USPAS Graduate Accelerator Physics Homework 7

Due date: Thursday February 1, 2024

1 ESS RF cavity cell count

Use the synchronism factor calculator at `http://www.toddsatogata.net/2024-USPAS/lab/Synchro.html` to help with the following problem.

The high energy end of an ESS-like proton linac is being designed using either 2 or 3 families of elliptical cell cavities to accelerate from a kinetic energy of 200 MeV to 2.0 GeV. The cavities in each family $i = 1, 2, (3)$ all have the same geometric beta, $\beta_{i,G}$. Family i accelerates from $\beta_{i,min}$ to $\beta_{i,max}$, so that $\beta_{1,max} = \beta_{2,min}$, et cetera. The synchronism factor $S(\beta/\beta_G, N)$ measures the efficiency with which each cavity accelerates, where N is the number of cells per cavity, with a maximum value of $S(1, N) = 1$.

(a) What are the values of γ and β at 200 MeV and 2.0 GeV?

(b) Assuming that $S(x, N) \approx S(1/x, N)$ for any N, what are the optimum β_{tran} values at which to transition from one family to the next, for both 2 and 3 families?

(c) What are the optimum β_G values for those 2 or 3 families?

(d) Using those β_{tran} and β_G values, what are the minimum values of the synchronism factor for 2 or 3 families, with $N = 5, 7$ or 9? How reasonable is the assumption that $S(x, N) \approx S(1/x, N)$?

(e) How would you decide whether to use 2 or 3 families? What are the competing cost and performance drivers?

2 Linearized RF Transport (Modified Peggs-Satogata 13.2, excluded after assignment)

Expand Equation 13.18,

$$\delta W_n = q(V_{A,n} \cos\phi_n - V_{A,r,n} \cos\phi_{r,n}) + \delta W_{n-1} \tag{2.1}$$

to derive the linearised matrix M_{TC} for a thin cavity:

$$\begin{pmatrix} \delta\phi \\ \delta W \end{pmatrix}_{\text{out}} = M_{TC} \begin{pmatrix} \delta\phi \\ \delta W \end{pmatrix}_{\text{in}} \tag{2.2}$$

Note that for a thin cavity, $\delta\phi$ does not change, much like transverse position x or y does not change going through a thin quadrupole.

3 Chicane Bunch Compressor (Modified Peggs-Satogata 14.1)

Figure 1 sketches the layout of a simple four-dipole chicane in which the bend angle θ is small, and a is the distance between the first and second dipoles.

Figure 1: A simple four-dipole chicane, with no quadrupoles.

(a) Following Equation 14.18 in the book, show that

$$M_{56} \equiv \frac{dz_2}{d\delta_1} = -2a\theta^2 \tag{3.1}$$

(b) **BONUS:** Following the Taylor expansion of the path length in Equation 14.19 in the book, show that

$$T_{566} = -\frac{3}{2}M_{56} \tag{3.2}$$

and

$$U_{5666} = 2M_{56} \tag{3.3}$$