Drink Beer Day!

Please set your cell phones to "vibrate" or "silent" mode. Thanks!

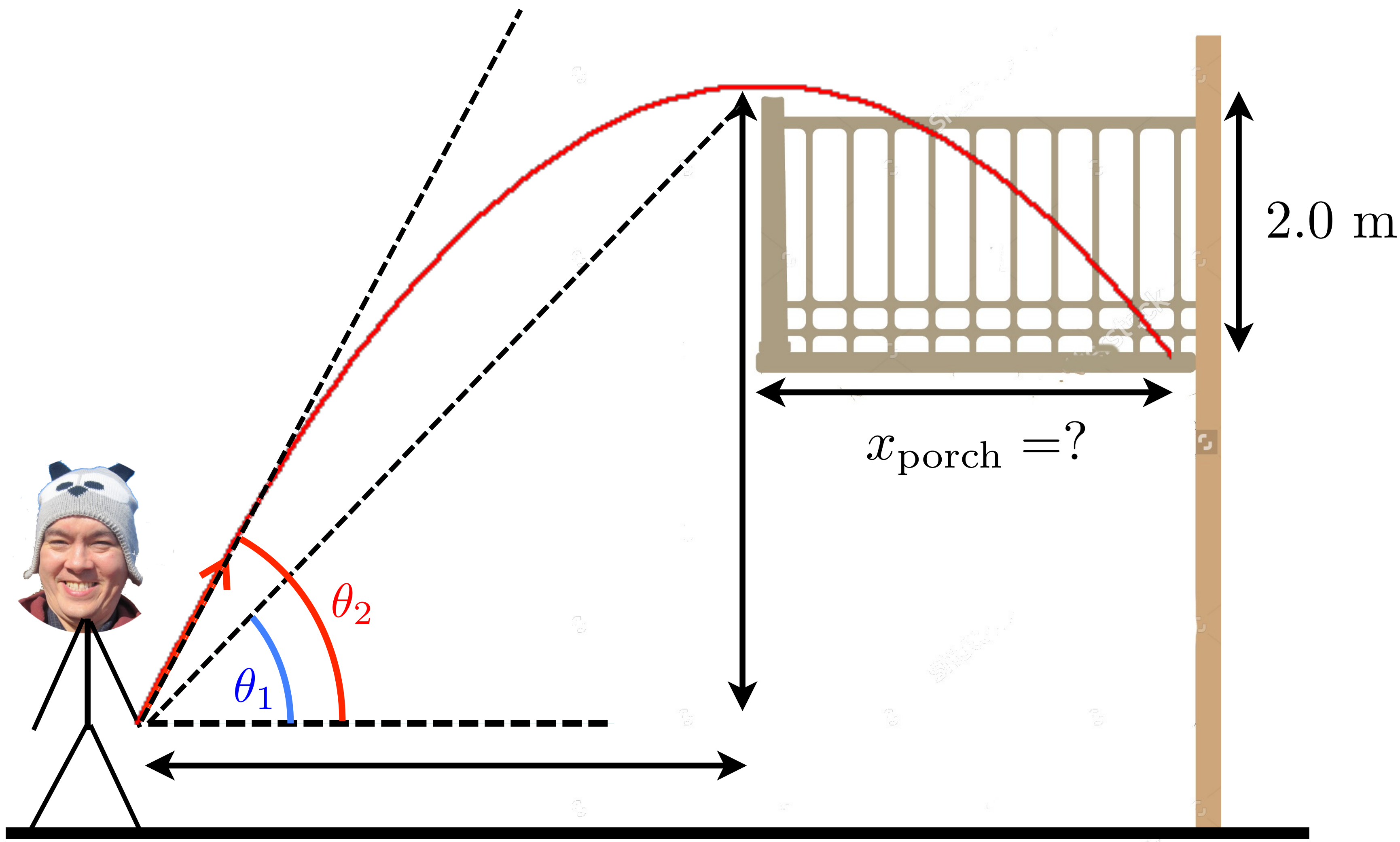
Exam Review

- In the first parts of class, I reviewed solutions for a few of the exam problems (in particular problems 1 and 5 a/b).
- Detailed solutions for these problems are found in the exam solution pdf that is linked to the email I sent out early Wed Sep 28 with your exam grade and detailed grading breakdown.

- Tip: Do not try to take shortcuts by calculating angles from distances in problem 5. The two angles in the diagram from problem 5 on the next page are **different**.

- It is important to separate pieces of problems into horizontal and vertical motion, and to not confuse the two.
  - Some people used gravitational acceleration in the horizontal direction….

