USPAS Accelerator Physics 2024
Hampton VA / Northern Illinois University

Introductions, Relativity, E&M, Accelerator Overview

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http://www.toddsatogata.net/2024-USPAS

Happy Birthday to Lev Landau (Nobel 1962), Alan Heeger (Nobel 2000), Lord Byron, and Guy Fieri!
Happy Come in From the Cold Day, Hot Sauce Day, and Answer Your Cat's Question Day!

Introductions and Outline

- A sign-in sheet is being passed around
- Introductions: Getting to know you, and us…
- Outline of this lecture
  - Course administrivia
  - Relativistic mechanics review
  - Relativistic E&M review, Cyclotrons
  - Survey of accelerators and accelerator concepts

Syllabus I

Date	Chapter/Who	Slides	Topic
M Jan 22 AM	1,2 / Todd	[9.1 Mb pdf]	Introductions, Relativity, Relativistic E&M, Linear Motion
M Jan 22 PM	2 / Steve		Linear Motion and Stability
T Jan 23 AM	3 / Todd	[10.2 Mb pdf]	Strong Focusing Transverse Optics
T Jan 23 PM	4 / Steve		Longitudinal and Off-Momentum Motion
W Jan 24 AM	5 / Steve		Emittances and Phase Space
W Jan 24 PM	6 / Todd	[5.5 Mb pdf]	Magnets and Magnet Design
H Jan 25 AM	7 / Steve		RF Cavities
H Jan 25 PM	8 / Steve		Linear Errors and Corrections
F Jan 26 AM	-- / Todd		Lattice Exercises and Insertions I
F Jan 26 PM	-- / Todd		Lattice Exercises and Insertions II

- First two weeks: Mostly transverse linear optics
  - Fundamentals and 6D equations of motion
  - Magnet design, fields, descriptions
  - Linear transverse optics and explorations with madx
  - Magnetic lattices and lattice design

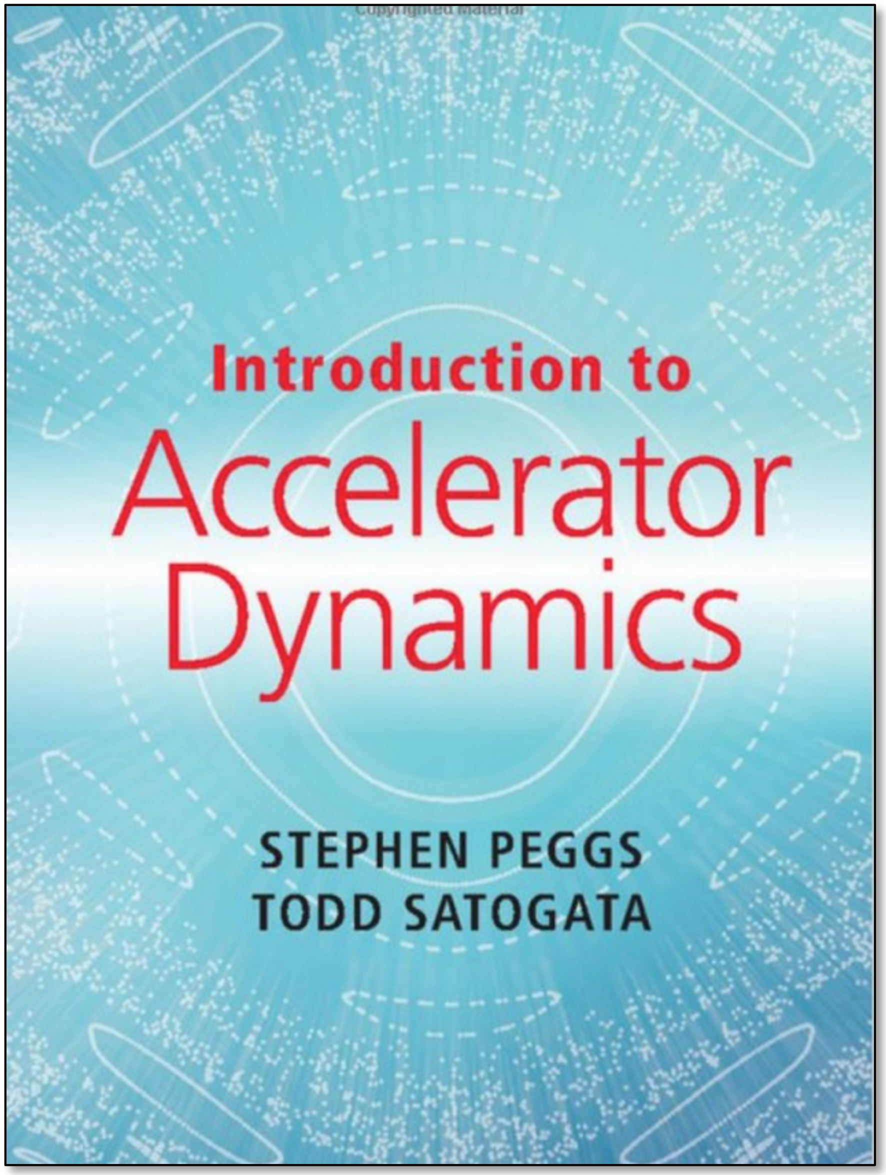
Syllabus II

Date	Chapter/Who	Slides	Topic
M Jan 29 AM	9 / Steve		Sextupoles and Chromaticity
M Jan 29 PM	10 / Todd		Octupoles, Detuning, and Slow Extraction
T Jan 30 AM	11 / Steve		Synchrotron Radiation -- Classical Damping
T Jan 30 PM	12+ / Todd		Synchrotron Radiation -- Quantum Excitation and Sync Light Facility Lattices
W Jan 31 AM	-- / Medani/Todd		Introduction to Impedances/Instabilities
W Jan 31 PM	13 / Steve		Linacs: Protons and Ions
H Feb 1 AM	15 / Steve		Beam-Beam Interaction: 1-D Resonances
H Feb 1 PM	16 / Todd		Nonlinear Dynamics and Chaos
F Feb 2 AM	14+ / Alex/Todd		Linacs: Electrons/ERLs/FFAs

- Second week: Everything else ☺
  - Synchrotron radiation and cooling
  - Introduction to instabilities/impedances
  - Nonlinear dynamics and collective effects
  - **We have flexibility to improvise and adjust**
    - e.g. brief student presentations instead of NL dynamics?

Class Textbook and Grading

- Peggs and Satogata
  - 1st edition
- We will cover quite a bit of this text
- One advantage over other texts:
  - authors are available for consultation
  - correction... complaint...
- Grading
  - 60% homework
  - 20% labs
  - 20% participation
  - No final exam

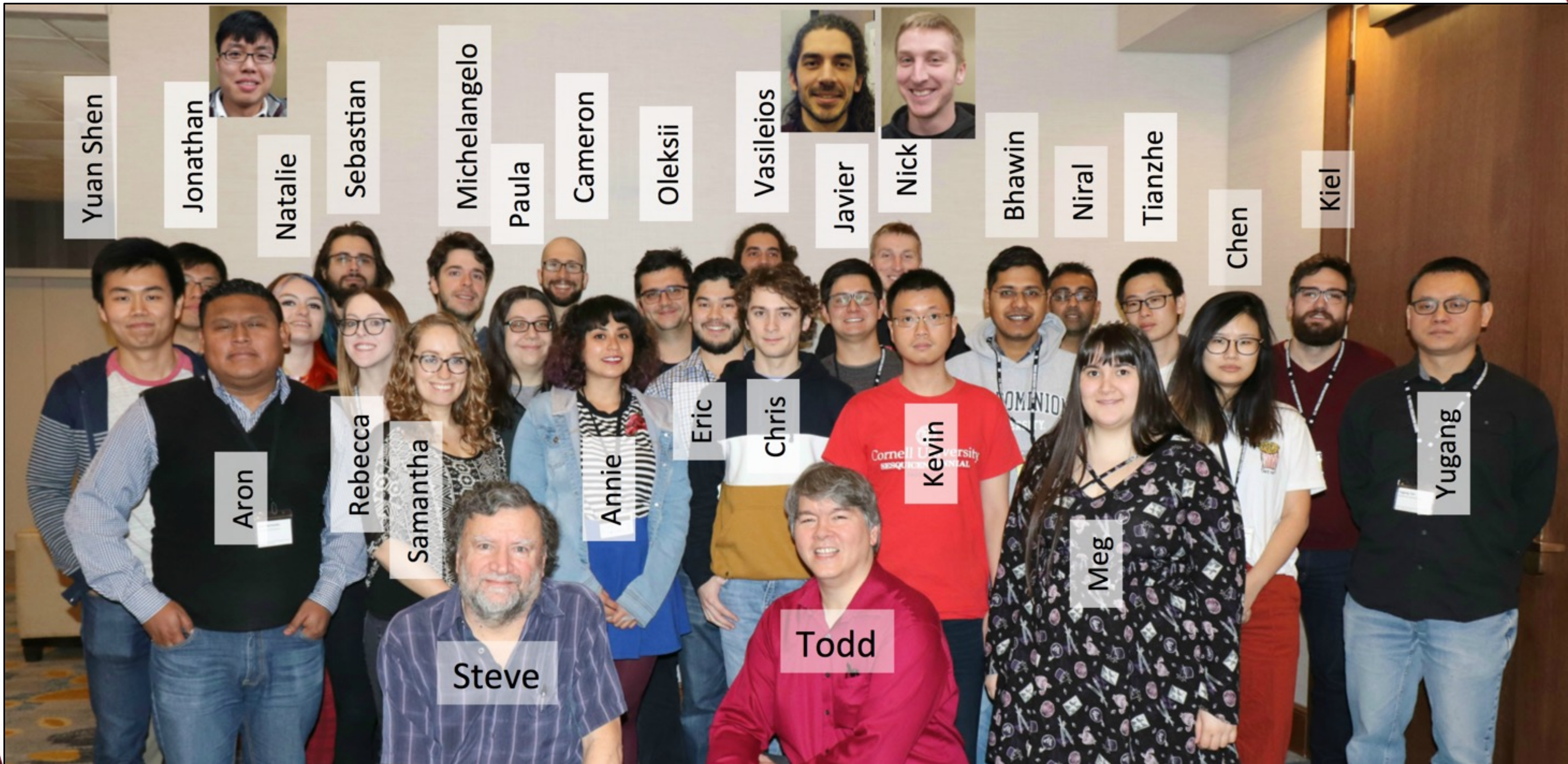


Homework and Schedule

- Homework is over half your grade!!
  - Medani and Alex are grading – be nice to them ☺
  - Collected at **start** of every morning class
    - hard copy please so Medani and Alex can scribble feedback
  - Their homework is to get it back to you by the next day
  - Lectures/lab times will run 09:00-12:00(ish), 13:30-16:30(ish)
    - One hour is a bit too short for walking to/from lunch
- Collaboration is encouraged! (Note: No final exam!)
  - In fact, it's a good part of the reason why you're here!
  - At least one of us will be available in the evenings
    - **Todd** is commuting but will often be here through dinner
- Please cite references, contributions of teammates, etc
  - Everyone must hand in **individual copies** of homework
  - Citations must include if you use **generative AI**, e.g. GPT4 etc

Growth Comes From Effort

Survival Existence Proof



January 2019 USPAS Graduate Accelerator Physics Course
Knoxville, TN

(January 2021 was all online)

Jean-Antoine Nollet

In 1746 he gathered about **two hundred monks** into a circle over a mile in circumference, with pieces of iron wire connecting them. He then discharged a battery of Leyden jars through the human chain and observed that **each man reacted at substantially the same time to the electric shock**, showing that the speed of electricity's propagation was very high.

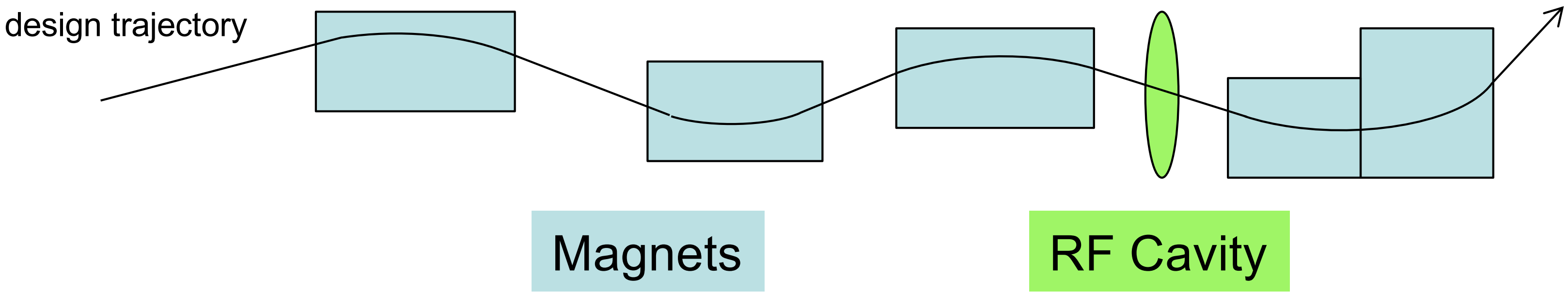
The Monkotron

- Nollet had
  - lots of charged particles
  - moving in a confined 2km ring (!)
  - at very high velocities
  - accelerated by high voltage

- Nollet didn't have
  - controlled magnets
  - controlled acceleration
  - proper instrumentation
  - many friends after this experiment

http://www.yproductions.com/writing/archives/twitch_token_of_such_things.html

Simplified Particle Motion



- Design trajectory
  - Particle motion will be perturbatively expanded around a **design trajectory** or **orbit**
  - This orbit can be over 10^{10} km in a storage ring
- Separation of fields: Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
  - Magnetic fields from static or slowly-changing magnets
    - transverse to design trajectory $\hat{x}, \hat{y}$
  - Electric fields from high-frequency RF cavities
    - in direction of design trajectory $\hat{s}$
- Relativistic charged particle velocities

Relativity Review

- Accelerators: applied E&M and special relativity
- Relativistic parameters:

$$\beta \equiv \frac{v}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \sqrt{1 - 1/\gamma^2}$$

  - Later β and γ will also be used for other quantities, but the context should usually make them clear
  - γ=1 (classical mechanics) to ~2.04×10^5 (to date) (where??)

- Total energy U, momentum p, and kinetic energy W

$$U = \gamma mc^2 \qquad p = (\beta\gamma)mc = \beta\left(\frac{U}{c}\right) \qquad W = (\gamma - 1)\, mc^2$$

Relative Relativity

WolframAl

LEP energy

Input interpretation:

LEP (Large Electron Positron Collider) ce

Result:

208 GeV (gigaelectronvolts)

Unit conversions:

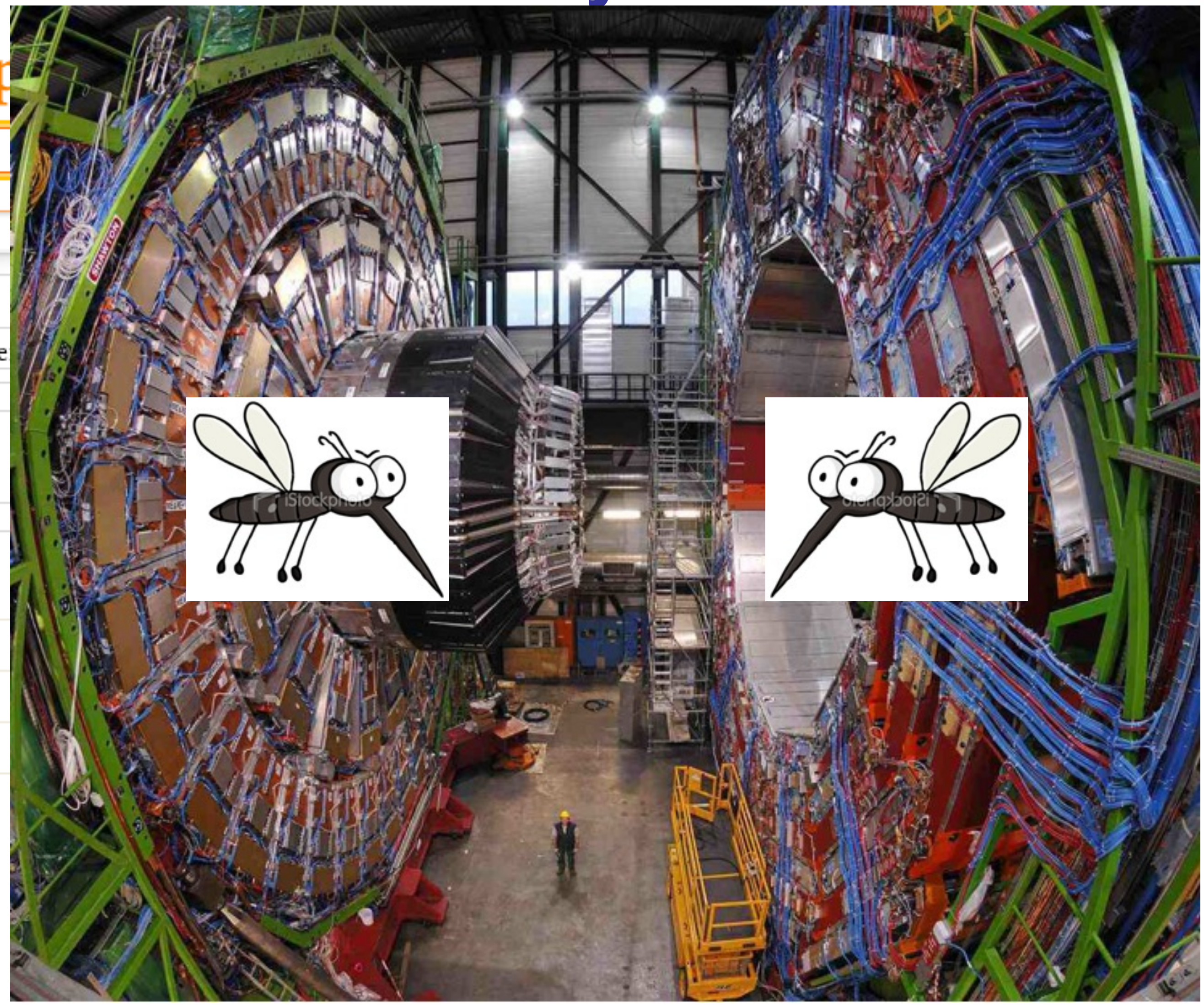
0.208 TeV (teraelectronvolts)

