

USPAS Graduate Accelerator Physics Homework 2

Due date: Wednesday January 24, 2024

1 Twiss Parameter Propagation

- (a) Prove Equations 3.34 and 3.35.

2 Drift Twiss Parameters

Consider a long field-free region adjacent to a collision point.

- (a) Use Equation 3.34 to show that the β -function evolves with s like $\beta(s) = \beta^* + \frac{(s-s_0)^2}{\beta^*}$.
- (b) How does $\alpha(s)$ evolve?
- (c) How does the phase $\phi(s)$ evolve?
- (d) What is the largest phase advance possible, across a field-free region?
- (e) Why are optics often displayed as $\sqrt{\beta}$, rather than β vs s ?

3 FODO Beta/Dispersion Ratio

Consider the close connection between η_x and β_x in well-matched cells.

- (a) Construct a lattice file (*eg* using MADX) to represent a FODO cell similar to that shown in textbook Figure 3.5, with thin quads of strength $\pm q$ separating 10 short dipoles of bend angle $\theta/10$. (It is not necessary to include any drifts, unless you wish to do so.)
- (b) Set the value of q to deliver a matched phase advance $\Delta\phi$ of 60 or 90 degrees.
- (c) Use *eg* MADX to generate a table of η_x and β_x values between $s = 0$, $s = 2L$
- (d) Does the ratio $\eta_x/\sqrt{\beta_x}$ for the lattice appear approximately constant? What is its approximate value (arithmetic mean, perhaps)?
- (e) What is the largest deviation of $\eta_x/\sqrt{\beta_x}$ from the mean value? What percentage of the mean value is this? At which lattice element does it occur?

4 Twiss Parameters from 6D M Matrix

Suppose that the following one-turn matrix M transforms motion from $s = 0$ to $s = C$ around the circumference of an accelerator:

$$\begin{pmatrix} -1.05746 & -3.59421 & 0.00000 & 0.00000 & 0.00000 & 35.44680 \\ 0.00189 & -0.93923 & 0.00000 & 0.00000 & 0.00000 & -0.82369 \\ 0.00000 & 0.00000 & 1.72622 & -72.45113 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.05149 & -1.58161 & 0.00000 & 0.00000 \\ -0.80399 & -36.25338 & 0.00000 & 0.00000 & 1.00000 & -50.03916 \\ 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 1.00000 \end{pmatrix}$$

- (a) What are the fractional parts of the horizontal and vertical tunes?
- (b) What are the horizontal Twiss functions (β, α, γ) at $s = 0$?
- (c) What is the dispersion function and its slope, η and η' , at $s = 0$?