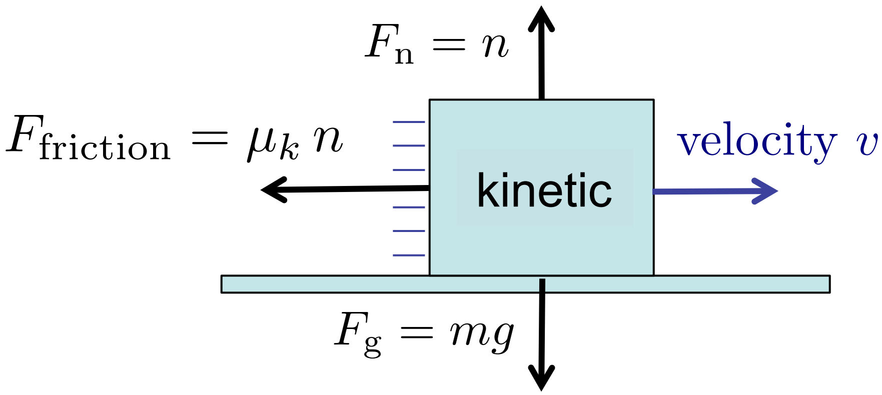
Problem 5: Angles Not The Same

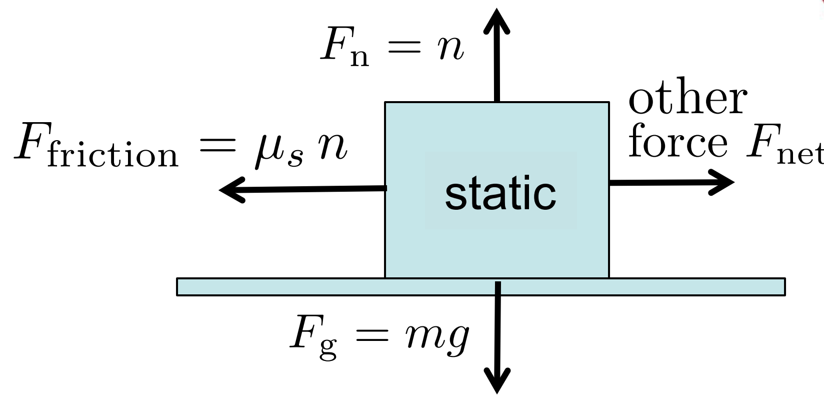


θ_1 and θ_2 are clearly not the same

You can only calculate θ_2 from the velocity vector components

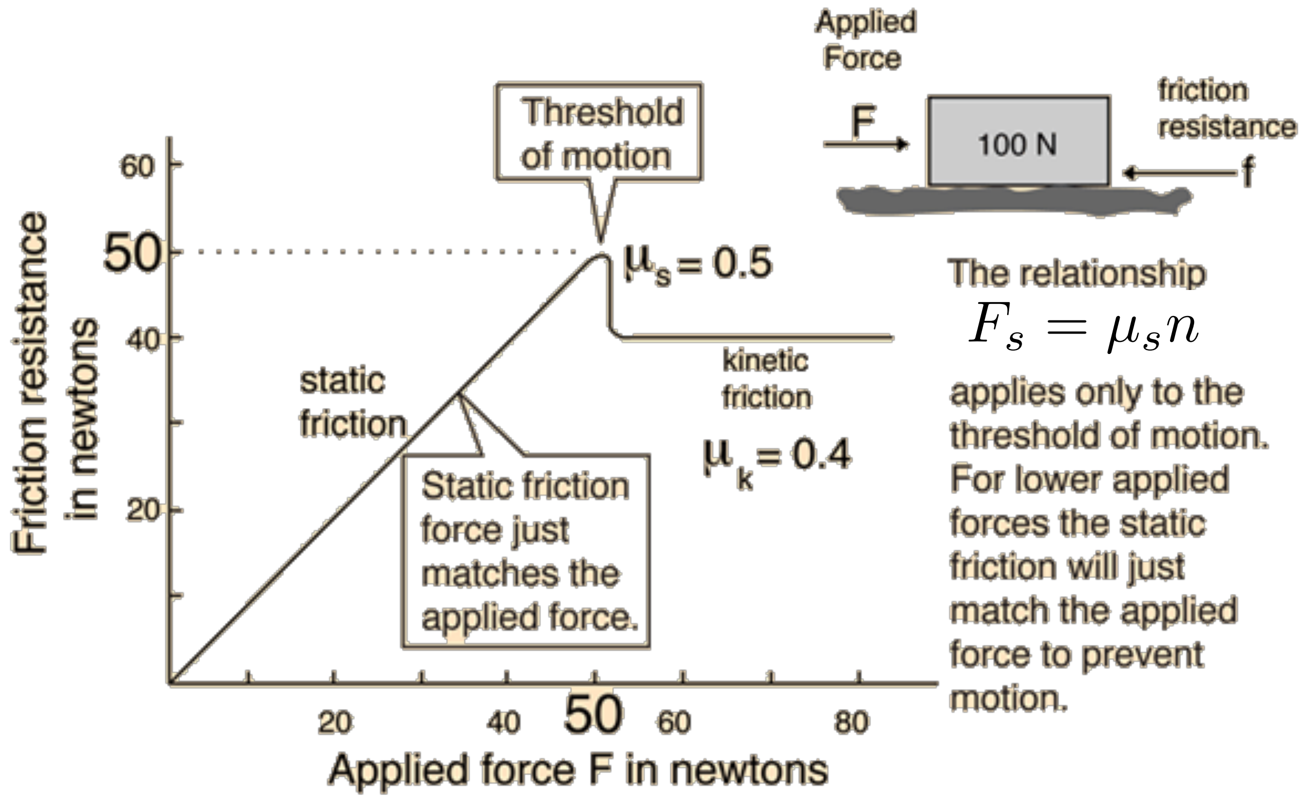
Frictional Forces





- **Friction** is a **force** (magnitude and direction!) that opposes the relative motion (velocity) of two contacting surfaces.
  - Newton's third law: Each surface feels equal and opposite force
- We have a pretty good basic model of frictional forces
  - Moving: **kinetic friction** $F_k = \mu_k n$ **against relative velocity**
  - Not moving: **static friction** $F_s \leq \mu_s n$ **against other net force**
  - This model of frictional force does **not** depend on velocity
  - Atmospheric friction (e.g. drag) is quite a lot more complicated
    - Depends on atmospheric density and viscosity, velocity, etc.

Static and Kinetic Frictional Forces



The relationship
$$F_s = \mu_s n$$
applies only to the threshold of motion. For lower applied forces the static friction will just match the applied force to prevent motion.