2.08×10^{11} eV (electronvolts)

0.03333 μJ (microjoules)

3.333×10^{-8} J (joules)

0.3333 ergs
(unit officially deprecated)



Comparisons as energy:

≈ (0.21 ≈ 1/5) ×
approximate kinetic energy of a flying mosquito ($\approx 1.6 \times 10^{-7}$ J)

≈ 2.2 × mass-energy equivalent of a Z boson ($\approx 1.5 \times 10^{-8}$ J)

Convenient Units

$$1\,\mathrm{eV} = (1.602\times10^{-19}\,\mathrm{C})(1\,\mathrm{V}) = 1.602\times10^{-19}\,\mathrm{J}$$

$$1\,\mathrm{MeV} = 1.602\times10^{-13}\,\mathrm{J}$$

$$1\,\mathrm{GeV} = 1.602\times10^{-10}\,\mathrm{J}$$

- How much is a TeV?
  - Energy to raise 1g about 16 μm against gravity
  - Energy to power 100W light bulb 1.6 ns
- But many accelerators have 10^{10-12} particles
  - Single bunch "instantaneous power" of tens of Terawatts
- Highest energy cosmic ray (1991)
  - ~300 EeV (3×10^{20} eV or 3×10^{8} TeV!) OMG particle

(125 g hamster at 100 km/hr)

Recent Records and Extremes

- 290 TeV ***neutrino*** observed at IceCube (Sep 22 2017)
  - Science, 13 Jul 2018, **361** pp. 147-151
  - Blazar neutrino factories: Sara Buson et al. 2022 ApJL 933 L43

- CERN FCC conceptual design reports
  - Published 15 Jan 2019: https://fcc-cdr.web.cern.ch
  - 90-365 GeV electron-positron collider (post-LEP)

- Future Planck-scale Ultimate Colliders: >10^{14} TeV
  - Steven Brooks (BNL) invited talk at IPAC'18
  - http://accelconf.web.cern.ch/AccelConf/ipac2018/papers/tuxgbd1.pdf

Relativity Review (Again)

- Accelerators: applied E&M and special relativity
- Relativistic parameters:

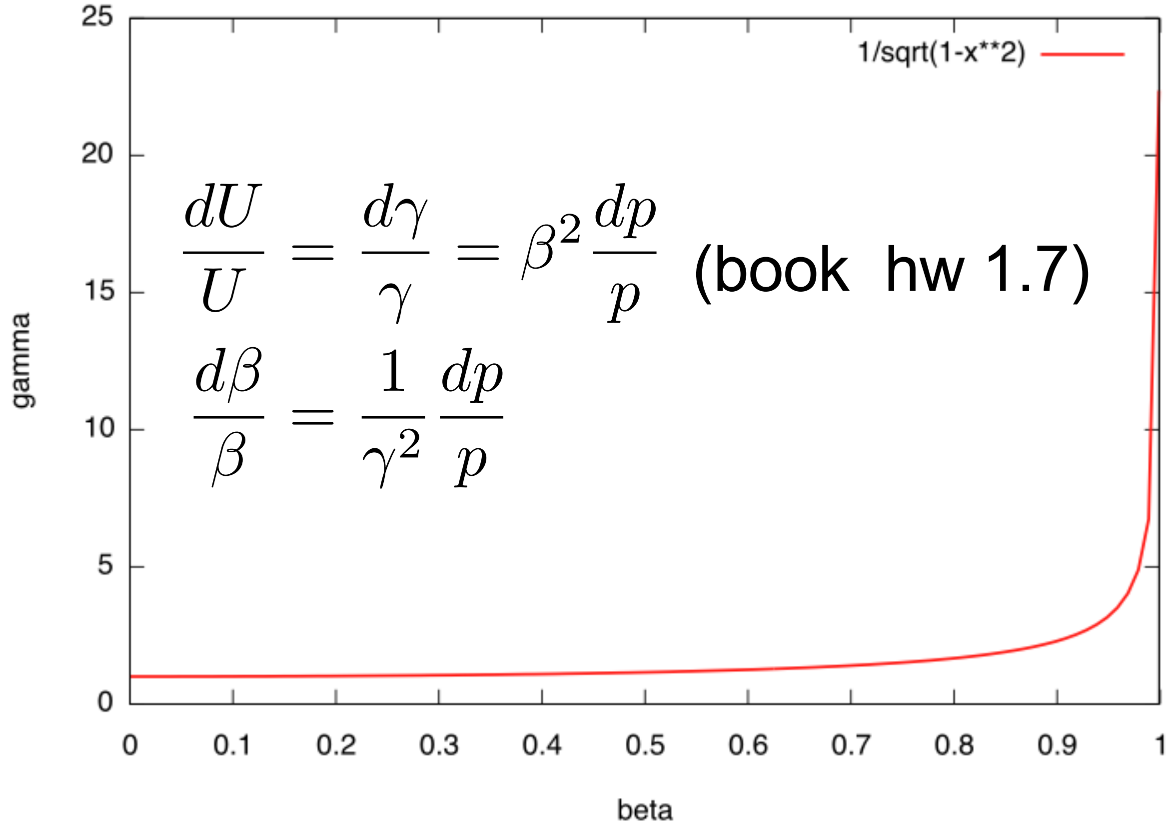
$$\beta \equiv \frac{v}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \sqrt{1 - 1/\gamma^2}$$

  - Later β and γ will also be used for other quantities, but the context should usually make them clear
  - γ=1 (classical mechanics) to ~2.04×10^5 (oh yeah, at LEP)

- Total energy U, momentum p, and kinetic energy W

$$U = \gamma mc^2 \qquad p = (\beta\gamma)mc = \beta\left(\frac{U}{c}\right) \qquad W = (\gamma - 1)\, mc^2$$

Convenient Relativity Relations



$$\frac{dU}{U} = \frac{d\gamma}{\gamma} = \beta^2 \frac{dp}{p}$$ (book hw 1.7)

$$\frac{d\beta}{\beta} = \frac{1}{\gamma^2}\frac{dp}{p}$$

- Differential relations are fun to derive, hold for all γ
- In "ultra" relativistic limit $\beta \approx 1$
  - Usually must be careful below $\gamma \approx 5$ or $U \approx 5\ mc^2$
    - For high energy electrons this is only $U \approx 2.5$ MeV
  - Many accelerator physics phenomena scale with γ^k or $(\beta\gamma)^k$

(Frames and Lorentz Transformations)

- The lab frame will dominate most of our discussions
  - But not always (synchrotron radiation, space charge…)

- Invariance of space-time interval (Minkowski)

$$(ct')^2 - x'^2 - y'^2 - z'^2 = (ct)^2 - x^2 - y^2 - z^2$$

- Lorentz transformation of four-vectors
  - For example, time/space coordinates in z velocity boost

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

(Four-Velocity and Four-Momentum)

- The proper time interval $d\tau = dt/\gamma$ is Lorentz invariant
- So we can make a velocity 4-vector

$$cu^\alpha \equiv \left(\frac{dct}{d\tau}, \frac{dx}{d\tau}, \frac{dy}{d\tau}, \frac{dz}{d\tau} \right) = c\gamma(1, \beta_x, \beta_y, \beta_z)$$

$$\text{Metric } \; g^{\mu\nu} = g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$$

- We can also make a 4-momentum

$$p^\alpha \equiv mcu^\alpha = mc\gamma(1, \beta_x, \beta_y, \beta_z)$$

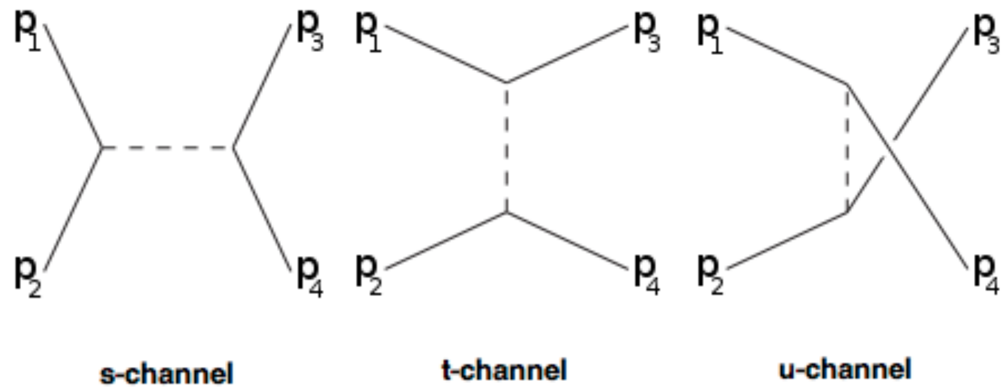
- Double-check that Minkowski norms are invariant

$$u^\alpha u_\alpha = u^\alpha g_{\alpha\beta} u^\beta = \gamma^2(1 - \beta^2) = 1$$

$$p^\alpha p_\alpha = m^2 c^2 u^\alpha u_\alpha = m^2 c^2$$

(Mandelstam Variables)

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$$



$$s + t + u = (m_1^2 + m_2^2 + m_3^2 + m_4^2)c^2$$

- Lorentz-invariant two-body kinematic variables
  - $p_{1\text{-}4}$ are four-momenta
- $\sqrt{s}$ is the total available center of mass energy
  - Often quoted for colliders
- Used in calculations of other two-body scattering processes
  - Moller scattering (e-e), Compton scattering (e-γ)

(Relativistic Newton)

$$\vec{F} = m\vec{a} = \frac{d\vec{p}}{dt}$$

- But now we can define a four-vector force in terms of four-momenta and proper time:

$$F^\alpha \equiv \frac{dp^\alpha}{d\tau}$$

- We are primarily concerned with electrodynamics so now we must make the classical electromagnetic Lorentz force obey Lorentz transformations

$$\boxed{\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)}$$

(book 1.8)

(Relativistic Electromagnetism)

- Classical electromagnetic potentials can be shown to combine to a four-potential (with c=1):

$$A^\alpha \equiv (\Phi, \vec{A})$$

- The field-strength tensor is related to the four-potential

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

  - E/B fields Lorentz transform with factors of γ, ($\beta\gamma$)

(Lorentz Lie Group Generators)

- Lorentz transformations can be described by a Lie group where a general Lorentz transformation is

$$A = e^{L} \qquad \det A = e^{\mathrm{Tr}\, L} = +1$$

where L is 4x4, real, and traceless. With metric g, the matrix gL is also antisymmetric, so L has the general six-parameter form

$$L = \begin{pmatrix} 0 & L_{01} & L_{02} & L_{03} \\ L_{01} & 0 & L_{12} & L_{13} \\ L_{02} & -L_{12} & 0 & L_{23} \\ L_{03} & -L_{13} & -L_{23} & 0 \end{pmatrix}$$

Deep and **profound** connection to EM tensor $F^{\alpha\beta}$

J.D. Jackson, Classical Electrodynamics 2nd Ed, Section 11.7

Relativistic Electromagnetism II

- The relativistic electromagnetic force equation becomes

$$\frac{dp^\alpha}{d\tau} = m\frac{du^\alpha}{d\tau} = \frac{q}{c}F^{\alpha\beta}u_\beta$$

- Thankfully we can write this in somewhat simpler terms

$$\frac{d(\gamma m\vec{v})}{dt} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

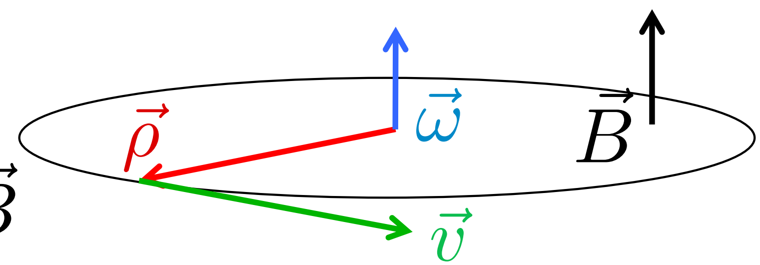
- That is, "classical" E&M force equations hold if we treat the momentum as relativistic, $\vec{p} = \gamma m\vec{v} = \gamma\vec{\beta}mc$
- If we dot in the velocity, we get relative energy transfer

$$\frac{d\gamma}{dt} = \frac{q\vec{E}\cdot\vec{v}}{mc^2}$$

- Unsurprisingly, we can only get energy changes from electric fields, not (conservative) magnetic fields

Constant Magnetic Field (Zero Electric Field)

- In a constant magnetic field, charged particles move in circular arcs of radius ρ with constant angular velocity ω:



$$\vec{F} = \frac{d}{dt}(\gamma m \vec{v}) = \gamma m \frac{d\vec{v}}{dt} = q\vec{v} \times \vec{B}$$

$$\vec{v} = \vec{\omega} \times \vec{\rho} \qquad \Rightarrow \qquad q\vec{v} \times \vec{B} = \gamma m \vec{\omega} \times \frac{d\vec{\rho}}{dt} = \gamma m \vec{\omega} \times \vec{v}$$

  - For $\vec{B} \perp \vec{v}$ we then have

$$qvB = \frac{\gamma m v^2}{\rho} \qquad p = \gamma m(\beta c) = q(B\rho) \qquad \frac{p}{q} = (B\rho)$$

$$\omega = \frac{v}{\rho} = \frac{qB}{\gamma m}$$

Rigidity: Bending Radius vs Momentum

Beam

$$\frac{p}{q} = (B\rho)$$

Accelerator (magnets, geometry)

$$p = \beta\gamma mc$$

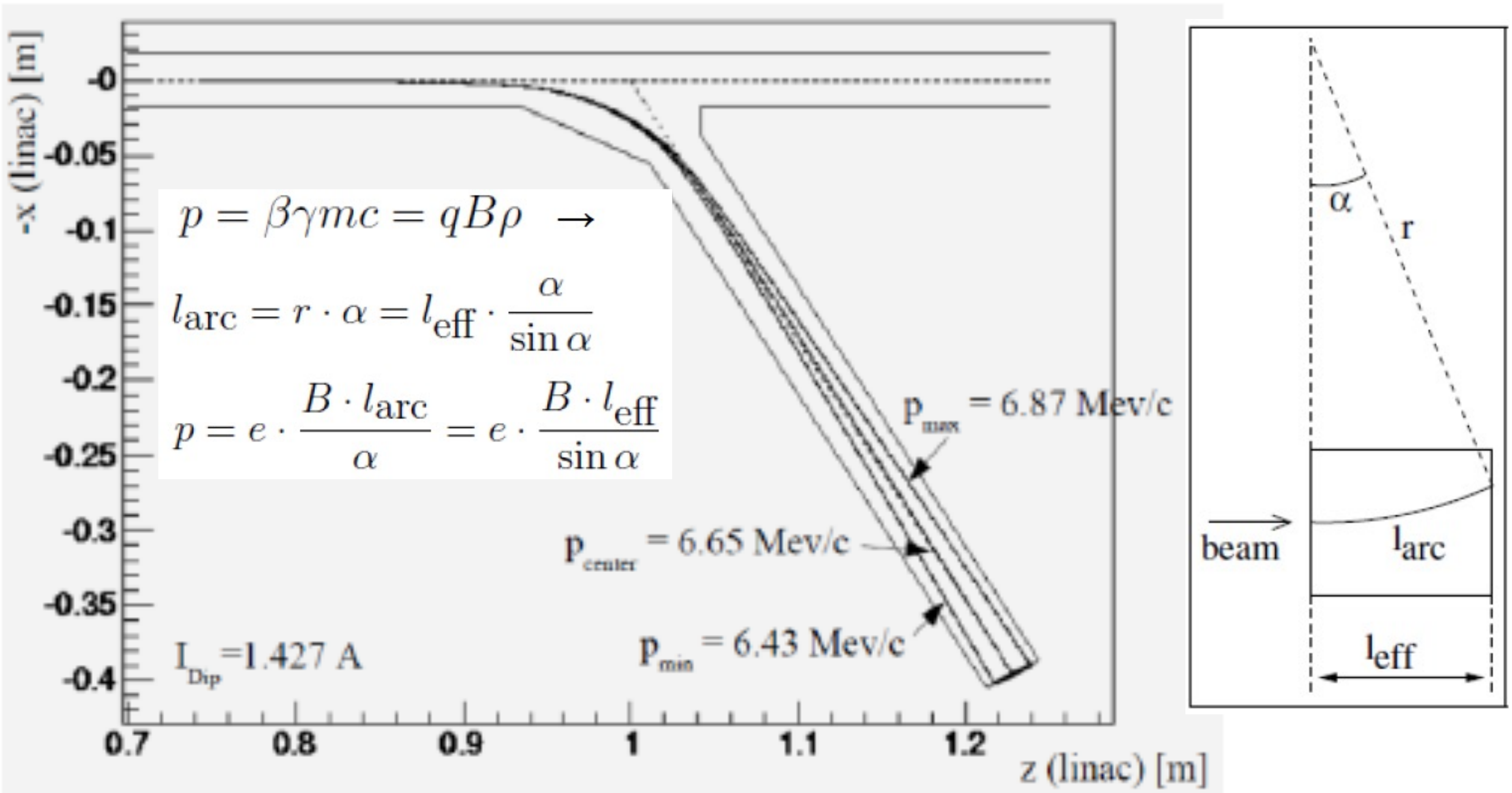
- This is such a useful expression in accelerator physics that it has its own name: **rigidity**
- Ratio of momentum to charge
  - How hard (or easy) is a particle to deflect?
  - Often expressed in [T-m] (easy to calculate B)
  - Be careful when q≠e!! (Homework question 2)
- A useful expression in "natural" units

$$\frac{p[\mathrm{GeV/c}]}{q[e]} \approx 0.3\, B[\mathrm{T}]\, \rho[\mathrm{m}]$$

Google for coefficient:
(1 Tesla)*(1 m)*(1 elementary charge) in GeV/c

Application: Particle Spectrometer

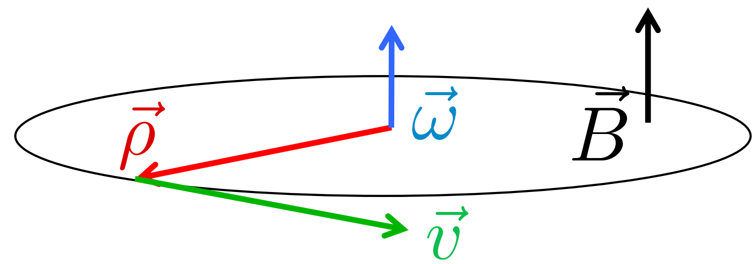
- Identify particle momentum by measuring bend angle α from a calibrated magnetic field B



Yujong Kim, 2010 accelerator physics lectures

Cyclotron Frequency

$$\omega = \frac{v}{\rho} = \frac{qB}{\gamma m}$$



- Another very useful expression for particle angular frequency in a constant field: **cyclotron frequency**

- In the nonrelativistic approximation

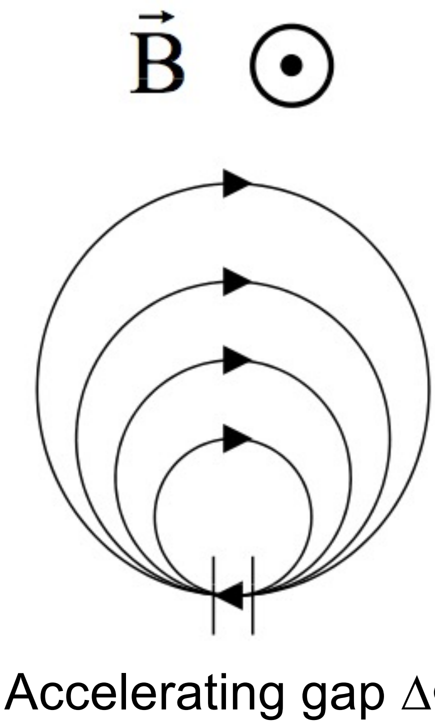
$$\omega_{\text{nonrelativistic}} \approx \frac{qB}{m}$$

Revolution frequency is independent of radius or energy!

