USPAS Graduate Accelerator Physics Homework 1

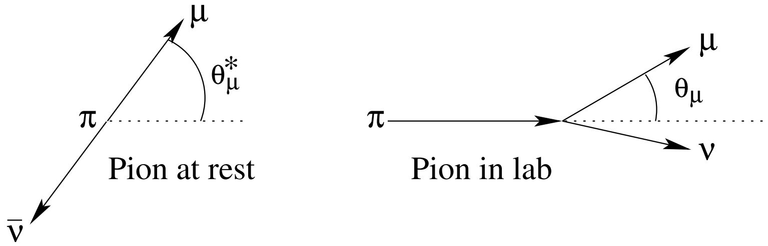
Due date: Tuesday January 23, 2024

1 Pion decay kinematics (Peggs/Satogata Modified Exercise 1.10)

Consider a charged pion decaying into a muon plus an antineutrino:

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

Use $m_{\pi^\pm} = 140$ MeV/c^2, $m_\mu = 106$ MeV/c^2, and $m_{\bar{\nu}} \approx 0$.



(a) (3 points) In the rest system of the pion, what are the energies and momenta of the muon and antineutrino?

(b) (3 points) Since neutrinos have now been discovered to have mass, how high must a pion beam energy be to produce some neutrinos at rest during their decays? Assume a rest mass of 0.01 eV/c^2 for muon neutrinos (and antineutrinos); you do not need to recalculate results from part (a).

(c) (4 points) For a moving pion with total energy $U_\pi = \gamma m_\pi c^2$ find an expression for the direction, θ_μ of the muon relative to the pion in the lab in terms of the angle θ^*_μ in the in the pion's rest system.

2 EIC frequency/field (Peggs/Satogata Modified Exercise 2.3)

The US electron-ion collider (EIC) hadron storage ring (HSR) will collide various ion species with polarized electrons. These ion species will include fully stripped gold ions (A=197, Z=79) at various gold ion *total* energies, including E_{Au}=110 GeV/nucleon. The HSR circumference for these gold ions will be 3832.915 m. Assume the rest mass of a fully stripped gold ion is 197×0.931336 GeV/c^2.

(a) (4 points) Calculate the revolution frequency f_{rf} of a gold ion at the injection energy of E_{inj}=10.5 GeV/nucleon, and at the collision energy of E_{coll}=110 GeV/nucleon. What is the change in revolution frequency for particles accelerated from E_{inj} to E_{coll}?

(b) (3 points) What are the gold ion rigidities ($B\rho$) at E_{inj} and E_{coll}?

(c) (3 points) If we assume that there are 192 identical dipoles per ring, each of length $L = 9.7$ m, what are the required dipole fields at E_{inj} and E_{coll}?

(flip the page...)

3 Lithium lens (Peggs/Satogata Modified Exercise 6.8: yes, you can do it even after the first day of lectures!)

(10 points) A lithium lens of length l and radius a has a constant total current I flowing through its end caps with uniform current density as pictured in Fig. 1. (So in this figure, the current is flowing from either left to right *or* right to left.) This current creates a magnetic induction $\vec{B}(r)$ within the lithium lens.

Consider a beam of antiprotons with momentum p that are passing left to right through this lithium lens. (Yes, the antiprotons actually pass through the lithium material fairly easily.)

(a) What is the magnetic induction $\vec{B}(r)$ in the lithium lens?

(b) What is the focal length of this lens for the antiprotons? (Recall that the focal length for a focusing lens is defined as the distance at which incoming parallel rays converge on the center axis.)

(c) Does the lithium lens current need to flow from left to right or right to left for the lens to focus this antiproton beam?

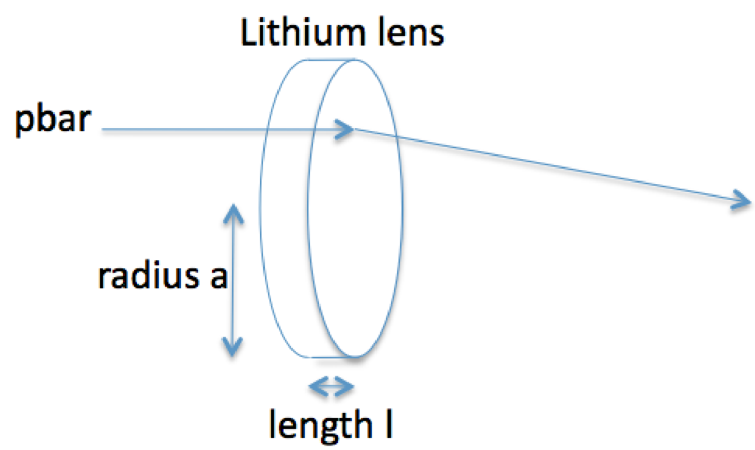


Figure 1: Lithium lens diagram. A uniform total current I is applied through the end caps (that is, pointing left to right or right to left) of the lithium cylinder to create a focusing lens.