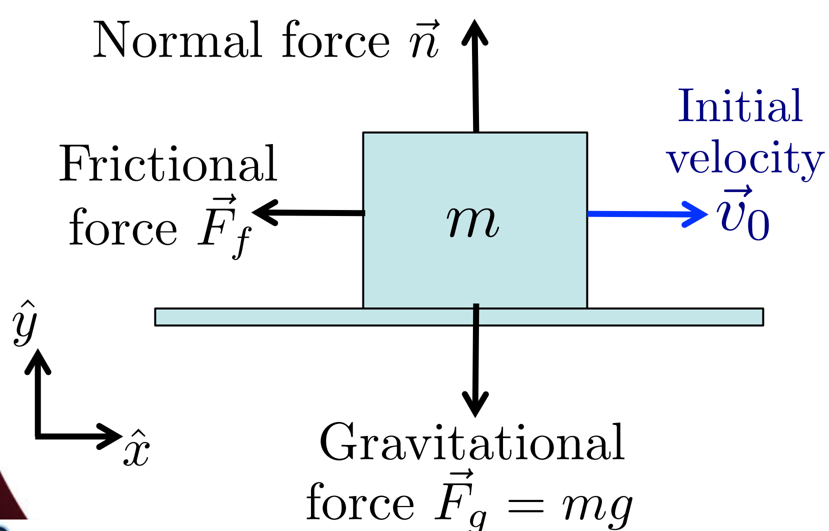
http://hyperphysics.phy-astr.gsu.edu/hbase/frict2.html

- Static friction acts to exactly cancel an applied force up to its maximum value, at which the object starts moving
  - The 100N object above does not start moving until the applied force F is greater than 50 N: $F_s = \mu_s n = (0.5)(100\text{ N}) = 50\text{ N}$
  - When the object starts moving, kinetic friction applies instead

Example Friction Problem

- Problems with friction are like all other Newton's law problems.
  - Identify the forces, draw a diagram, identify vector components, write Newton's law and solve for unknowns.
  - You'll need to relate the force components in two perpendicular directions, corresponding to the normal force and the frictional force.

- **Example:** A box sliding to a stop due to friction on a surface



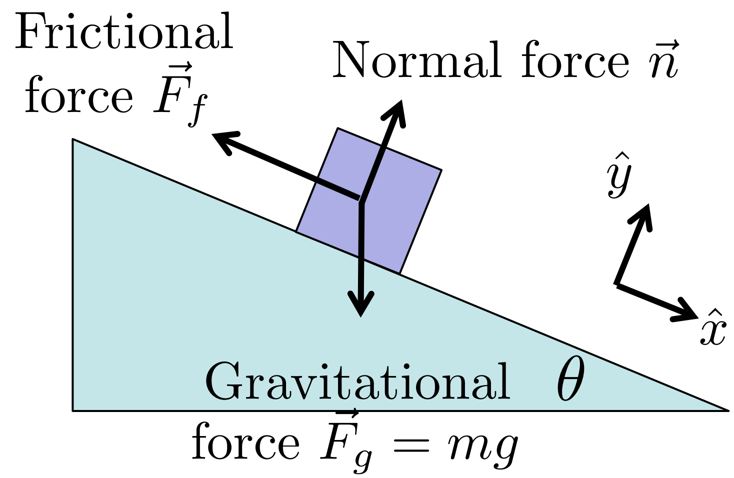
Coefficients of friction $\mu_s,\ \mu_k$

$$\text{Vertical}: F_{\text{net}} = n - mg = 0$$
$$n = mg$$
$$\text{Horizontal}: F_{\text{net}} = -F_f = -\mu_s n$$
$$F_{\text{net}} = -\mu_s mg = ma_x$$
$$a_x = -\mu_s g$$
$$\text{Time to stop}: t = \frac{v - v_0}{a_x} = \boxed{\frac{v_0}{\mu_s g} = t}$$

A More Practical Friction Problem

A box of mass m sits on a surface. We incline the surface until the box just starts slipping down the surface, and measure this angle of incline θ.

What is μ_s?