Lawrence and the Cyclotron

Ernest Orlando Lawrence

- Can we repeatedly spiral and accelerate particles through the same potential gap?



Cyclotron Frequency Again

- Recall that for a constant B field

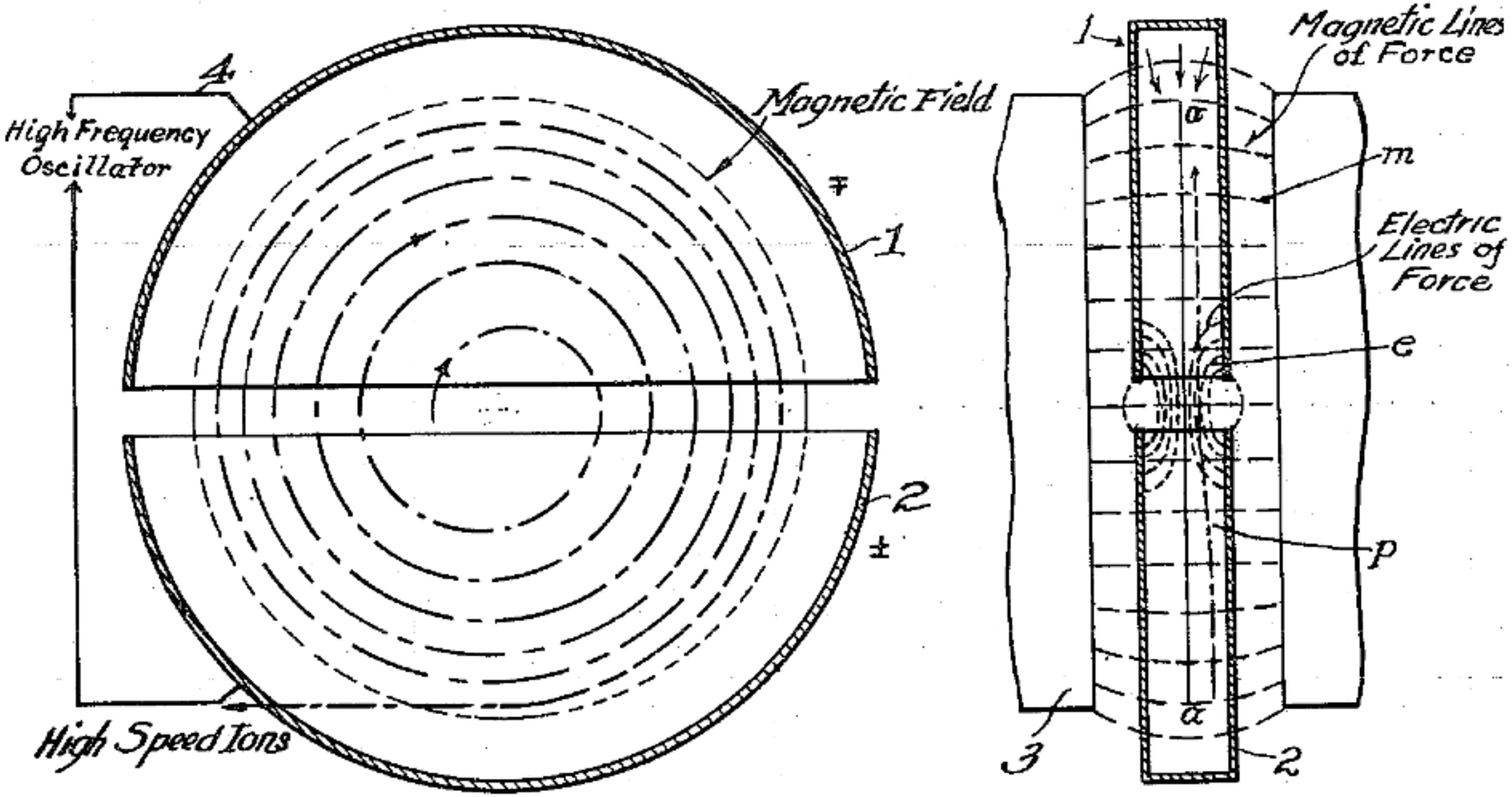
$$p = \gamma m v = q(B\rho) \quad \Rightarrow \quad \rho = \left(\frac{\gamma m}{qB}\right) v$$

  - Radius/circumference of orbit scale with velocity
    - Circulation time (and frequency) are independent of v
  - Apply AC electric field in the gap at frequency f_{rf}
    - Particles accelerate until they drop out of resonance

$$\omega = \frac{v}{\rho} = \frac{qB}{\gamma m} \qquad f_{\mathrm{rf}} = \frac{\omega}{2\pi} = \frac{qB}{2\pi\gamma m}$$

    - Note a first appearance of "bunches", not DC beam
    - Works best with heavy particles (hadrons, not electrons)

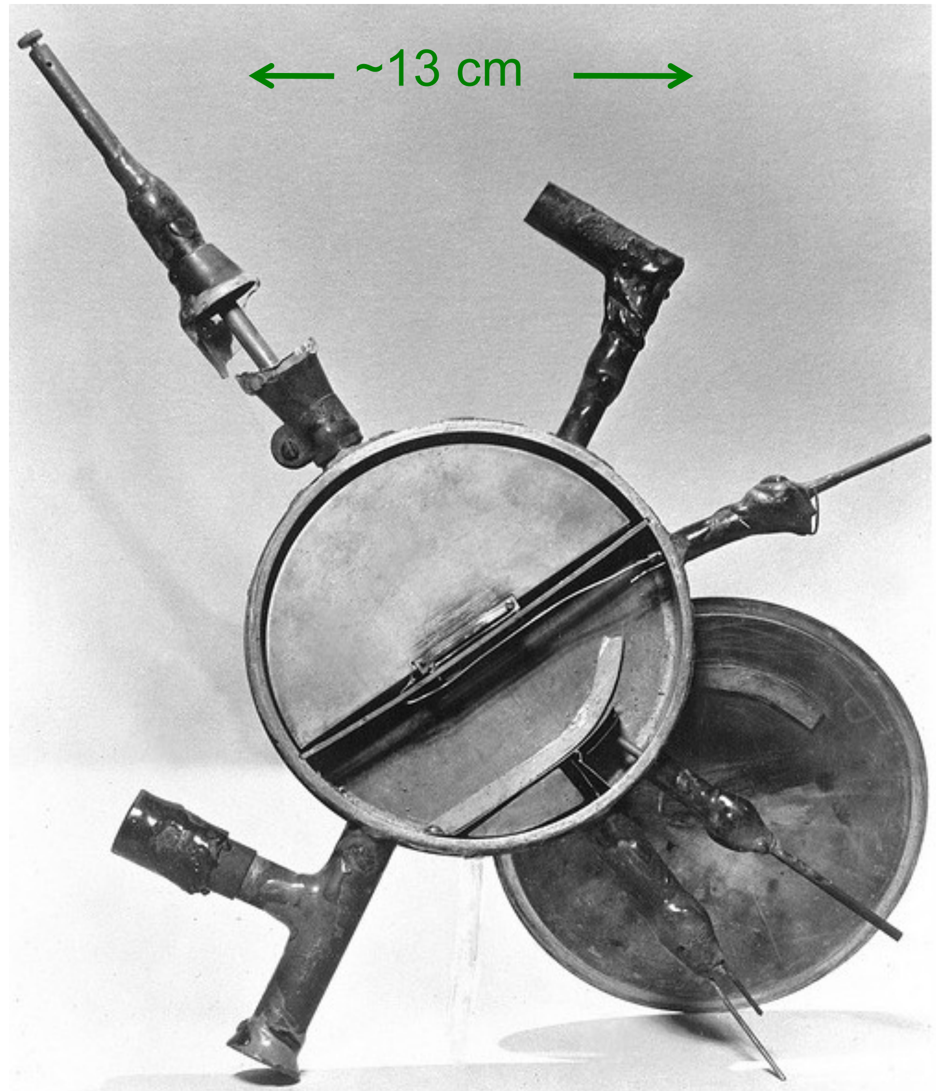
A Patentable Idea



- 1934 patent 1948384
  - Two accelerating gaps per turn!

All The Fundamentals of an Accelerator

- Large static magnetic fields for guiding (~1T)
  - But no vertical focusing
- HV RF electric fields for accelerating
  - (No phase focusing)
  - (Precise f control)
- p/H source, injection, extraction, vacuum
- 13 cm: 80 keV
- 28 cm: 1 MeV
- 69 cm: ~5 MeV
- … 223 cm: ~55 MeV



(Berkeley)

Livingston, Lawrence, 27”/69 cm Cyclotron

M.S. Livingston and E.O. Lawrence, 1934

The Joy of Physics

- Describing the events of January 9, 1932, Livingston is quoted saying:

"I recall the day when *I had adjusted the oscillator to a new high frequency*, and, with *Lawrence looking over my shoulder*, tuned the magnet through resonance. As the galvanometer spot swung across the scale, indicating that protons of 1-MeV energy were reaching the collector, *Lawrence literally danced around the room with glee*. The news quickly spread through the Berkeley laboratory, and we were busy all that day demonstrating million-volt protons to eager viewers."

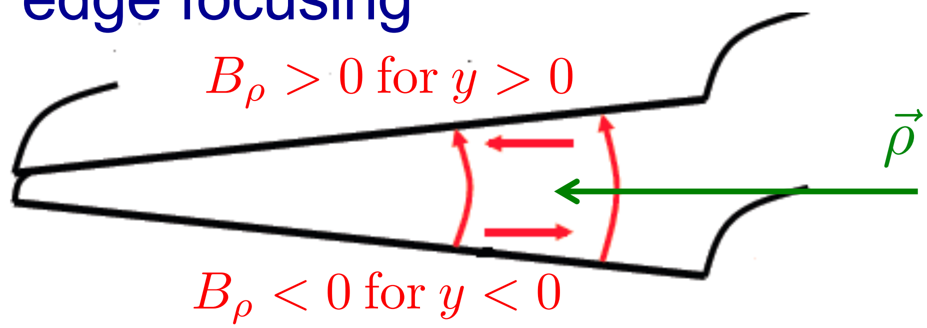
APS Physics History, Ernest Lawrence and M. Stanley Livingston

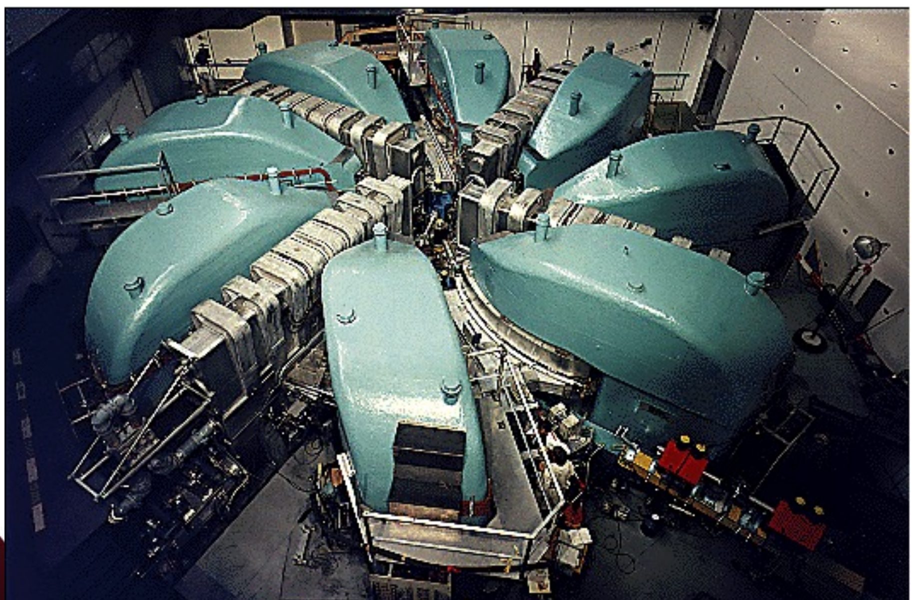
Modern Isochronous Cyclotrons

- Higher bending field at higher energies
  - But also introduces vertical defocusing
  - Use bending magnet "edge focusing"

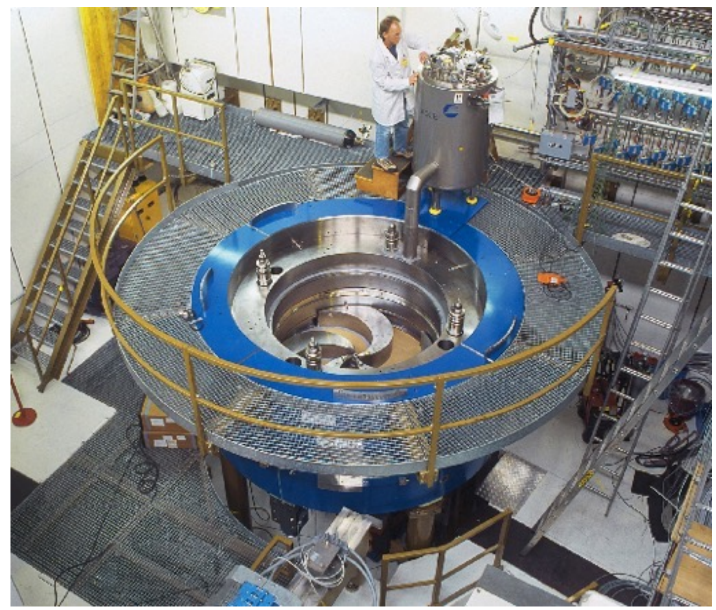
    (Weds magnet lecture)

$$f_{\rm rf} = \frac{qB(\rho)}{2\pi\gamma(\rho)m}$$





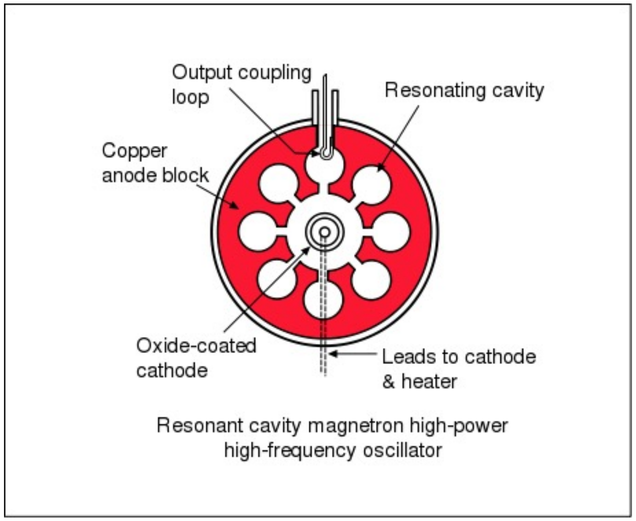
590 MeV PSI Isochronous Cyclotron (1974)

