

USPAS Graduate Accelerator Physics Homework 5

Due date: Tuesday January 30, 2024

1 Hénon triangle near $Q = 1/3$ (Peggs/Satogata 9.4)

Consider the equilateral triangle in (x, x') normalised phase space predicted by text Equations 9.27 and 9.28.

- What is the radius of the largest circle that can be inscribed inside the triangle?
- What is the orientation of the triangle?
- What happens to the area and the orientation of the triangle as the tune Q is (slowly) swept through the value of $1/3$?

2 Hénon dynamic aperture simulation (Modified Peggs/Satogata 9.5)

Use the simulation at <http://www.toddsatogata.net/2024-USPAS/homework/Henon.html> to investigate motion under the Hénon map by adjusting the two control parameters: tune Q and the number of turns tracked T . Launch multiple trajectories at many initial locations in phase space. Consider the plot of Hénon dynamic aperture (DA) versus tune shown in text Figure 9.4.

- How small must $|Q - 1/3|$ be, for the triangular predictions of Exercise 9.4 (question 1 in this homework) to be reasonably valid?
- Devise and define a convenient quantitative measure of the size of the stable region – the DA. (There are many ways to do this.)
- How does the DA in the range $0.5 < Q < 1.0$ relate to the DA in the range below $Q = 0.5$? Why?

3 Dodecapole Detuning (Modified Peggs/Satogata 10.2)

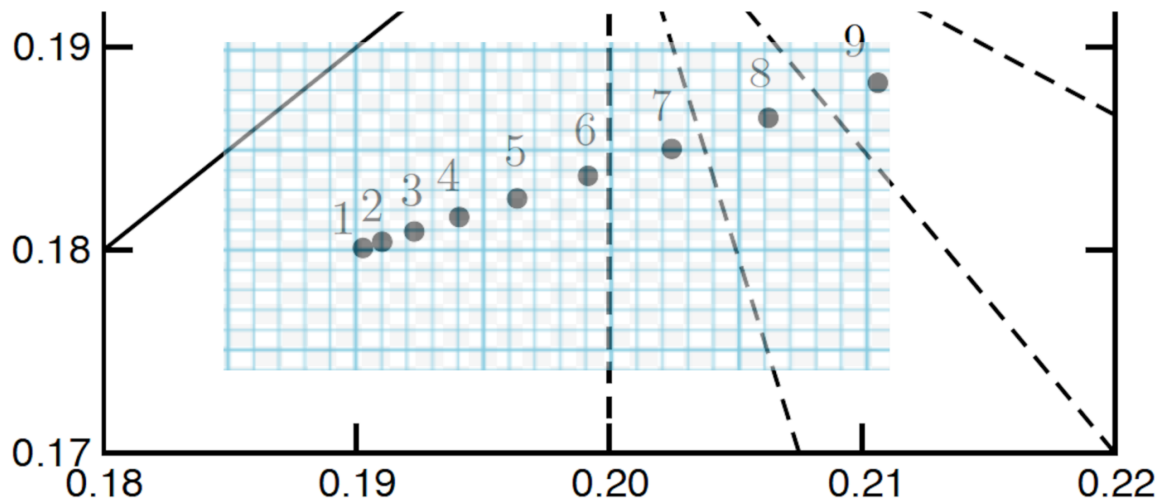
If a single dodecapole (12-pole) magnet delivers an angular kick of

$$\Delta x' = -g_{12}x^5 \tag{3.1}$$

and causes normalized phase space detuning

$$Q_x = Q_{0x} + Ag_{12}a^B \tag{3.2}$$

what are the numerical values of coefficients A and B ? Note that here you must only calculate the effect of the detuning to *first order* in dodecapole strength.



4 Nonlinear Tune Tracking Data (Modified Peggs/Satogata 10.1)

You have simulated the RHIC accelerator with a set of nine particles launched with design momentum ($\delta = 0$), $x' = 0$, and initial x offsets of 1, 2, \dots 9 mm at a location with horizontal beta function $\beta_x = 40$ m. You “measure” the fractional tunes of these particles from the plot shown above to be:

x [mm]	Q_x	Q_y
1	0.1903	0.1800
2	0.1910	0.1802
3	0.1923	0.1809
4	0.1941	0.1816
5	0.1963	0.1825
6	0.1991	0.1837
7	0.2024	0.1851
8	0.2061	0.1866
9	0.2105	0.1884

- plot Q_x and Q_y vs. J_x from the above table.
- What is the simplest fit to the tune vs. action data?
- What is the simplest and most likely dominant nonlinearity based on the variation of the tunes $Q_{x,y}$ with their actions $J_{x,y}$?