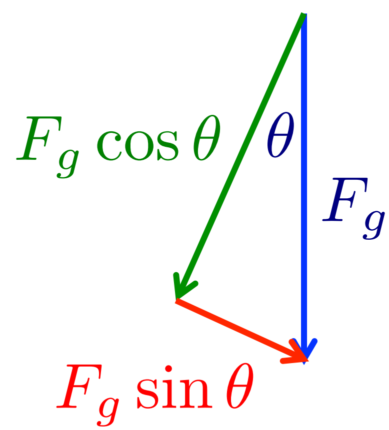


$$\text{Vertical}: F_{\text{net}} = 0 = n - F_g \cos\theta$$

$$n = F_g \cos\theta$$

$$\text{Horizontal}: F_{\text{net}} = 0 = F_g \sin\theta - F_f$$

$$F_f = F_g \sin\theta$$

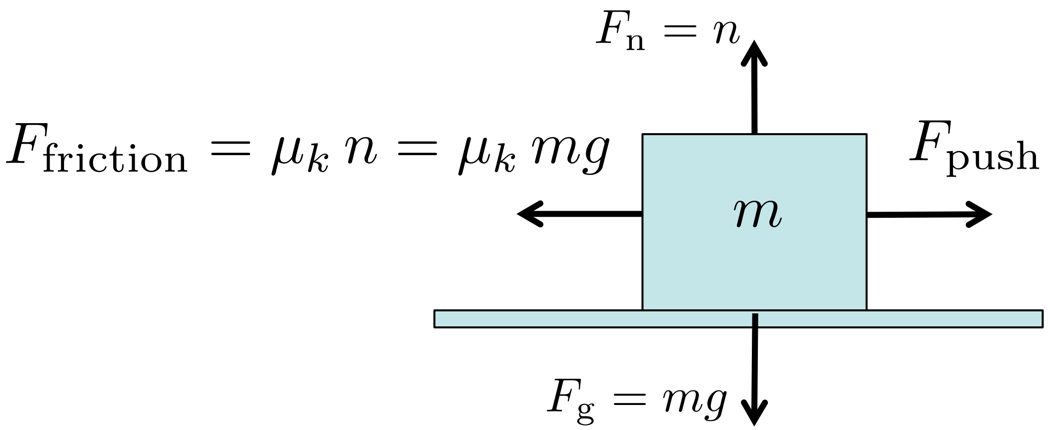


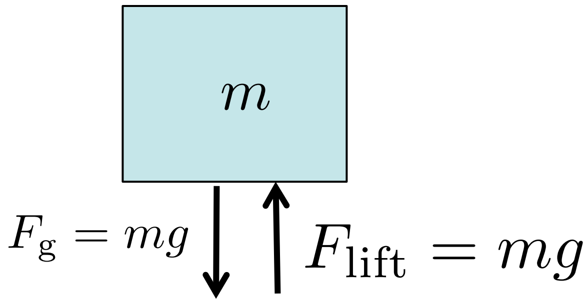
Force being decomposed is hypoteneuse of triangle

$$F_f = \mu_s n = \mu_s F_g \cos\theta$$

$$\boxed{\mu_s = \tan\theta}$$

Tangible/Ponderable (10 minutes)





- Put a flat object on the table in front of you (e.g. cell phone, notebook..)
  - Use an object that does not roll (we haven't discussed rolling yet)
  - Compare the forces to push it **horizontally** at constant speed, and to hold it **vertically** still against the pull of gravity
    - Estimate the coefficient of kinetic friction between your object and the surface
    - Try it again on a different level flat surface (e.g. a white board)
    - Can coefficients of kinetic or static friction be greater than 1?
    - If you tilt the surface, can you measure the angle where it starts slipping and get μ_s that way?

Some Coefficients of Friction

First Material	Second Material	Static	Kinetic
Cast Iron	Cast Iron	1.1	0.15
Aluminum	Aluminum	1.05-1.35	1.4
Rubber	Asphalt (Dry)	--	0.5-0.8
Rubber	Asphalt (Wet)	--	0.25-0.75
Rubber	Concrete (Dry)	--	0.6-0.85
Rubber	Concrete (Wet)	--	0.45-0.75
Oak	Oak (parallel grain)	0.62	0.48
Oak	Oak (cross grain)	0.54	0.32
Ice	Ice	0.05-0.5	0.02-0.09
Teflon	Steel	0.2	--
Teflon	Teflon	0.04	--

http://physics.info/friction

Quantities and their Relationships: Impulse

- We've covered several physics relationships so far in this course
  - Many are relationships between position, velocity, acceleration, time, and force
  - We've even **defined** the vectors velocity, acceleration, force, and momentum this way

$$\vec{v} \equiv \frac{d\vec{x}}{dt} \qquad \vec{a} \equiv \frac{d\vec{v}}{dt} \qquad \vec{F} \equiv m\vec{a} \qquad \vec{p} \equiv m\vec{v}$$

  - We can define change in **impulse** as a force times the time it's applied

$$\Delta\vec{I} = \vec{F}\Delta t$$

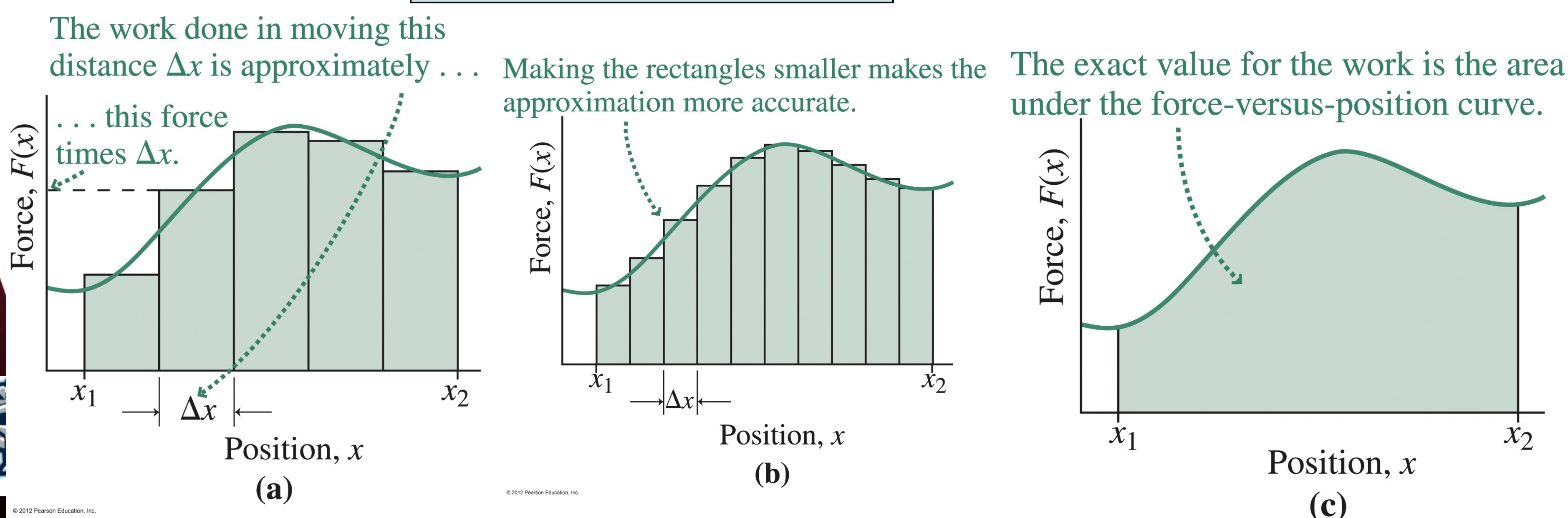
  - Adding these up (or integrating!) gives the total impulse