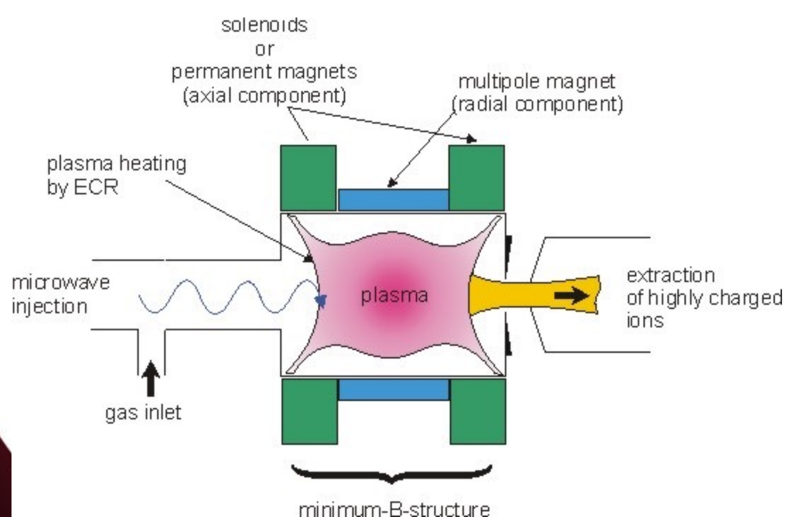


250 MeV PSI Isochronous Cyclotron (2004)

Electrons, Magnetrons, ECRs

Radar/microwave magnetron





ECR plasma/ion source

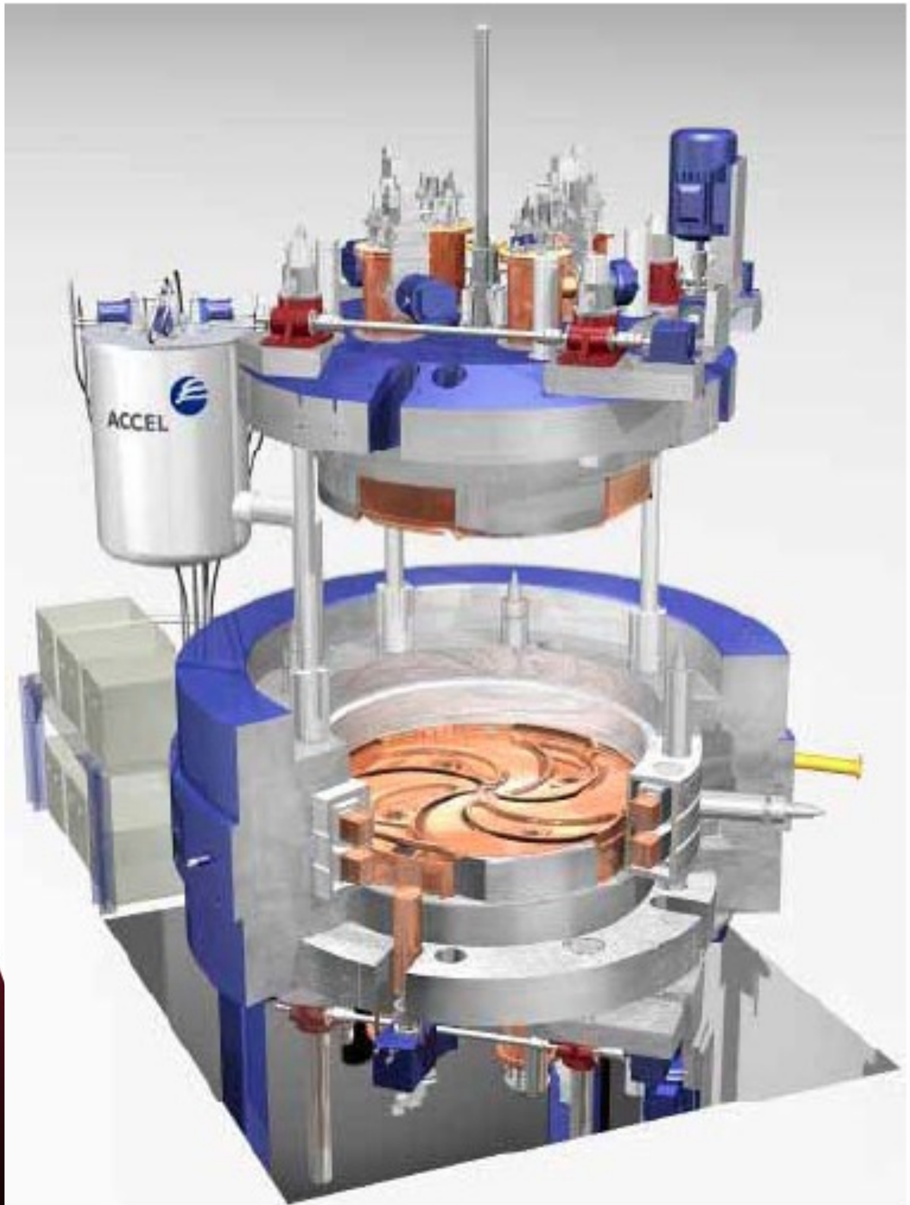
- Cyclotrons aren't good for accelerating electrons
  - Very quickly relativistic!
- But narrow-band response has advantages and uses
  - Magnetrons

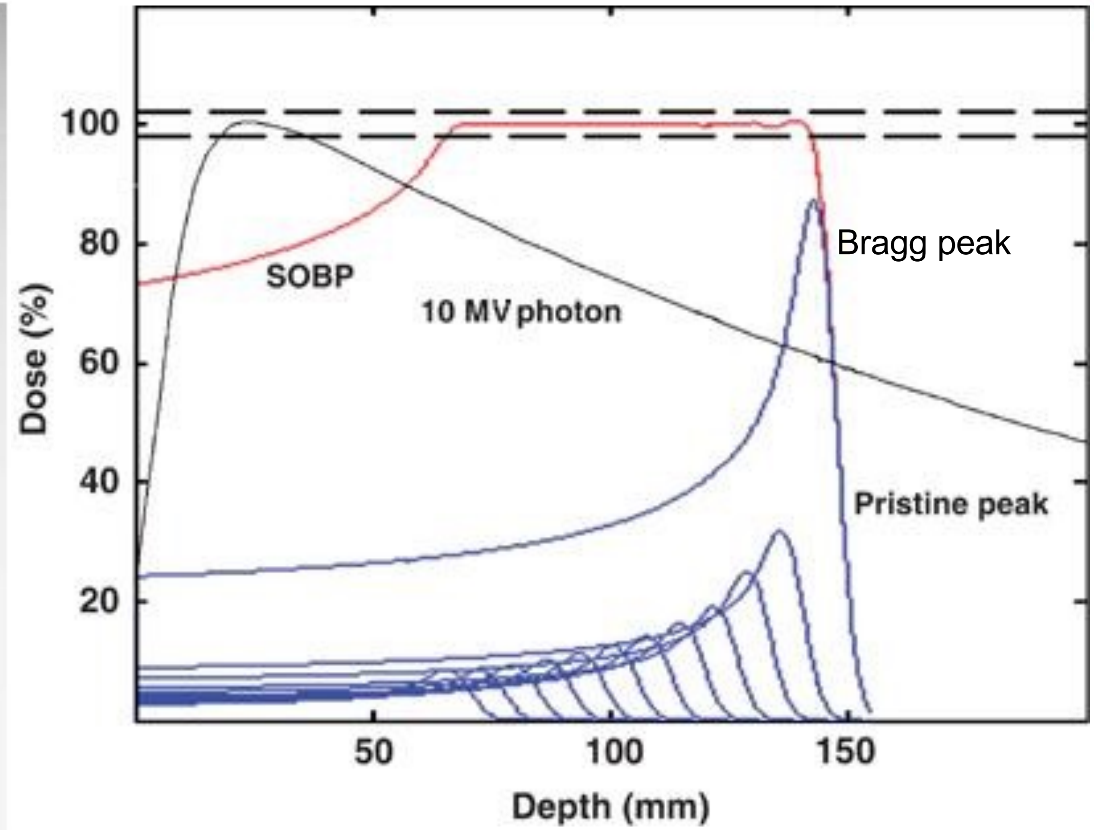
    generate resonant high-power microwaves from circulating electron current
  - ECRs
    - generate high-intensity ion beams and plasmas by resonantly stripping electrons with microwaves

Cyclotrons Today

- Cyclotrons continue to evolve
  - Many contemporary developments
    - Superconducting cyclotrons
    - Synchrocyclotrons (FM modulated RF)
    - Isochronous/Alternating Vertical Focusing (AVF)
    - FFAGs (Fixed Field Alternating Gradient)
  - Versatile with many applications even below ~500 MeV
    - High power (>1MW) neutron production
    - Reliable (medical isotope production, ion radiotherapy)
    - Power+reliability: ~5 MW p beam for ADSR (accelerator driven subcritical reactors, e.g. Thorium reactors)

Accel Radiotherapy Cyclotron





Distinct dose localization advantage for hadrons over X-rays

Also present work on proton and carbon radiotherapy fast-cycling synchrotrons

=========== Break ===========

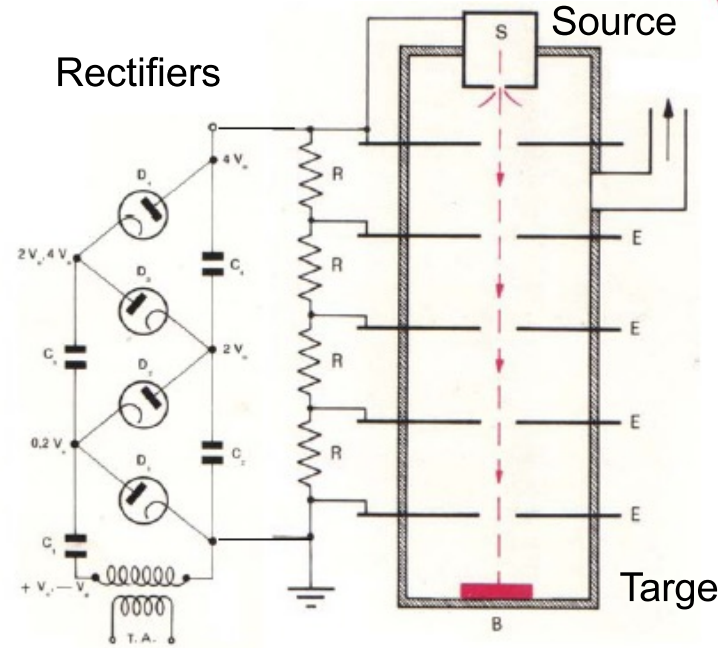
(Brief) Survey of Accelerator Concepts

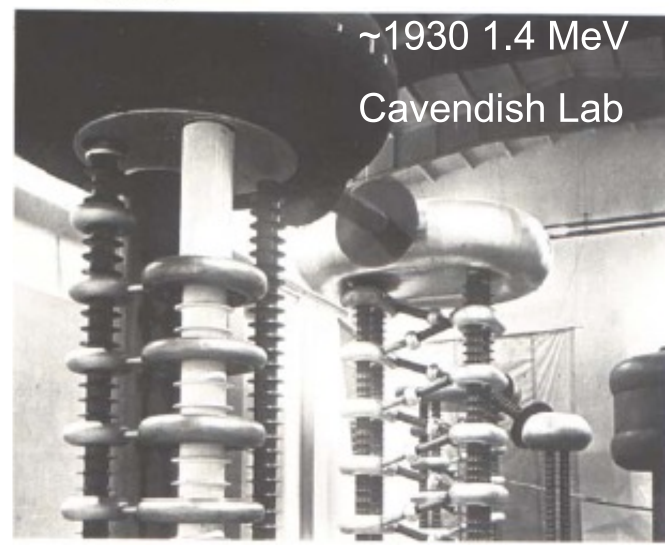
- Producing accelerating gaps and fields (DC/AC)
- Microtrons and their descendants
- Betatrons (and betatron motion)
- Synchrotrons
  - Fixed Target Experiments
  - Colliders and Luminosity (Livingston Plots)
  - Light Sources (FELs, Compton Sources)

- Others include
  - Medical Applications (radiotherapy, isotope production)
  - Spallation Sources (SNS, ESS)
  - Power Production (ADSR)

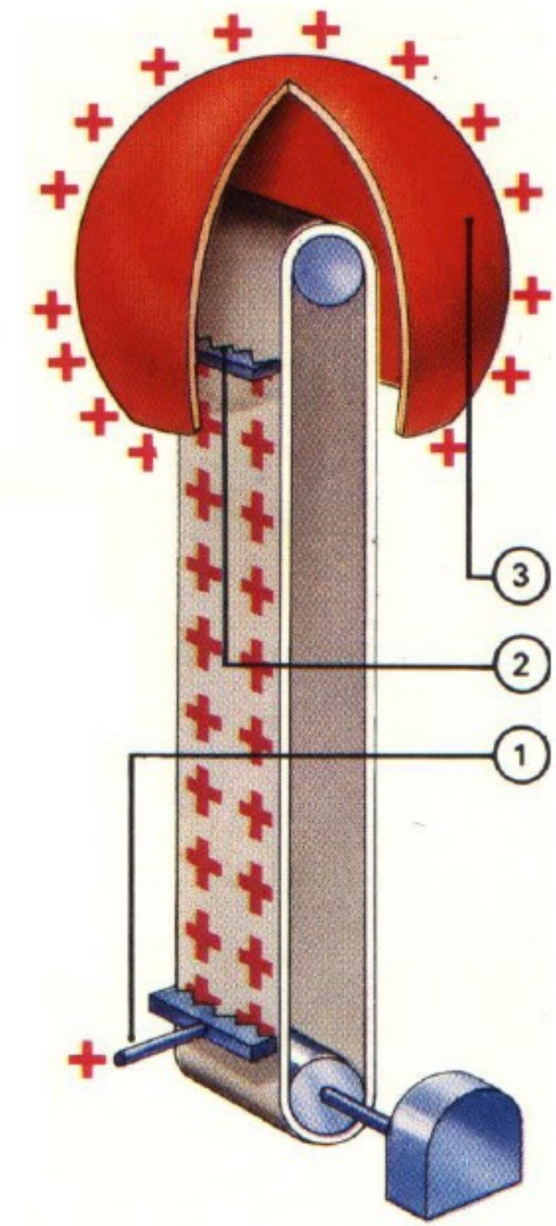
DC Accelerating Gaps: Cockcroft-Walton

- Accelerates ions through successive electrostatic voltages
  - First to get protons to >MeV
  - Continuous HV applied through intermediate electrodes
  - Rectifier-multipliers (voltage dividers)
  - Limited by HV sparking/breakdown
  - FNAL still uses a 750 kV C-W
- Also example of early ion source
  - H gas ionized with HV current
  - Provides high current DC beam





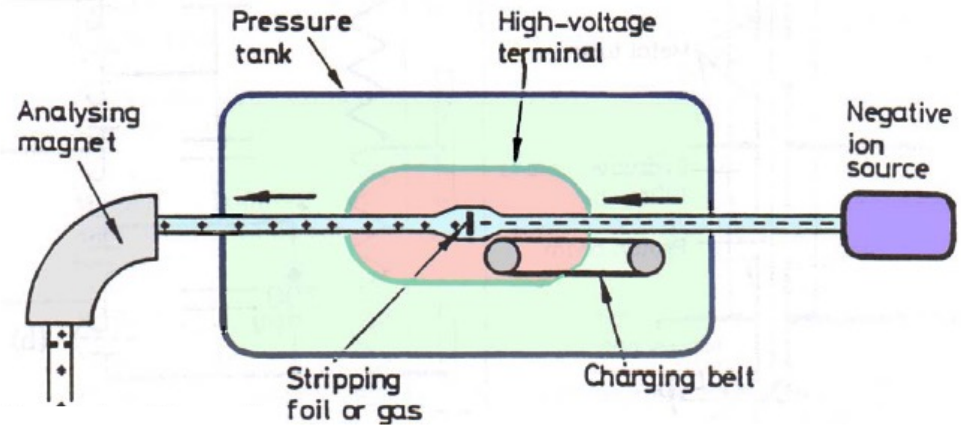
DC Accelerating Gaps: Van de Graaff



- How to increase voltage?
  - R.J. Van de Graaff: charge transport
  - Electrode (1) sprays HV charge onto insulated belt
  - Carried up to spherical Faraday cage
  - Removed by second electrode and distributed over sphere
- Limited by discharge breakdown
  - ~2MV in air
  - Up to 20+ MV in SF_6!
  - Ancestors of Pelletrons (chains)/Laddertrons (stripes)

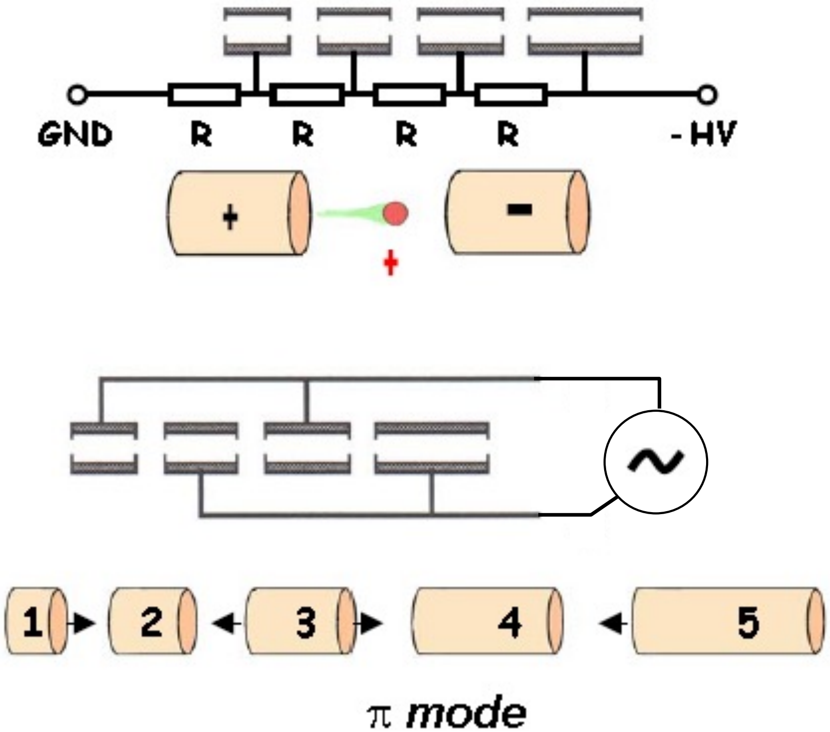
Van de Graaff Popularity

DC Accelerating Gaps: Tandem Van de Graaff

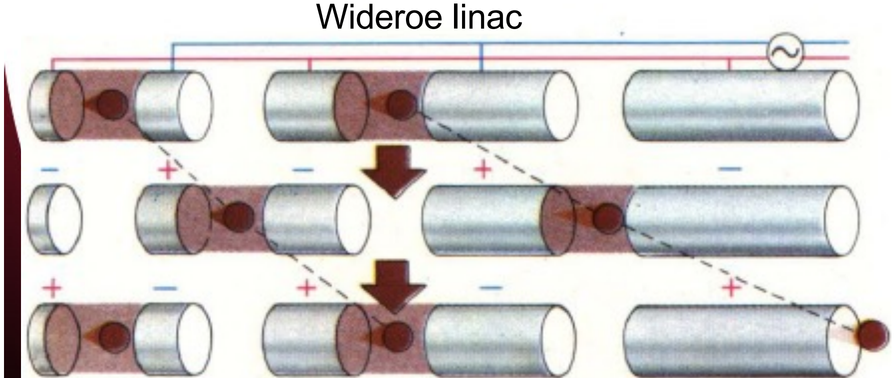


- Reverse ion charge state in middle of Van de Graaff allows over twice the energy gain
  - Source is at ground
- This only works for negative ions
- However, stripping need not be symmetric
  - Second stage accelerates more efficiently
- BNL: two Tandems (1970, 14 MV, 24m)
  - Au^{-1} to $Au^{+10}/Au^{+11}/Au^{+12}$ to Au^{+32} for RHIC
  - About a total of 0.85 MeV/nucleon total energy

