

USPAS Graduate Accelerator Physics Homework 3

Due date: Thursday January 25, 2024

1 Courant-Snyder invariant/action derivation (Peggs/Satogata 5.1)

Derive Equation 5.4:

$$2J_x = \beta_x x'^2 + 2\alpha_x x x' + \gamma_x x^2 \quad (1.1)$$

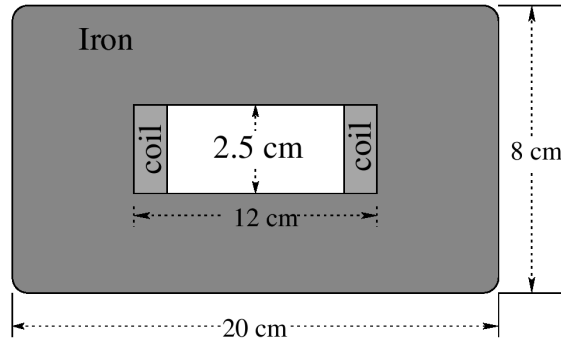
2 Action distribution for a Gaussian beam (Peggs/Satogata 5.2)

If the horizontal projection of the beam distribution function, $\rho(x)$, is Gaussian at a point with zero dispersion,

$$\rho(x) \sim e^{-x^2/2\sigma^2} \quad (2.1)$$

what is the shape of $\rho(J)$, the action distribution?

3 Magnet Design (Peggs/Satogata 6.6)



Consider the window-frame dipole magnet shown in the figure, with gap width $w = 12$ cm, gap height $g = 2.5$ cm, and length $l = 1$ m.

(a) Show that the magnetic field in the gap is

$$B = \mu_0 \frac{NI}{g} \quad (3.1)$$

where N is the number of turns, and I is the current in each turn, if the iron has infinite relative permeability $\mu_r \equiv (\mu/\mu_0) = \infty$.

(b) Show that the inductance of the magnet is

$$L = \mu_0 N^2 \frac{lw}{g} \quad (3.2)$$

- (c) If the resistivity of copper is ρ , and the cross sectional area of the copper in each of the two coils is A , what is the total power dissipation in the magnet?
- (d) How many ampere-turns (NI) are necessary to achieve a field of 0.6 T in the gap, if the relative permeability of the iron is $\mu_r = 5000$. What, approximately, is the maximum field in the iron? Where?
- (e) Air-cooled copper coils carry a maximum current density of about 1.5 A/mm^2 , while water-cooled copper coils can carry almost 10 times as much. However, water cooling adds the potential for water leaks and is more expensive, so it is avoided if possible. Would you recommend water cooled or air cooled coils for this magnet? how much horizontal space is available between the coils for a beampipe?
- (f) If the magnet power supply has a maximum current of 1000 A, how many turns should be used in the coil?
- (g) Calculate the inductance of the magnet. How much energy is stored in the gap? How much power is dissipated in the coils?
- (h) Assuming constant field in the iron, estimate the additional energy stored in the iron yoke.