$$\vec{I} = \int \vec{F}\, dt$$

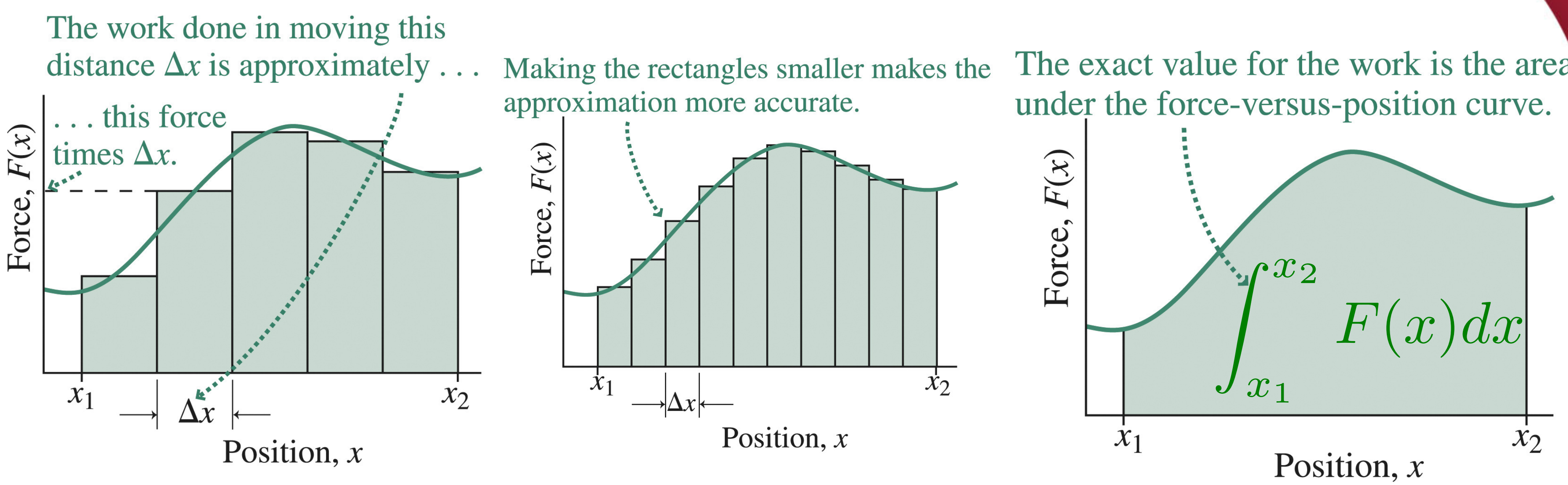
Quantities and their Relationships: Work

- Impulse is useful, but another combination is even moreso
  - We define **work** W as **net force applied times the distance its applied in**
    - The force can be different at various spots so we really need to add together (roughly constant) force over small distances
    - This is really another integral, like impulse. It's a scaler (units: 1 J=1 N-m)

$$W \equiv \int F\,dx$$ work in one dimension

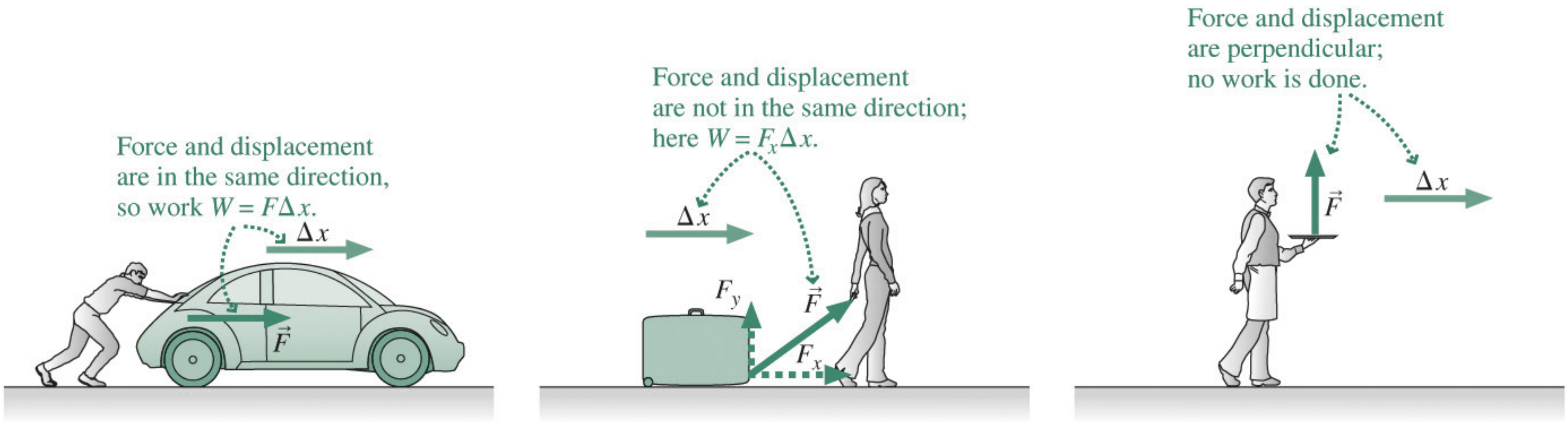


A Quick Aside: Integrals



- An integral is really a combination of two things:
  - A **sum of rectangular areas** that are sketched "under" a curve
    - Since these are areas, the integral has the same units as the units of the x-axis times the units of the y-axis – whatever those are.
    - In calculus they often ignore those limits. In physics, we can't.
  - A **limit** of that sum of areas as the width of each rectangle gets smaller and smaller (and approaches zero)
    - We add together more and more rectangles in this process
    - There are **definite** and **indefinite** integrals