From Electrostatic to RF Acceleration



Wideroe linac



Pagani and Mueller 2002

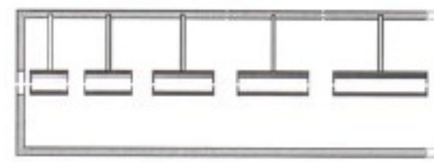
- Cockcroft-Waltons and Van de Graaffs have DC voltages, E fields
- What about putting on AC voltage?
  - Attach consecutive electrodes to opposite polarities of ACV generator
  - Electric fields between successive electrodes vary sinusoidally
  - Consecutive electrodes are 180 degrees out of phase (π mode)
- At the right drive frequency, particles are accelerated in each gap
  - While polarity change occurs, particles are shielded in drift tubes
  - To stay in phase with the RF, drift tube length or RF frequency must increase at higher energies

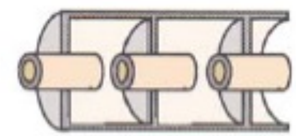
Resonant Linac Structures

- Wideroe linac: π mode

- Alvarez linac: 2π mode

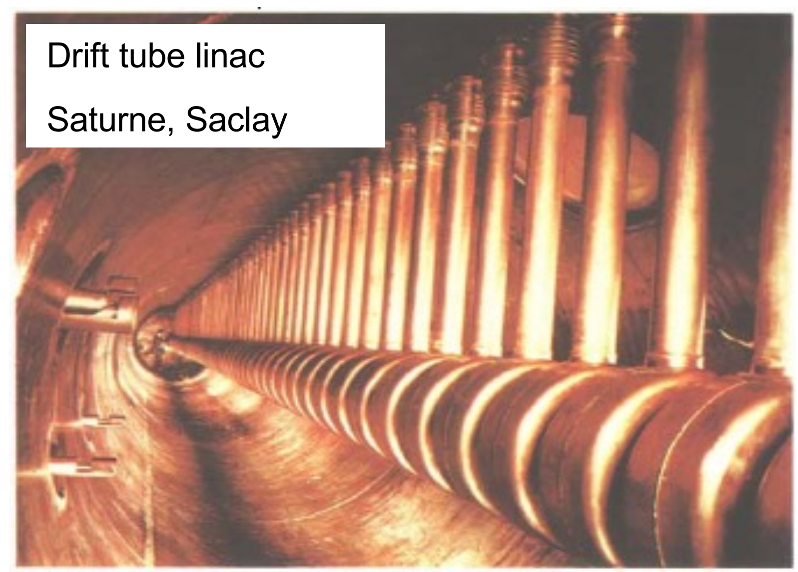
- Need to minimize excess RF power (heating)
  - Make drift tubes/gaps resonant to RF frequency
  - In 2π mode, currents in walls separating two subsequent cavities cancel; tubes are passive
  - We'll cover RF and longitudinal motion next week…





Wideroe linac

ALICE HI injector, IPN Orsay

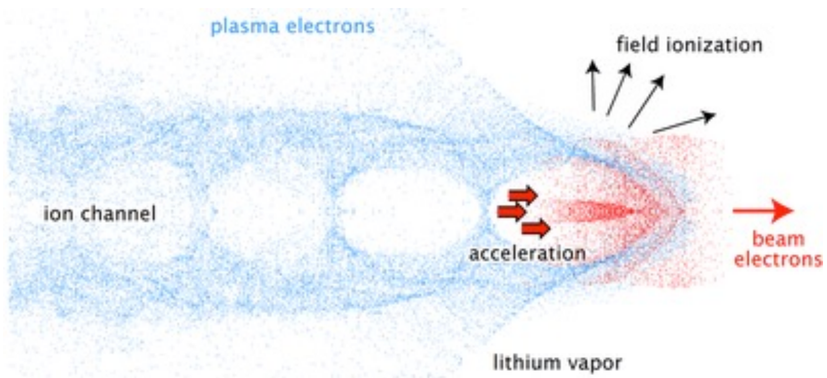


Drift tube linac

Saturne, Saclay

Advanced Acceleration Methods

- How far do accelerating gradients go?
  - Superconducting RF acceleration: ~40 MV/m
  - CLIC: ~100 MV/m
    - Two-beam accelerator: drive beam couples to main beam
  - Dielectric wall acceleration: ~100 MV/m
    - Induction accelerator, very high gradient insulators
  - Dielectric wakefield acceleration: ~GV/m
  - Laser plasma acceleration: ~40 GV/m
    - electrons to 1 GeV in 3.3 cm
    - particles ride in wake of plasma charge separation wave



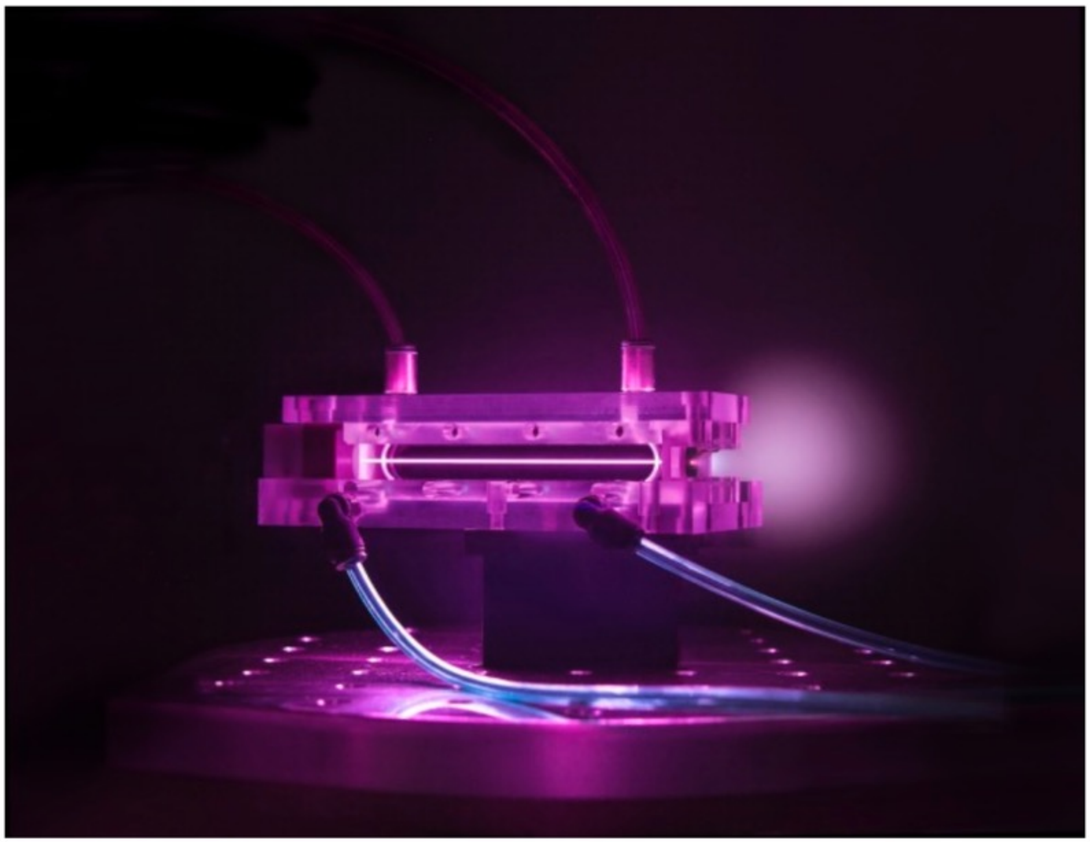
BELLA (LBL) Makes the News

World Record for Compact Particle Accelerator

Researchers at Berkeley Lab ramp up energy of laser-plasma “tabletop” accelerator.

News Release Kate Greene 510-486-4404 • DECEMBER 8, 2014

Tweet 480 | g+1 207 | Like 271 | Share 16 | Email 24



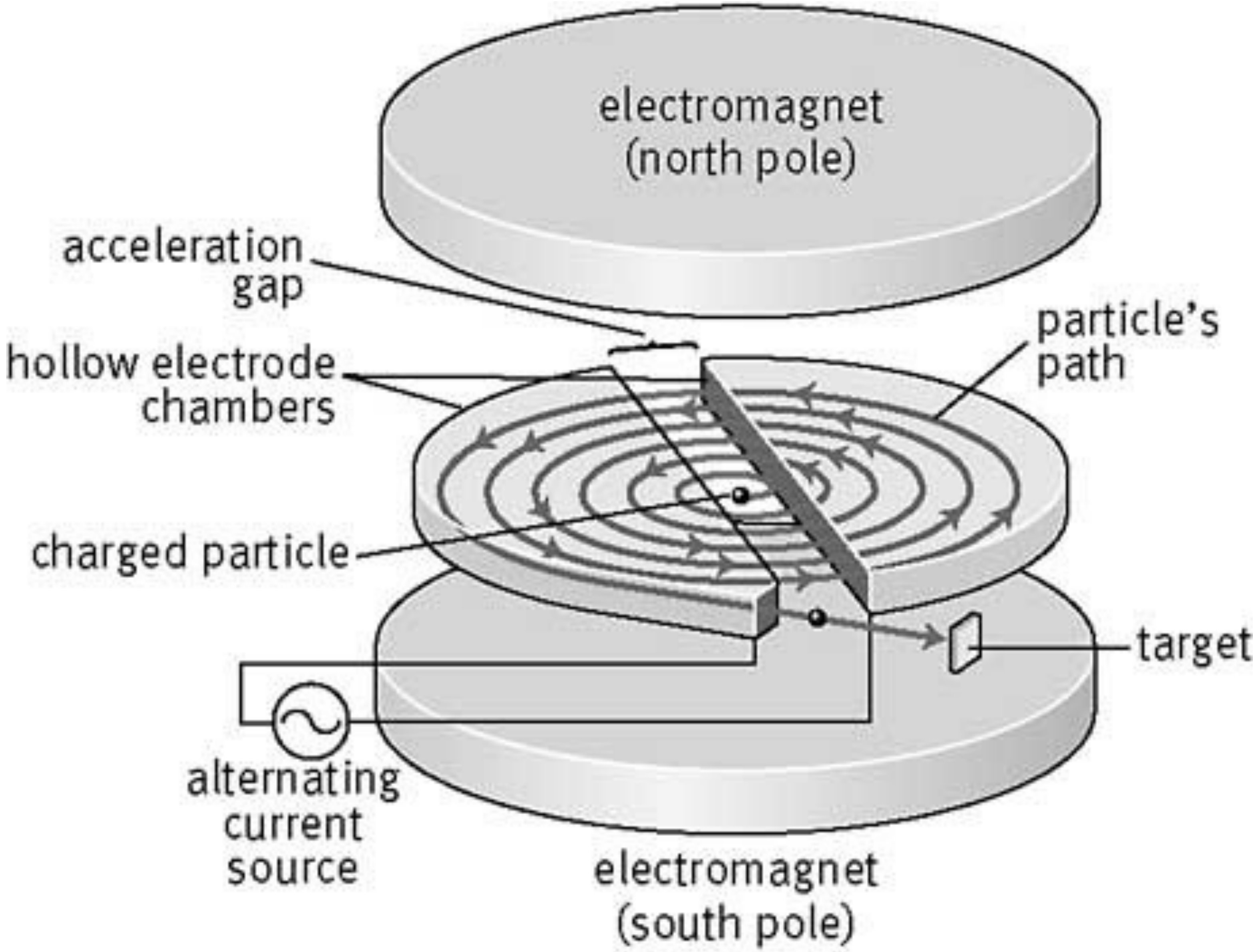
4.2 GeV electrons
in 9 cm

40 fs (=12 μm), 0.3 PW peak power drive laser

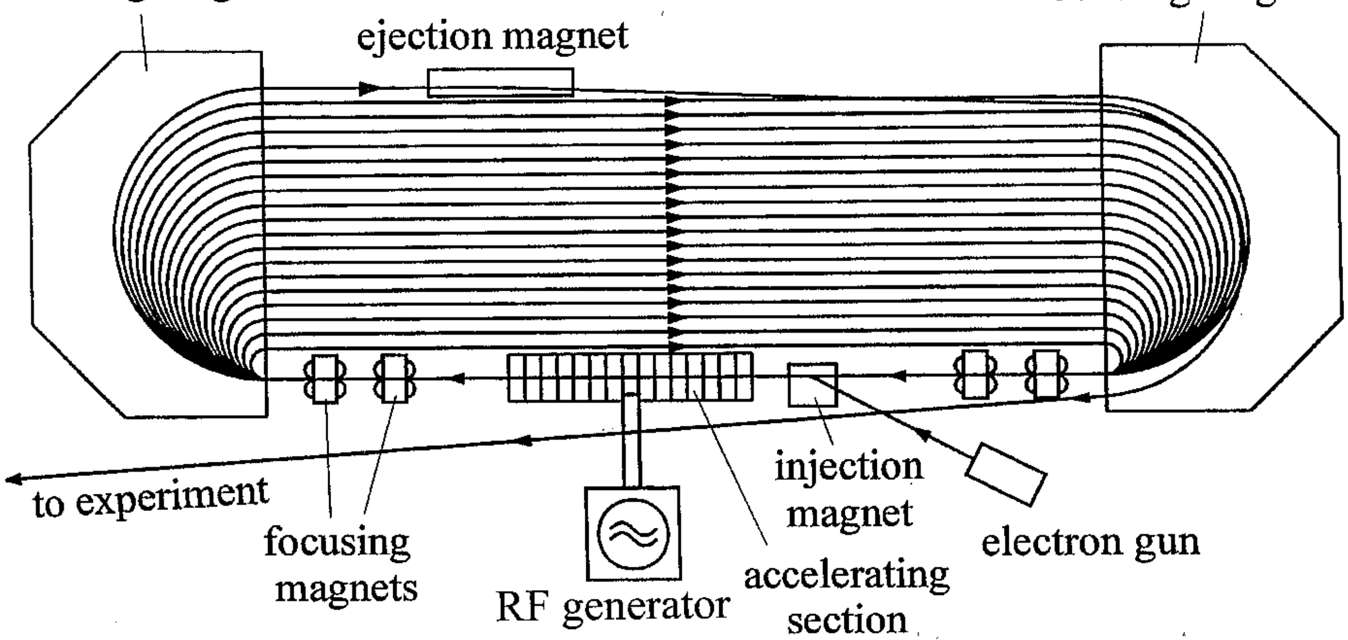
Multi-GeV Electron Beams from Capillary-Discharge-Guided Subpetawatt Laser Pulses in the Self- Trapping Regime

W. P. Leemans, et al., PRL **113**, 245002 2014 (December 8, 2014)

Cyclotrons (Again)

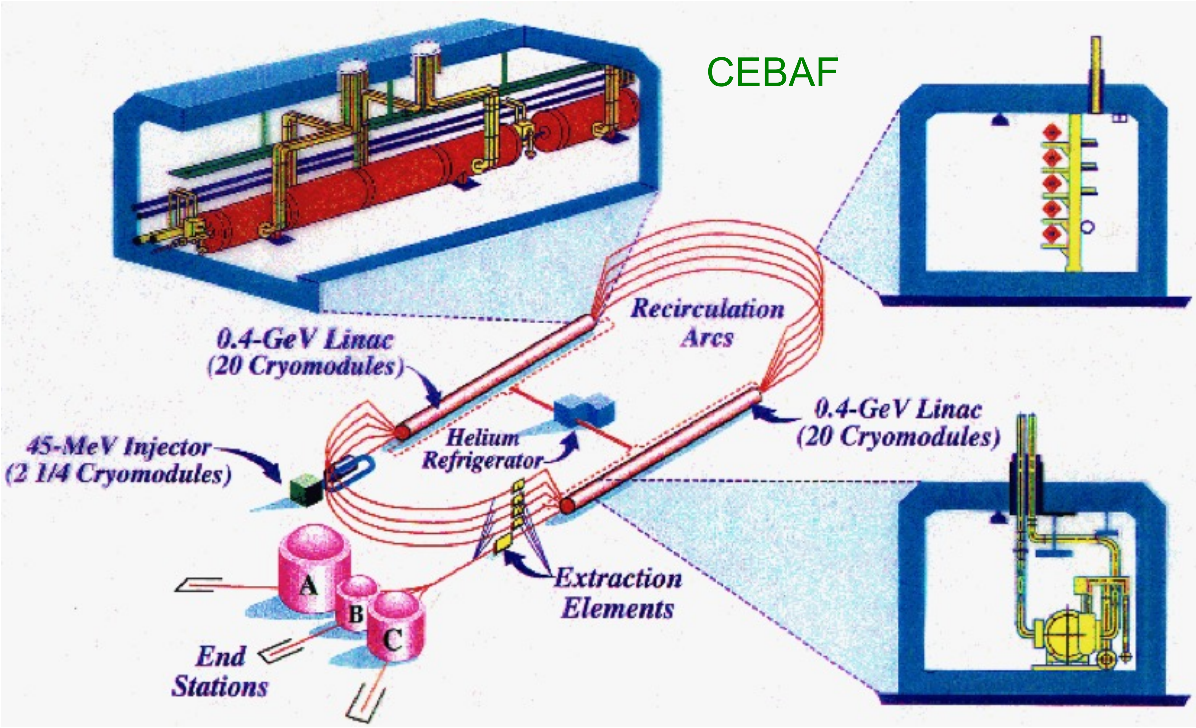


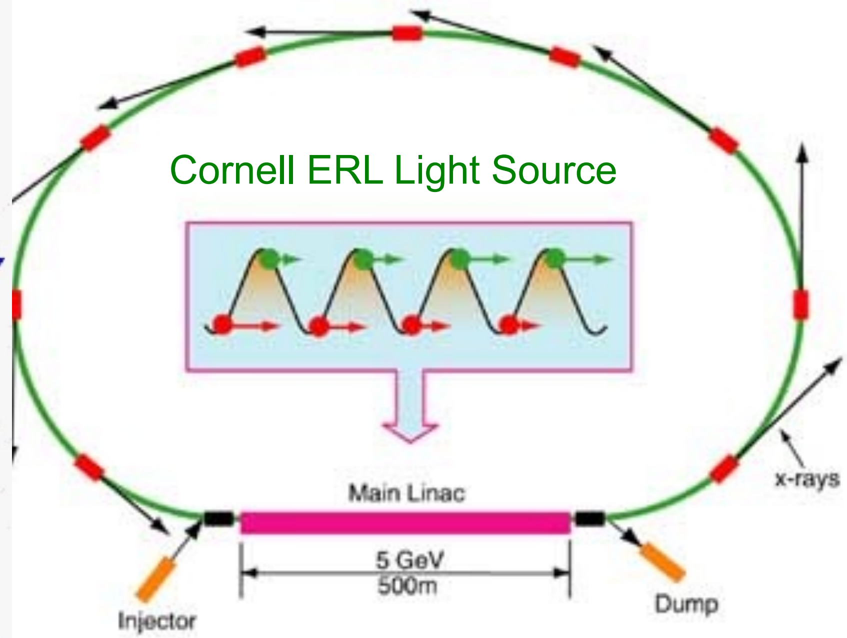
Microtrons



- What about electrons? Microtrons are like cyclotrons
  - but each revolution electrons "slip" by integer # of RF cycles
  - Trades off large # of revs for minimal RF generation cost
  - Bends must have very large momentum aperture
  - Used for medical applications today (20 MeV, 1 big magnet)
  - Mainz MAMI: 855 MeV, used for nuclear physics

