Computer Lab 2

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Due **Before class** Friday, February 2 2024

1. **Double Bend Achromat**
   (a) Construct a double bend achromat with the parameters given in class (slide 21 of the Friday PM class notes) in madx: $L = l = 2\,\mathrm{m}$, $\theta = 0.01\,\mathrm{rad}$, $f = (L+2l)/4 = 1.5\,\mathrm{m}$, $(KL)_{\mathrm{quad}} = 0.677\,\mathrm{m}^{-1}$.
   (b) Track $\eta_x = 0, \eta'_x = 0$ through the achromat and demonstrate that it is an achromat by reproducing the figure of η_x vs s on slide 21.
   (c) Can a system like this be periodic in both β_x and β_y? That is, can you construct an achromatic bending "cell" out of just a DBA?
   (d) Add an extra defocusing quadrupole on each end of the DBA of a "reasonable" strength (e.g. with focusing length on the order of 2-3 times the spacing of the elements to avoid overfocusing), then a drift and a half a focusing quadrupole (with the same strength as the defocusing quadrupole) to each end. Use madx to find the periodic cell boundary conditions. Plot the periodic lattice functions $(\beta_x, \beta_y, \eta_x)$ for your new DBA lattice cell.

2. **The Henon Map with Variable Strength**

   `http://www.toddsatogata.net/2024-USPAS/homework/Henon2.html`

   Open the Henon map in the URL above and play with it a bit. Clicking near coordinates $(0, 0)$ will "launch" particles to be tracked through a simple motion that is similar to a linear ring with a single sextupole. The normalized coordinates (x, x') are displayed once per iteration of the map in a Poincaré section. This is among the simplest of nonlinear maps, a rotation with a kick, so it has been extensively studied by dynamicists. Set the number of iterations to 1000; this will track 1000 "turns" for each launched particle.

   (a) Set $b_2 = 0$ and track a particles for several different tune values Q. Why does the motion always appear pretty much the same? What happens when you set the tune to a low-order rational number, like 0.25?
   (b) Keep $Q = 0.25$ and start raising b_2. Around what values of b_2 do you start seeing something new happening for large-amplitude particles? Are the tunes for these particles increasing or decreasing from 0.25?
   (c) Set the tune near $Q = 1/3$ and raise the sextupole strength to about $b_2 = 0.2$. Compare the motion to motino that we discussed in class. What happens when you move the tune to the other side of $Q = 1/3$, and why?

(d) Set the tune to $Q = 0.252$ and gradually raise b_2 from near zero by increments of 0.2 from 0 to 2. How does the phase space change? Bonus: what order in sextupole strength drives the observed resonance?

(e) Set the sextupole strength to $b_2 = 0.5$ and find the separatrix. Change the number of iterations to 100 and track near and away from the separatrix. From this can you infer the period of the particle motion around the separatrix?

3. **The Henon Map with Damping**

   `http://www.toddsatogata.net/2024-USPAS/homework/Henon3.html`

   Open the damped Henon map example in the URL above and play with it a bit. In electron rings, there is synchrotron radiation damping, so particles tend to damp to the closed orbit. This can be included in the nonlinear dynamics simulation, along with sources of noise, to show how particles can damp into resonance islands instead of onto the closed orbit.

   (a) Set the tune Q to 0.254 and δ to 0.001. Launch a few particles to observe that you have damping turned on. Here the damping delta is very strong.

   (b) Gradually raise b_2 from near zero by increments of 0.2 from 0–2. How does the phase space change now? Note that the resonance islands can become attractors even in the presence of relatively strong damping, so they are really stable fixed points.

4. Chirikov Resonance Overlap and Beam-Beam

   `http://www.toddsatogata.net/2024-USPAS/homework/BeamBeam.html`

   Open the beam-beam map example in the URL above and play with it a bit. This is a simplified model of the beam-beam interaction of two beams as we will discuss in class; you can think of it as a nonlinear kick from an oncoming Gaussian beam. Keeping the tune at $Q = 0.331$, set the beam-beam tune shift ξ to zero, and click within the plot area to launch particles and produce Poincaré plots. With no nonlinearity, these are all horizontal lines, consistent with constant action.

   (a) Produce phase space plots by gradually increasing ξ by 0.001. With what ξ do you start to see resonance islands?

   (b) Vary ξ up to about 0.03. How do the resonance island locations and widths seem to scale with ξ? What other harmonics of phase space distortion appear at small, medium, and large amplitudes?

   (c) As ξ gets even larger, you will see more and more resonance islands appear at small amplitudes. These islands remain small, but at some point ξ is large enough that resonances start to overlap, and stochastic motion occurs consistent with the Chirikov overlap criterion. Experimentally find the lowest value of ξ where stochasticity occurs to two significant figures. (Hint: It's between 0 and 1.)