Examples of Work (from a Physicist ☺)



- Work is really a bookkeeping tool
  - We'll relate it to how the **energy** of an object or system changes
  - We only count force and displacement in the same direction
  - Forces applied perpendicular to Dx or when Dx=0 do no work!
    - Holding an object still against gravity, or moving it horizontally
    - Static frictional forces (since there is no Dx)

Ponderable (10 minutes)

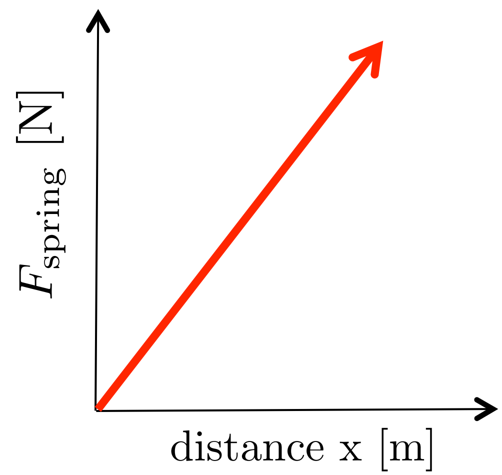
- Remember, **work W** in one dimension is **defined** as

$$W \equiv \int F dx = F \Delta x \quad \text{for constant } F$$

$$1 \text{ Joule} = 1 \text{ N} - \text{m}$$

  - What is the work done to lift an object of mass m over a distance h against the force of gravity? (Assume $v_{final}=v_0=0$ m/s)
    - This is quite straightforward: work is force times distance

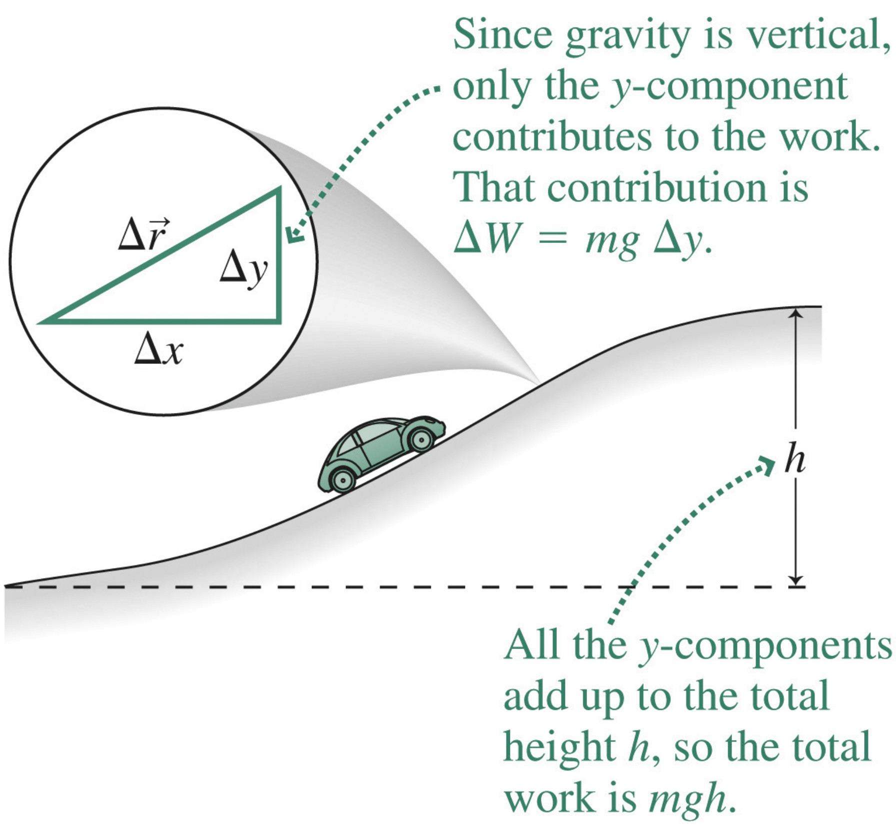
- A harder one: the force of spring relates to one end's displacement by $F_{\text{spring}} = -kx$
  - What is the work done to displace a spring by a distance x in terms of the spring constant k and the distance x?
    - This is a bit less straightforward: work is force times distance but the force is not constant; it's larger as we compress or stretch the spring



Work Done Against Gravity

- The work done by an agent lifting an object of mass *m* against gravity depends only on the vertical distance *h*:

$$W = mgh$$



- The work is **positive** if the object is raised (moved against the force of gravity) and **negative** if it's lowered (moved with the force of gravity).