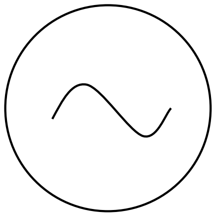
Recirculating Linacs and ERLs



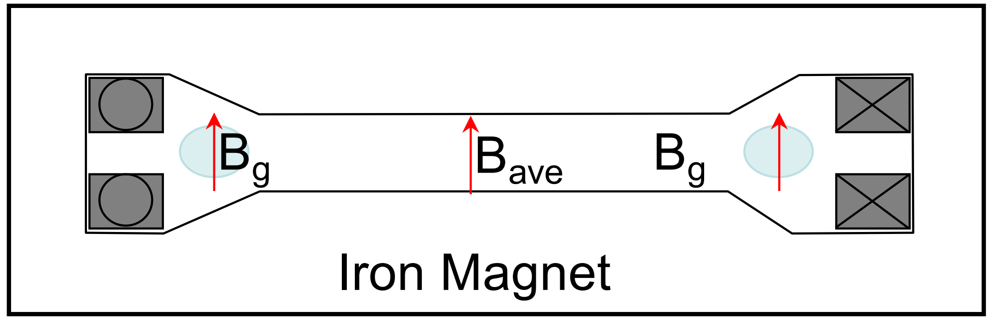


- Recirculating linacs have separate arcs, longer linacs
  - CEBAF: 4->6->12 GeV polarized electrons, 2 SRF linacs
  - Higher energy at cost of more linac, separated bends
- Energy recovery linacs recirculate exactly out of phase
  - Raise energy efficiency of linac, less beam power to dump
  - Requires high-Q SRF to recapture energy efficiently

Betatrons



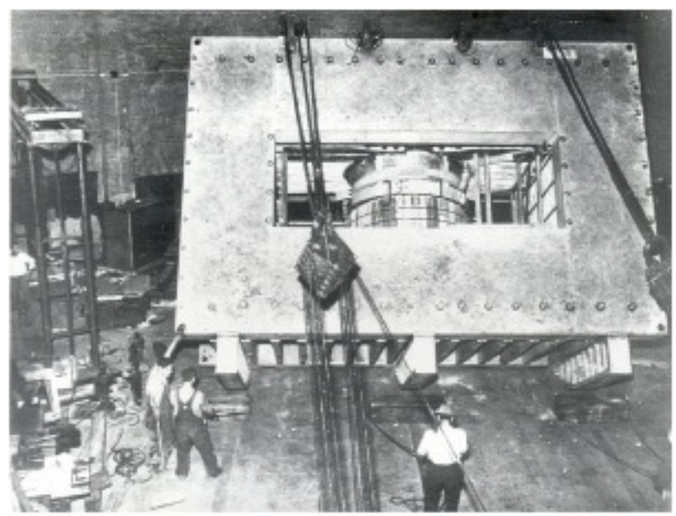
$I(t)=I_0 \cos(2\pi\omega_I t)$



- Apply Faraday's law with time-varying current in coils
- Beam sees time-varying electric field – accelerate half the time!
- Early proofs of stability: focusing and betatron motion

Donald Kerst
UIUC 2.5 MeV
Betatron, 1940

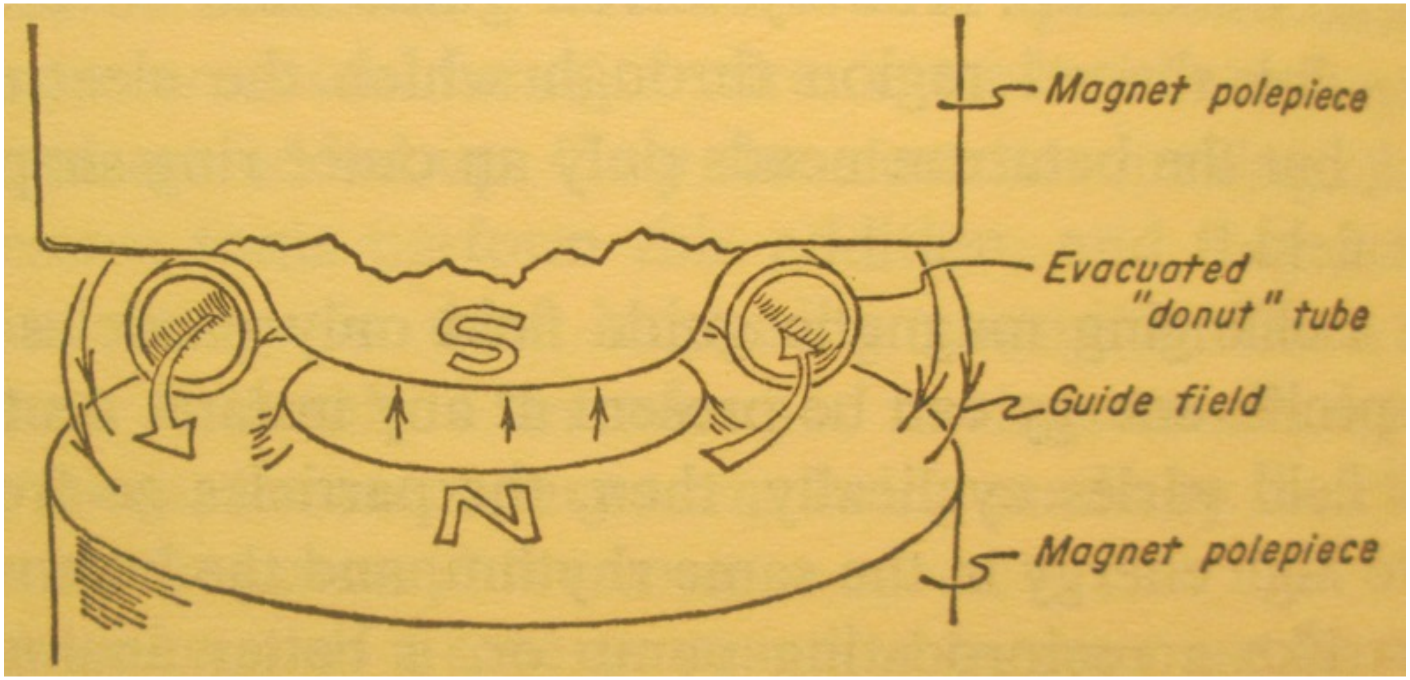
Don't try this at home!!

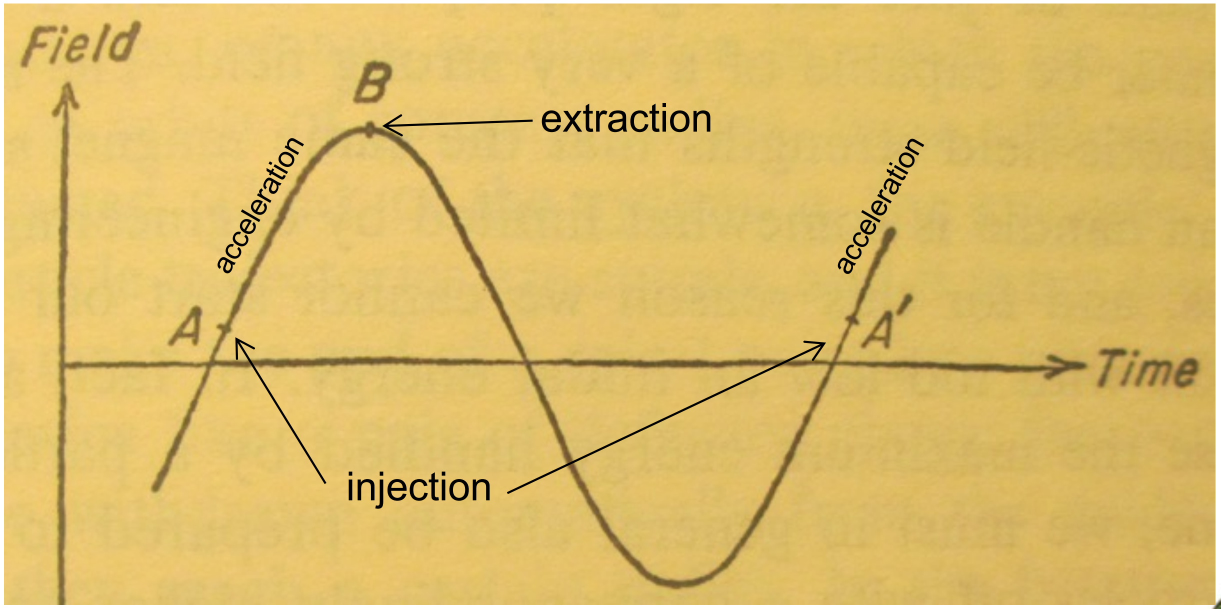


UIUC 312 MeV
betatron, 1949

Really don't try this at home!!

Betatrons

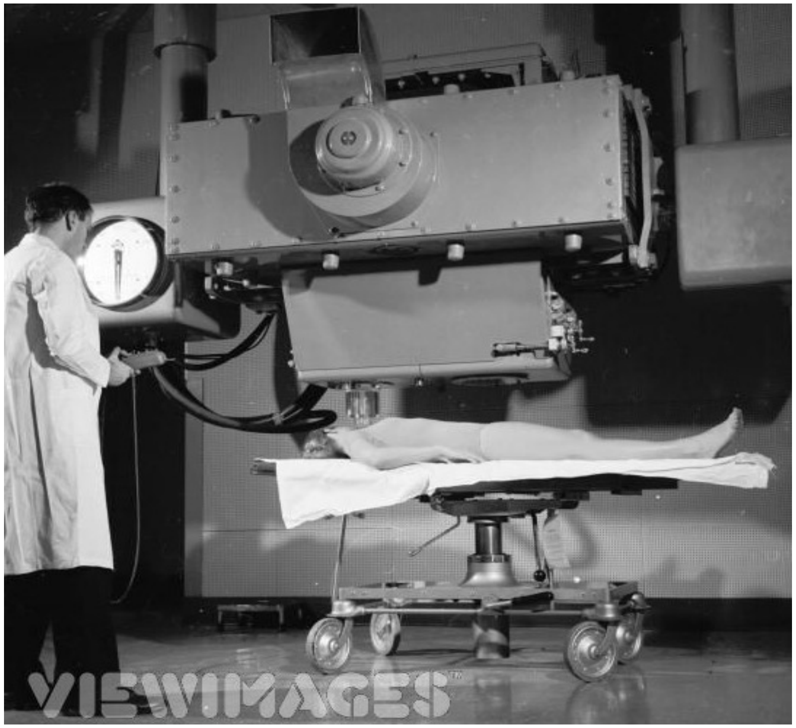




Wilson and Littauer: "Accelerators, Machines of Nuclear Physics", 1960

Betatrons

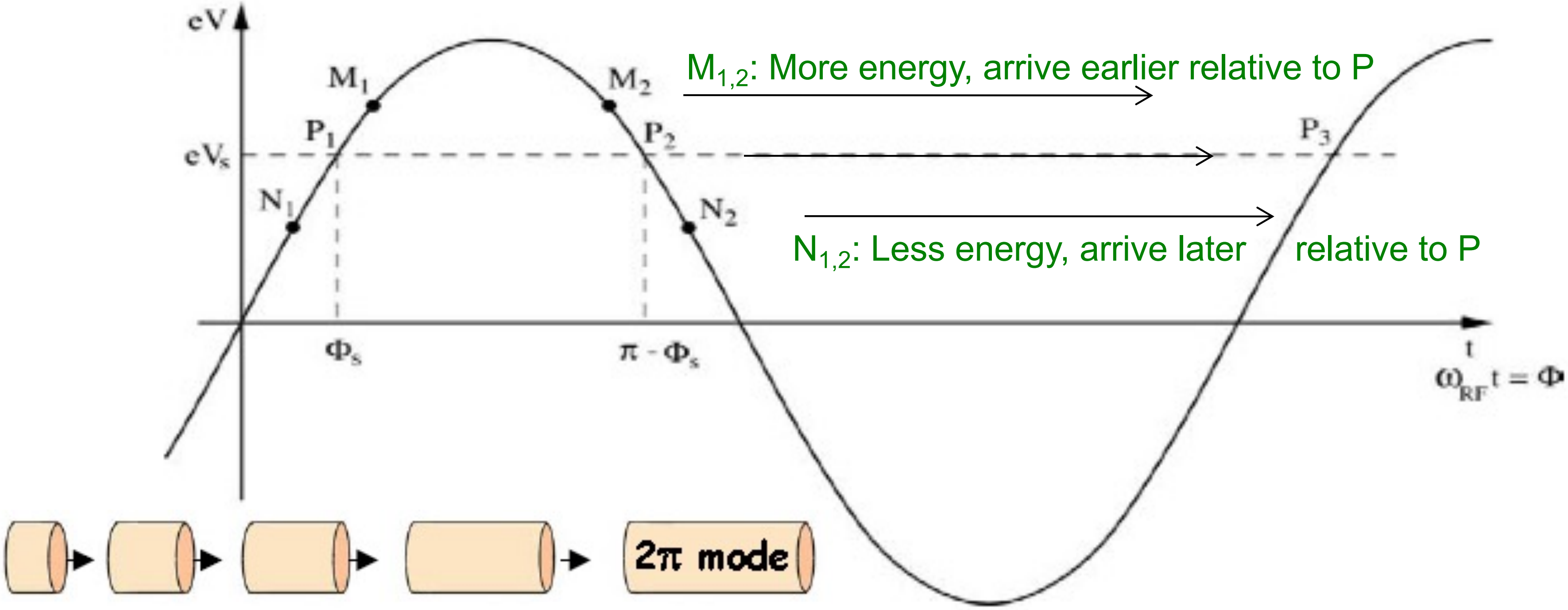
- Betatrons produced electrons up to 300+ MeV
  - Early materials and medical research
  - Also produced medical hard X-rays and gamma rays
- Betatrons have their challenges
  - Linear aperture scaling
  - Large stored energy/impedance
  - Synchrotron radiation losses
  - Quarter duty cycle
  - Ramping magnetic field quality



This will only hurt a bit…

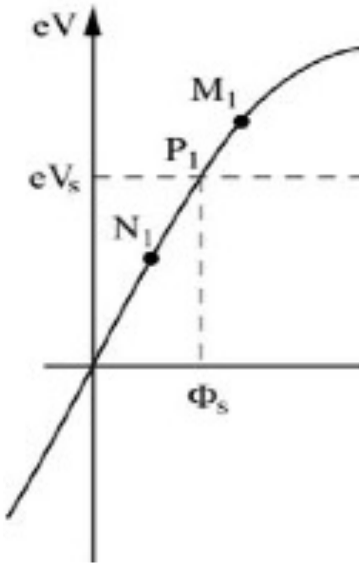
- **More on betatrons/weak focusing this afternoon**

Phase Stability



- Consider a series of accelerating gaps (or a ring with one gap)
  - **By design** there is a synchronous phase Φ_s that gains just enough energy to hit phase Φ_s in the next gap
  - $P_{1,2}$ are fixed points: they "ride the wave" exactly in phase
- If increased energy means increased velocity ("below transition")
  - M_1,N_1 will move towards P_1 (local stability) => **phase stability**
  - $M_{2,}N_2$ will move away from P_2 (local instability)

Phase Stability Implies Transverse Instability



$$\frac{\partial V}{\partial t} > 0 \quad \Rightarrow \quad \frac{\partial E_z}{\partial z} < 0$$

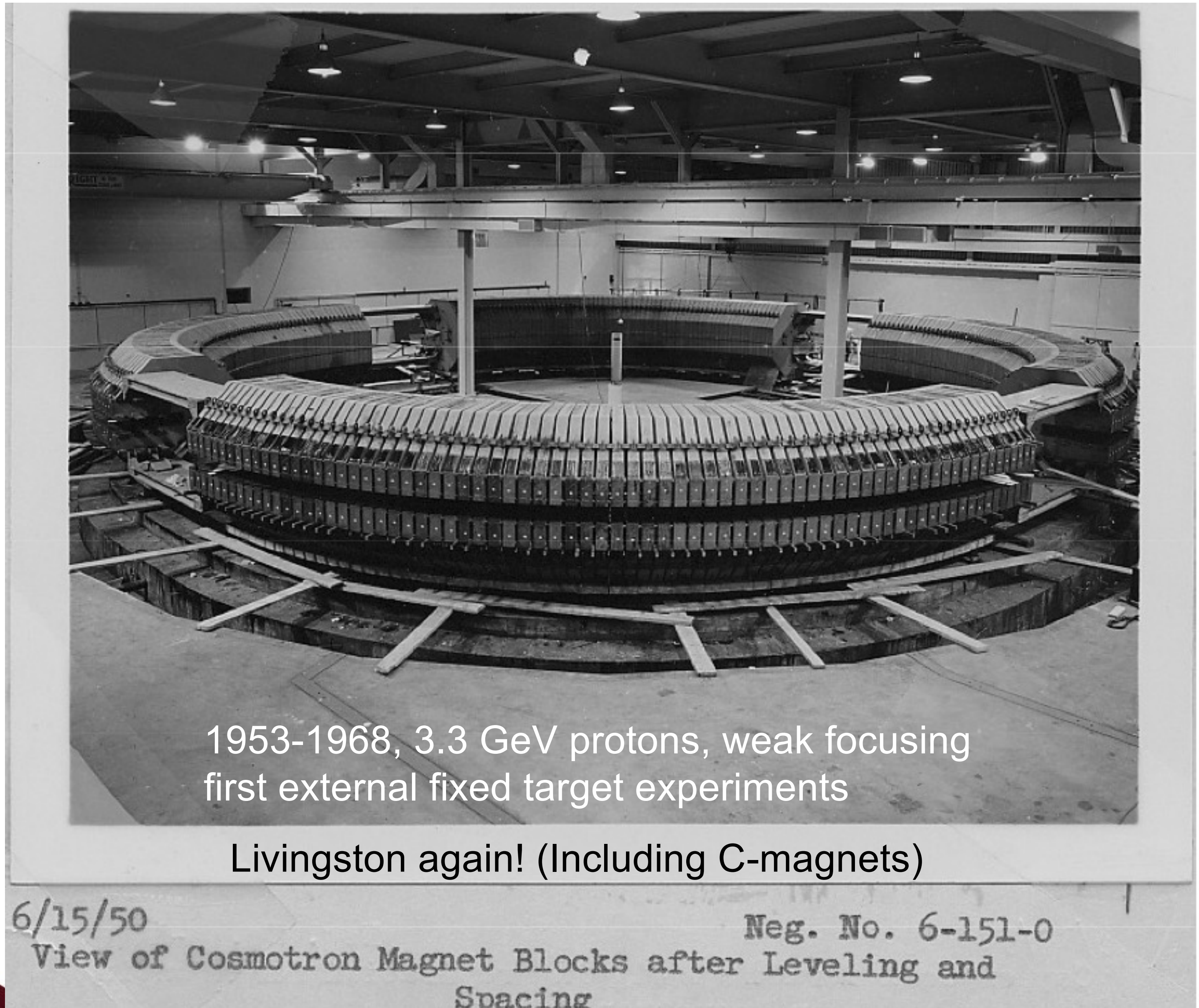
- For phase stability, longitudinal electric field must have a negative gradient. But then (source-free) Maxwell says

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \Rightarrow \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \quad \Rightarrow \quad \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} > 0$$

There must be some transverse defocusing/diverging force!