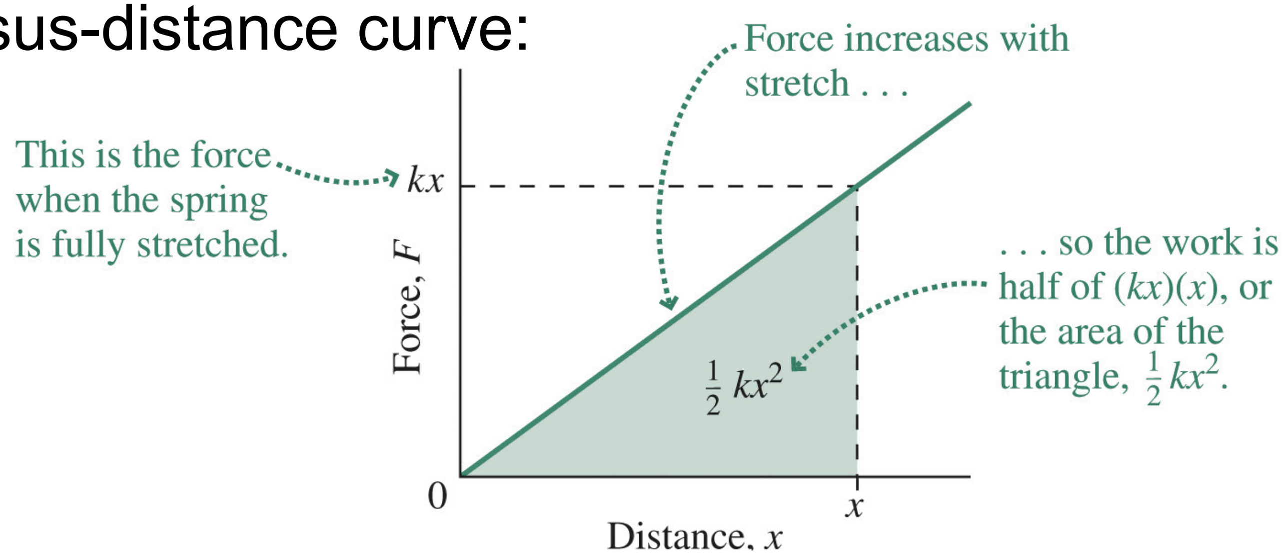
- The horizontal motion

Work Done in Stretching a Spring

- A spring exerts a force $F_{\text{spring}} = -kx$
- Someone stretching a spring exerts a force $F_{\text{stretch}} = +kx$, and the work done is

$$W = \int_0^x F(x)\,dx = k\int_0^x x\,dx = \left(\frac{1}{2}kx^2\right)\Big|_0^x = \boxed{\frac{1}{2}kx^2 = W}$$

- In this case the work is the area under the triangular force-versus-distance curve:



(This is where I ended Mon Sep 28)

Energy

- **Energy**: the capacity of an object to perform work
  - **Energy** is what we add up when we do our bookkeeping
  - **Work** is how energy moves through application of forces
- How do we do the energy bookkeeping for a system?
  - Add up energy from a variety of different sources and things that we know can do work
  - **Conservation of energy:** total energy for a system is constant

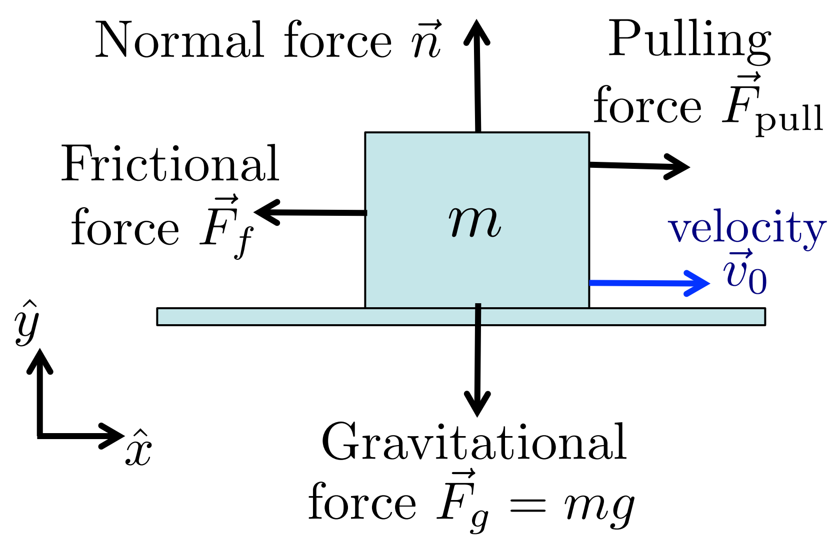
- Kinetic energy: energy of object's motion, $KE = ½ mv^2$
- Gravitational potential energy: energy from the potential of falling a certain distance under constant gravity: $PE_g=mgDy$
- Spring potential energy: $PE_s = ½ kx^2$
- Energy lost to friction over distance Dx: $E_f = m_x nDx$
- Chemical energy, nuclear energy, and others…

Work and Net Work

- **Energy**: the capacity of an object to perform work
  - **Energy** is what we add up when we do our bookkeeping
  - **Work** is how energy moves through application of forces

- Since work involves transfer of energy, and we want to account for all energy, it's important to account for all forces

- Example: Pulling a box against friction at constant velocity
  - Net sum of forces on box is zero
  - So **work done on box** is zero
  - But I still do work (I'm exerting a force over a distance)
  - The energy of my work goes into **frictional losses**