Any accelerator with RF phase stability (longitudinal focusing) needs transverse focusing! (solenoids, quads…)

BNL Cosmotron



1953-1968, 3.3 GeV protons, weak focusing
first external fixed target experiments

Livingston again! (Including C-magnets)

National Academy of Sciences, Biographical Memoir
of M. Stanley Livingston by Ernest D. Courant

LBL Bevatron

Ed McMillan and Ed Lofgren

- Last and largest weak-focusing proton synchrotron
- 1954, Beam aperture about 4' square!, beam energy to 6.2 GeV
- Discovered antiproton 1955, 1959 Nobel for Segre/Chamberlain

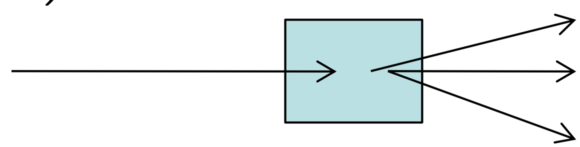
(Became Bevelac, decommissioned 1993, demolished recently)

Fixed Target Experiments

- Why did the Bevatron need 6.2 GeV protons?
  - Antiprotons are “only” 930 MeV/c^2 (times 2…)
  - Bevatron used Cu target, p+n->p+n+p+pbar
  - Mandelstam variables give:

$$\frac{E_{\mathrm{cm}}^2}{c^2} = 2\left(\frac{E_1 E_2}{c^2} + p_{\mathrm{z}1} p_{\mathrm{z}2}\right) + (m_{01}c)^2 + (m_{02}c)^2$$

- Fixed Target experiment



$$(4m_{\mathrm{p}0}c)^2 < \frac{E_{\mathrm{cm}}^2}{c^2} = 2\frac{E_1 m_{\mathrm{p}0}}{c^2} + 2(m_{\mathrm{p}0}c)^2 \quad \Rightarrow \quad E_1 > 7m_{\mathrm{p}0}c^2$$

$$E_{\mathrm{cm}} = \sqrt{2E_1(m_{02}\, c^2)}$$

  - Available CM energy scales with root of beam energy
    - Main issue: forward momentum conservation steals energy

Two Serious Problems

- These machines were getting way too big
  - Bevatron magnet was 10,000 tons
  - Apertures scale linearly with machine size, energy

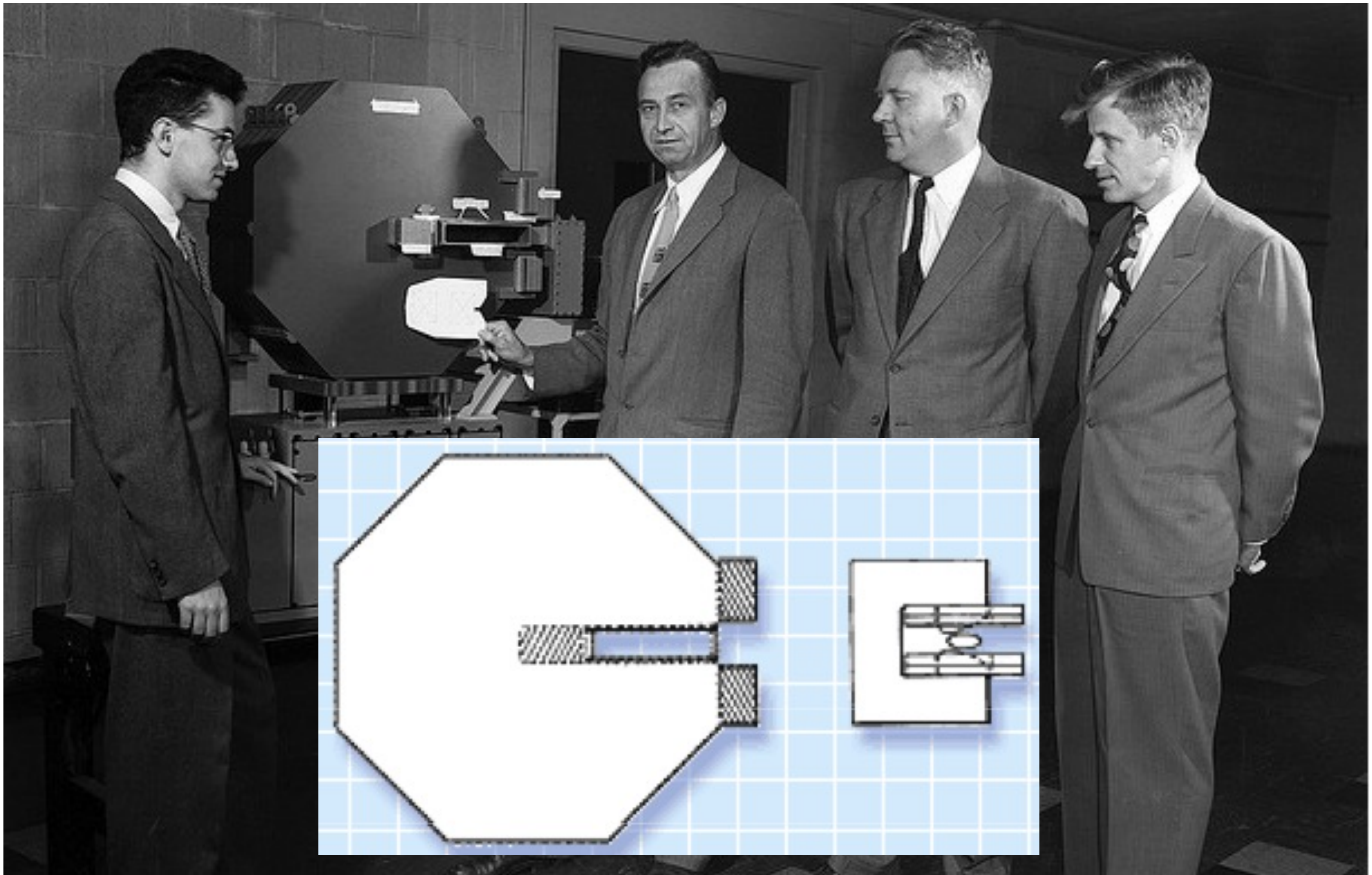
    (Length/circumference scales linearly with energy at fixed field strength too…)

- Fixed target energy scaling is painful
  - Available CM energy only scales with $\sqrt{E_{beam}}$

- Accelerator size grew with the square of desired CM energy
  - Something had to be done!!!

Strong Focusing (1952) and Colliders (1958-62ish) to the rescue!!!

Livingston *Again*?

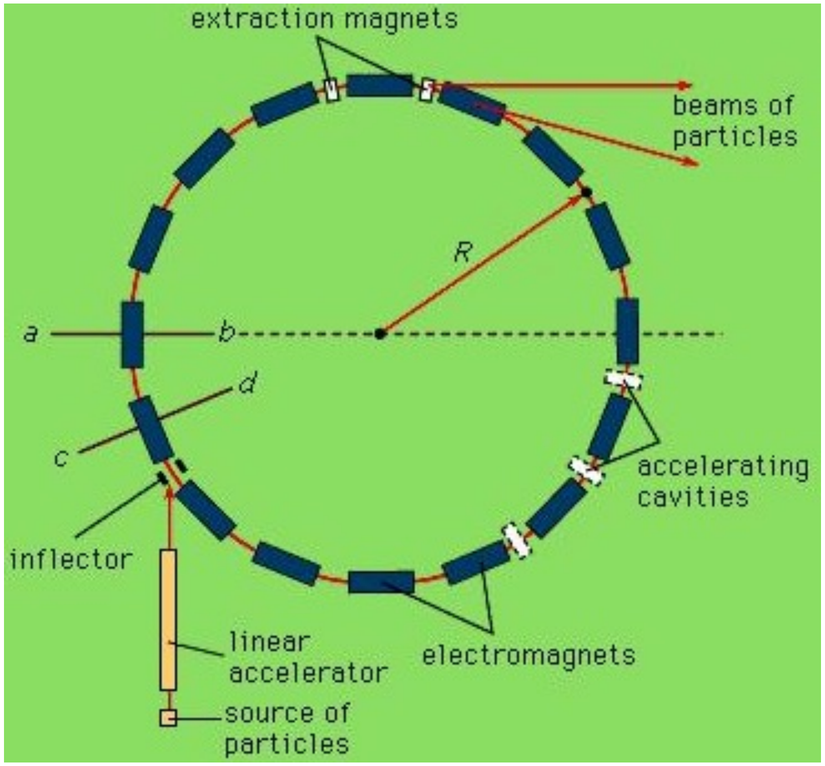


E. Courant, M.S. Livingston, H. Snyder, and J.P. Blewett

- Strong focusing (and its mathematical treatment) is really the focus (ha) of the rest of this week

Would you buy a used Cosmotron lamination from this man?

The Synchrotron



- The best of both worlds (1944)
  - Cyclotron accelerating system (RF gaps)
    - (Not inductive betatron acceleration)
  - Variable Betatron magnetic bending field
    - (Not constant cyclotron bending field)

- "Synch"-rotron
  - Particle bend radius is close to **constant**

$$B\rho = \frac{p}{q} \quad \Rightarrow \quad \rho = \frac{1}{q}\left(\frac{p}{B}\right)$$

    - B field changes with particle momentum p
  - Circumference is also close to **constant**
    - Revolution frequency and RF frequency also changes with particle velocity β and particle momentum p

$$f_{\rm rev} = \frac{\beta c}{2\pi R}$$

$$f_{\rm rf} = 2\pi h f_{\rm rev} = \frac{h\beta c}{R}$$

$$h \equiv \text{harmonic number}$$

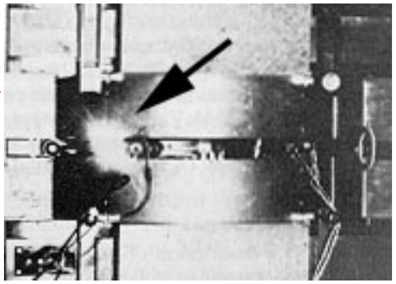
Why is this such a big deal?

- The **big deal** is that both existing technologies scaled very badly with particle energy
  - **Betatrons**: central induction magnet area (flux) scales quadratically with accelerator radius (energy); beam size also scales badly
  - **Cyclotrons**: main magnet scales quadratically with energy radius (energy); problems with relativistic hadrons
  - (High gradient linacs weren't quite developed yet)

- Large, high-energy accelerator cost was **completely** dominated by scaling of **large** magnets
  - The synchrotron permitted the **decoupling** of peak accelerator energy and magnetic field apertures
  - Higher energies required more magnets (linear scaling) but not larger aperture magnets (quadratic scaling, or worse)

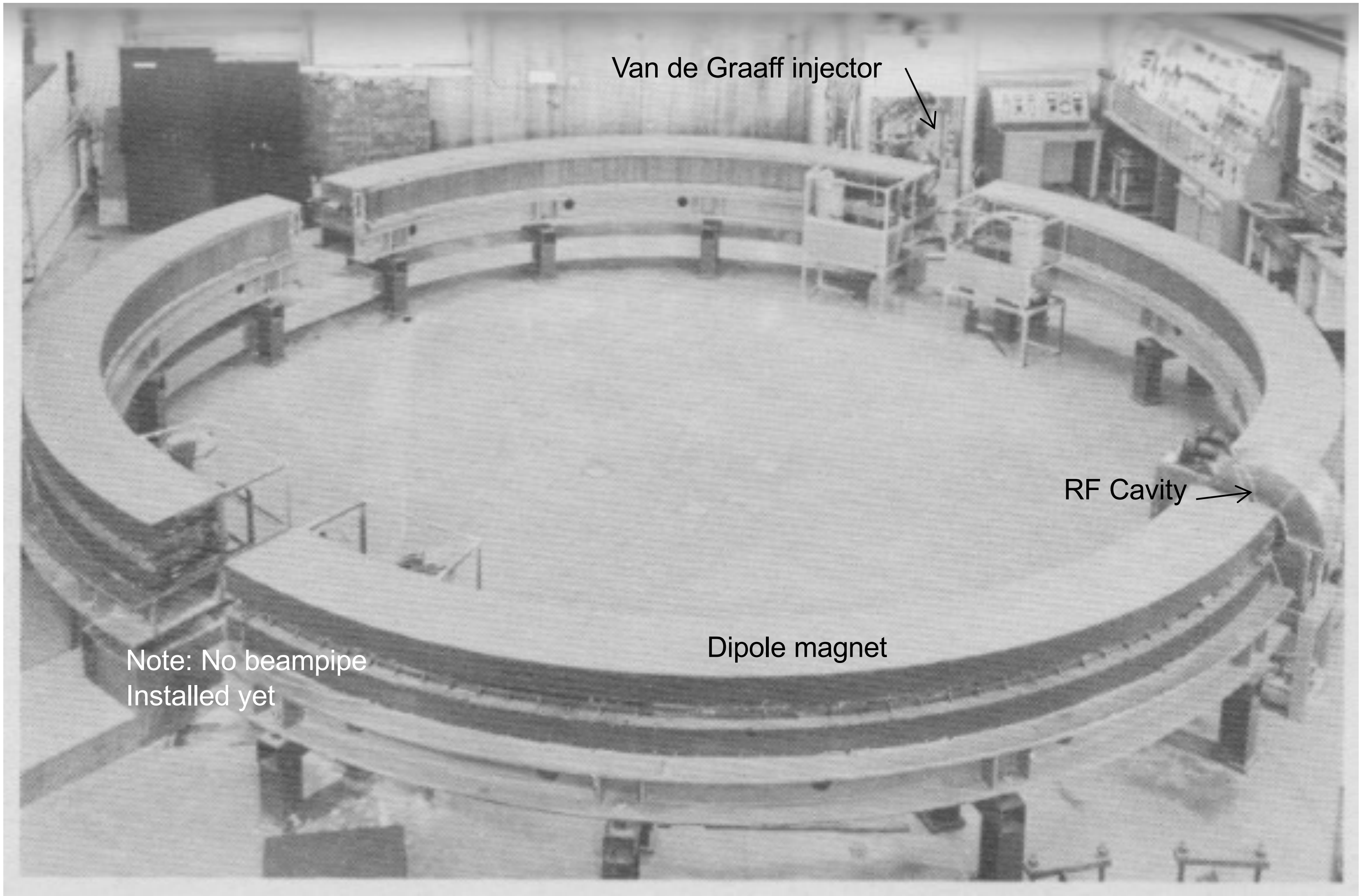
Synchrotron and Phase Stability

- The synchrotron depends on our "old" friend, longitudinal phase stability
  - We'll review phase stability in RF/longitudinal lectures
- Historical context
  - Phase stability: V. Veksler (Russia, 1944) and E.M. McMillan (Los Alamos/LBL, 1945) (1951 Nobel w/G. Seaborg, first transuranic element)
  - First synchrotrons were electron accelerators (~1947)
    - Eliminate bulky core induction magnet of betatrons
    - Easily ultrarelativistic ➔ f_{rev}, f_{RF} nearly constant
    - Energy E~pc so ρ constant ➔ Energy/B = constant
    - 50 MeV betatron (GE, Schenectady) ➔ 70 MeV synchrotron
      - First observation of **synchrotron radiation**
    - Cornell electron synchrotron (1.3 GeV, 1954)
  - Proton synchrotrons came soon after (1950)



Accelerators: Machines of Nuclear Physics (Wilson/Littauer 1960)

Cornell Electron Synchrotron (1954)



- 1.3 “BeV” (GeV) with van de Graaff injector
  - First strong focusing synchrotron, 16 tons of magnets, 4 cm beam pipe

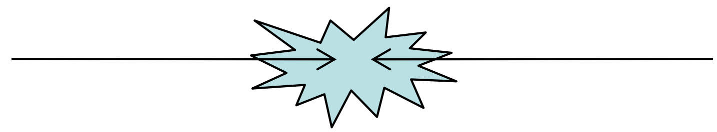
Accelerators: Machines of Nuclear Physics (Wilson/Littauer 1960)

Collider Experiments

- What if the Bevatron was a collider?
  - Antiprotons are "only" 930 MeV/c^2 (times 2…)
  - Two-body system (Mandelstam variables) gives (again):

$$\frac{E_{\rm cm}^2}{c^2} = 2\left(\frac{E_1 E_2}{c^2}\right) + p_{\rm z1}p_{\rm z2} + (m_{01}c)^2 + (m_{02}c)^2$$