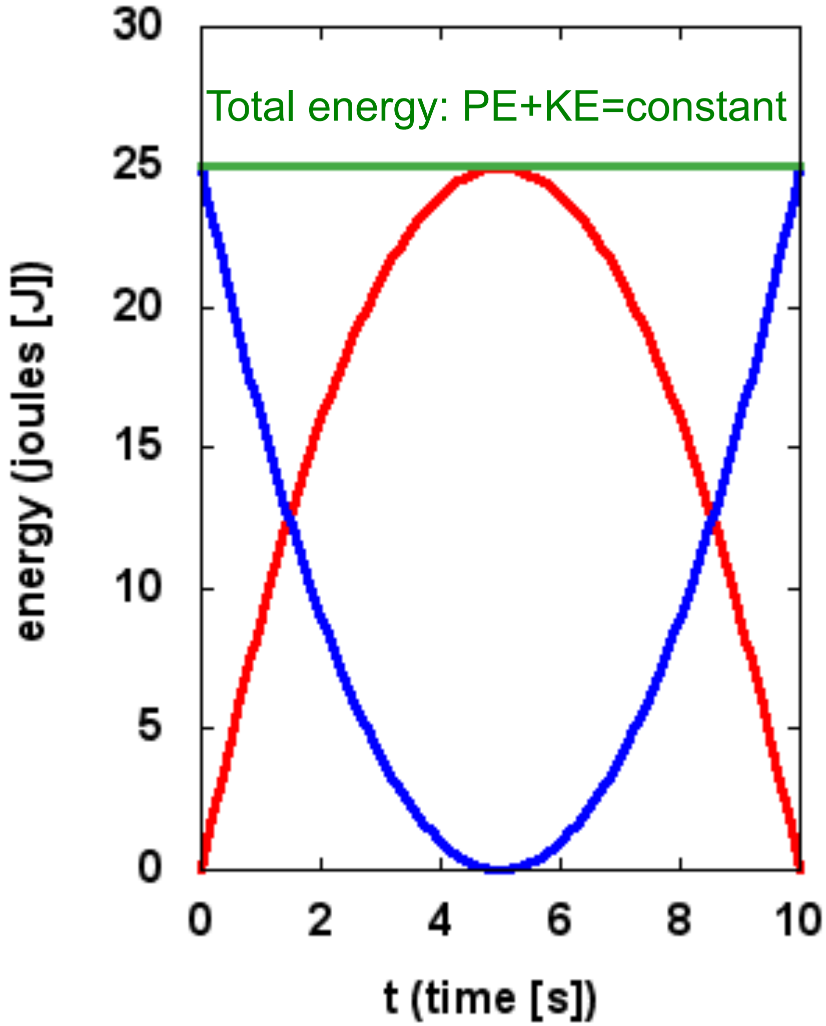


Coefficients of friction $\mu_s,\ \mu_k$

Example: Ball Toss

- Consider your professor tossing a juggling ball upwards
  - I do some work on it to add energy to the system
  - The system is now the ball!

- At start, h=0 m and all energy is kinetic energy
- As the ball moves up, potential energy grows and kinetic energy goes down
- At top, all energy is potential energy since v=0 m/s and KE=0 J
- As the ball comes back down, potential energy is released and kinetic energy grows again



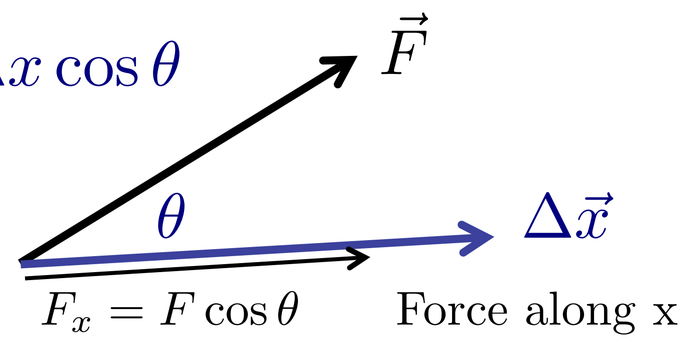
Kinetic energy: $KE = \frac{1}{2} mv^2$

Potential energy: $PE_g = mgh$

Work in Multiple Dimensions

- Work is adding up (integrating) force along a distance
  - But remember that force perpendicular to the distance moved does no work
  - So what we're really doing is adding up the **component** of force that is **along** the direction of motion

$$W = F_x \Delta x = F \Delta x \cos\theta$$

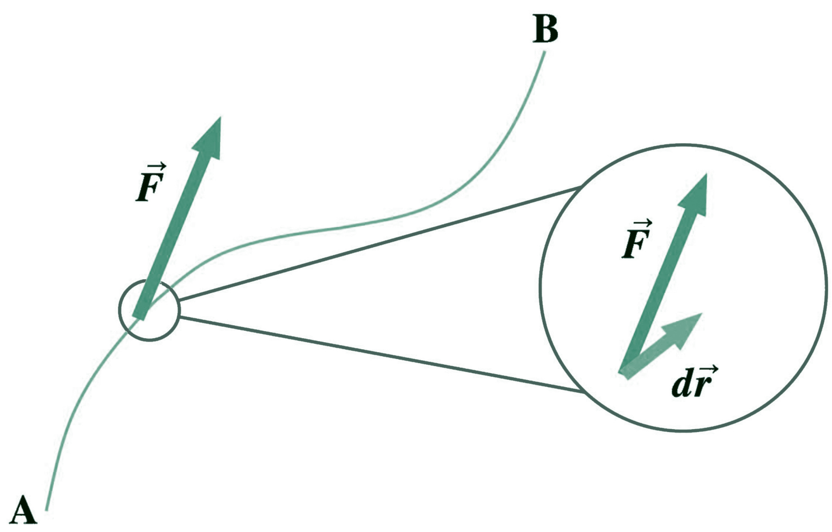


- This process of taking the "product" of two vectors and getting a scaler by looking at the component is a **dot product**

$$W = \int \vec{F} \cdot d\vec{x} = \int F dx \cos\theta$$

A Varying Force in Multiple Dimensions

- In the most general case, an object moves on an arbitrary path subject to a force whose magnitude and whose direction relative to the path may vary with position.
- In that case the integral for the work becomes a **line integral**, the limit of the sum of scalar products of infinitesimally small displacements with the force at each point.



$$W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot \vec{r}$$