- Case 2: Collider



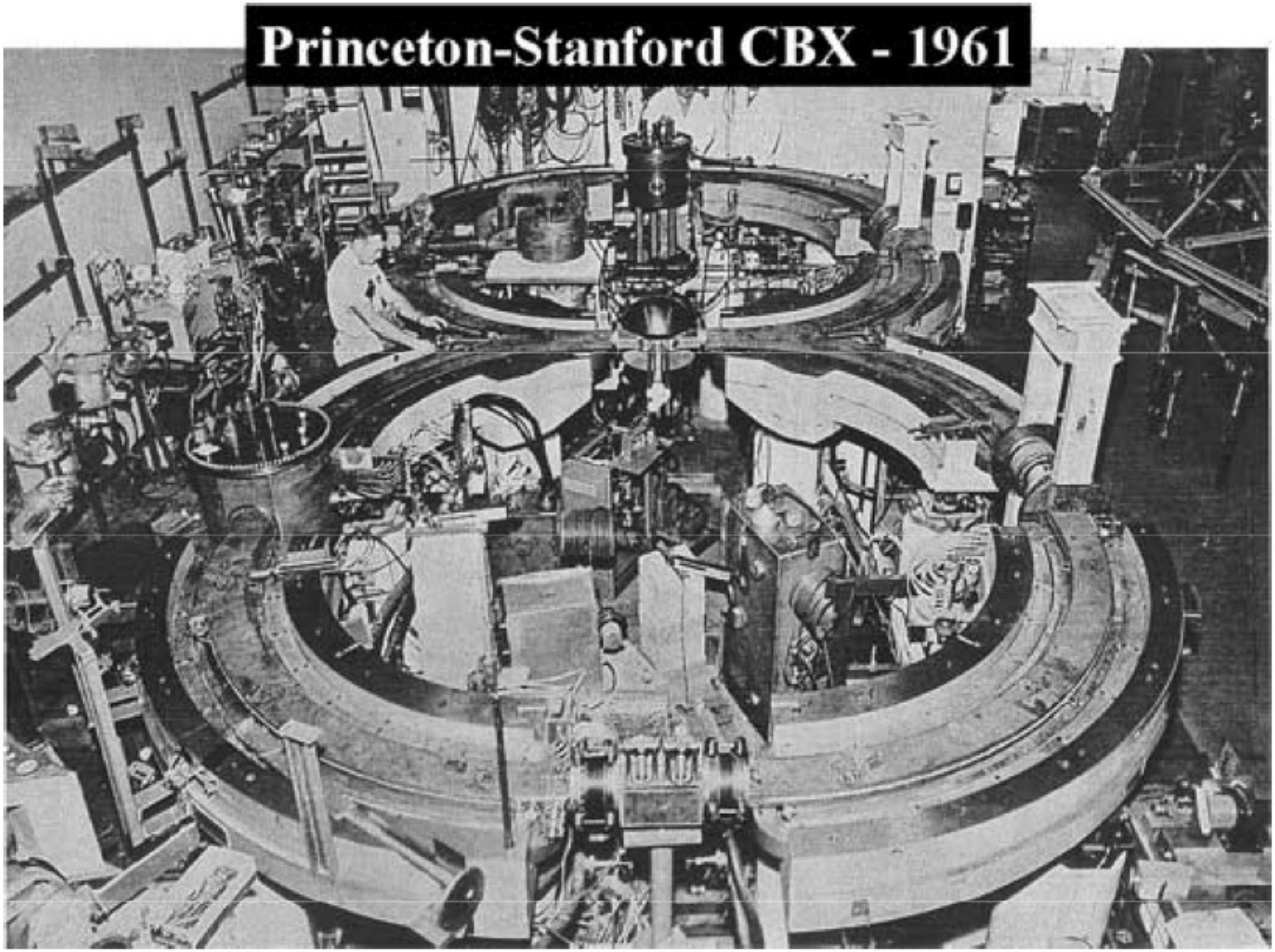
$$E_1 \gg m_{01}c^2 \qquad E_2 \gg m_{02}c^2$$

$$E_{\rm cm} = 2\sqrt{E_1 E_2} = 2E \quad \text{if } E_1 = E_2$$

  - Linear scaling with beam energy!
  - For Bevacollidatron, e- + e+ -> p+pbar is possible!

(Although the cross section is probably pretty small)

First Electron Collider



Cambridge Electron Accelerator

THE CEA TEAM, 1959. The group that led the Cambridge Electron Accelerator (CEA) in Cambridge, Massachusetts. The machine was later converted for colliding beam experiments, testing the technique of 'low-beta' that proved so important in storage rings. Seated from left: Thomas Collins and David Jacobus. Standing from left: Fred Barrington, CEA Director Stanley Livinston, Robert Cummings, Lee Young, John Rees, William Jones, Janez Dekkra, and Kenneth Robinson (deceased).

SLAC Beam Line, “Colliding Beam Storage Rings”, John Rees, Mar 1986

Luminosity

- **Luminosity** L is a measure of how many interactions of **cross section** σ can be created per unit time

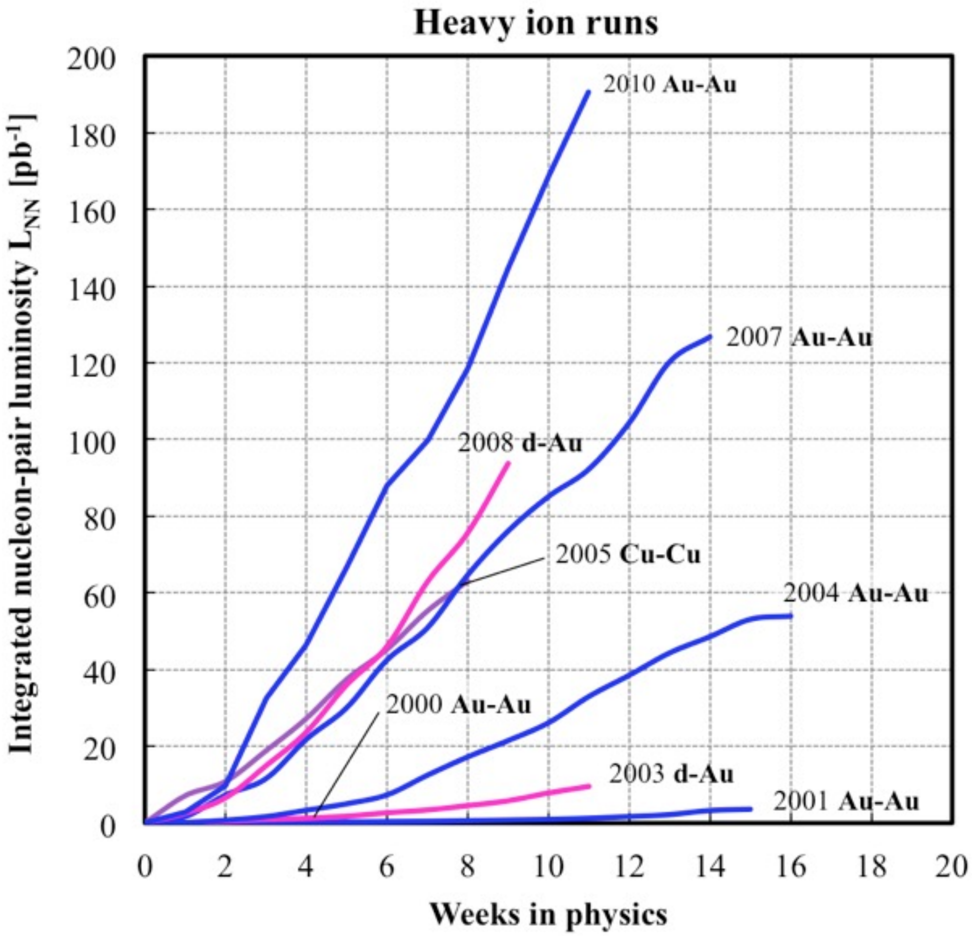
$$L\sigma = \frac{dN}{dt} \qquad N = \sigma \int L\, dt = \sigma\, L_{\mathrm{int}}$$

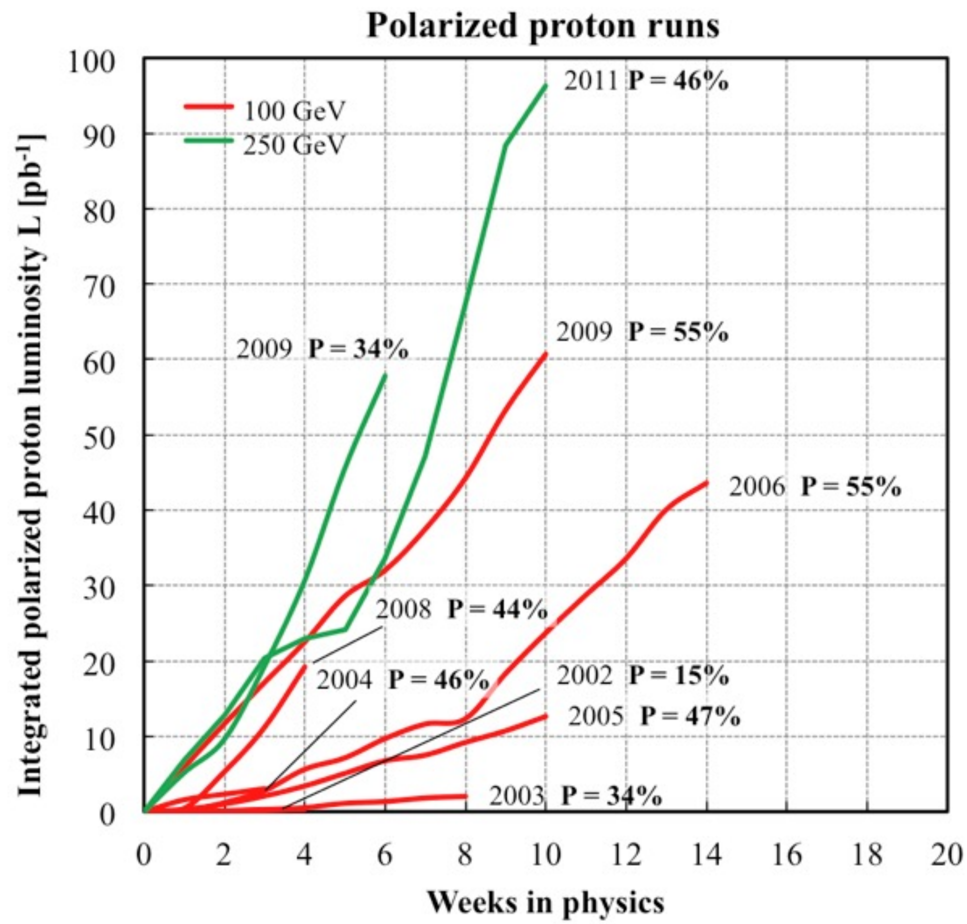
  - L_{int} is integrated luminosity, an important factor of production for colliders
  - $[L]=cm^{-2}\,s^{-1}$, $[L_{int}]=cm^{-2}$ (1 ba=10^{-24} cm; 1 $pb^{-1}=10^{36}\,cm^{-2}$)
- For equal-sized head-on Gaussian beams in a collider

$$L = \frac{f_{\mathrm{rev}}\, h\, N_1\, N_2}{4\pi\sigma_x\sigma_y} \qquad \text{(\textasciitilde book 1.11)}$$

  - $\sigma_{x,y}$ are rms beam sizes, h is number of bunches
    - Colliding 100 μm 7.5e9 proton bunches at 100 kHz for 1 year gives about 1 pb^{-1} of integrated luminosity - independent of energy (kind of)

Evolution of RHIC Collider Luminosities

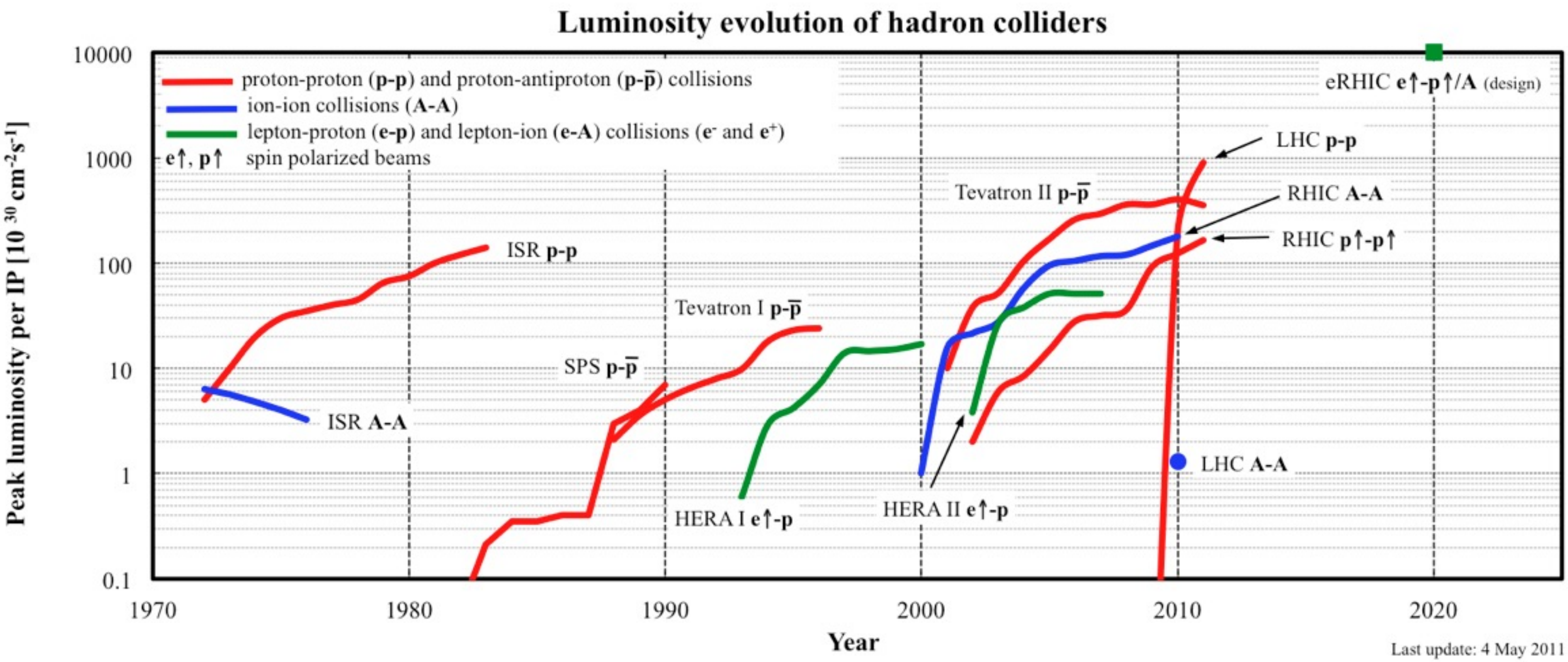




Note: The nucleon-pair luminosity is defined as $L_{NN} = A_1 A_2 L$, where L is the luminosity, and A_1 and A_2 are the number of nucleons of the ions in the two beam respectively.

W. Fischer, http://www.rhichome.bnl.gov/RHIC/Runs

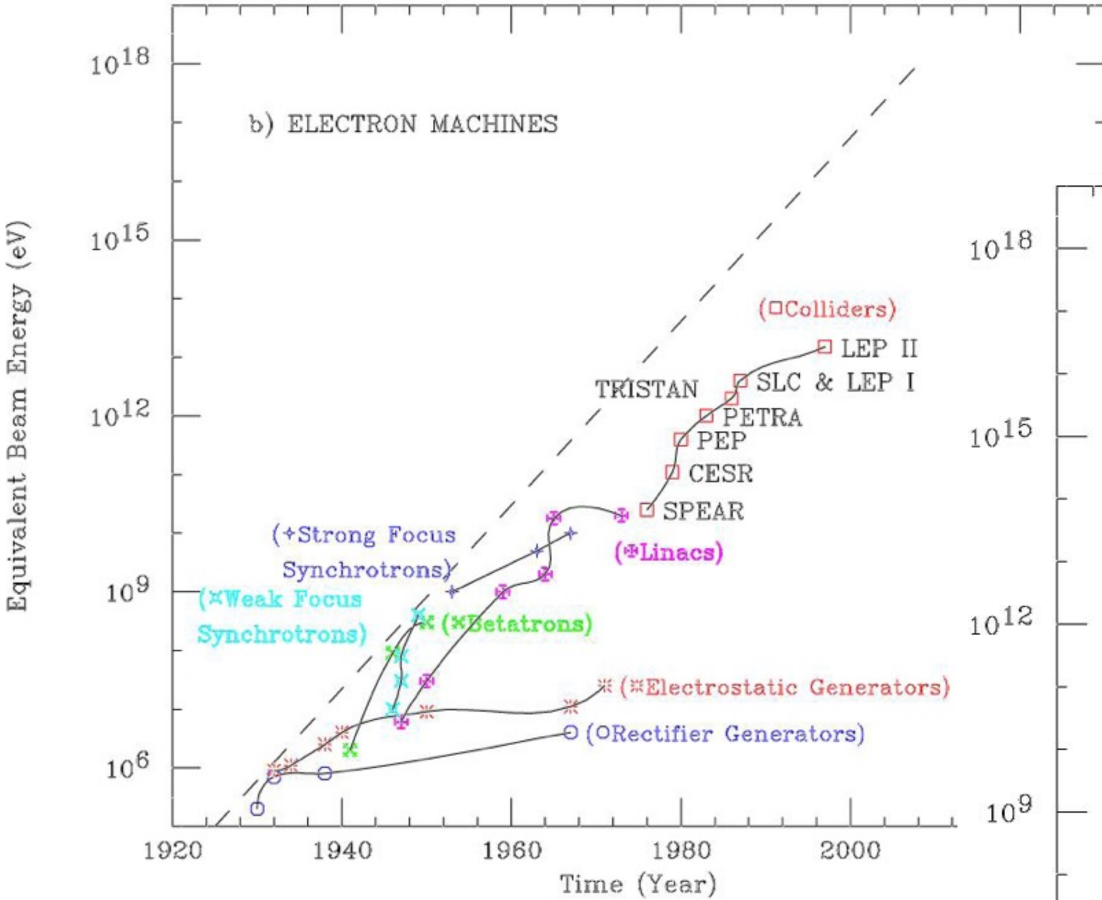
Evolution of Hadron Collider Luminosities



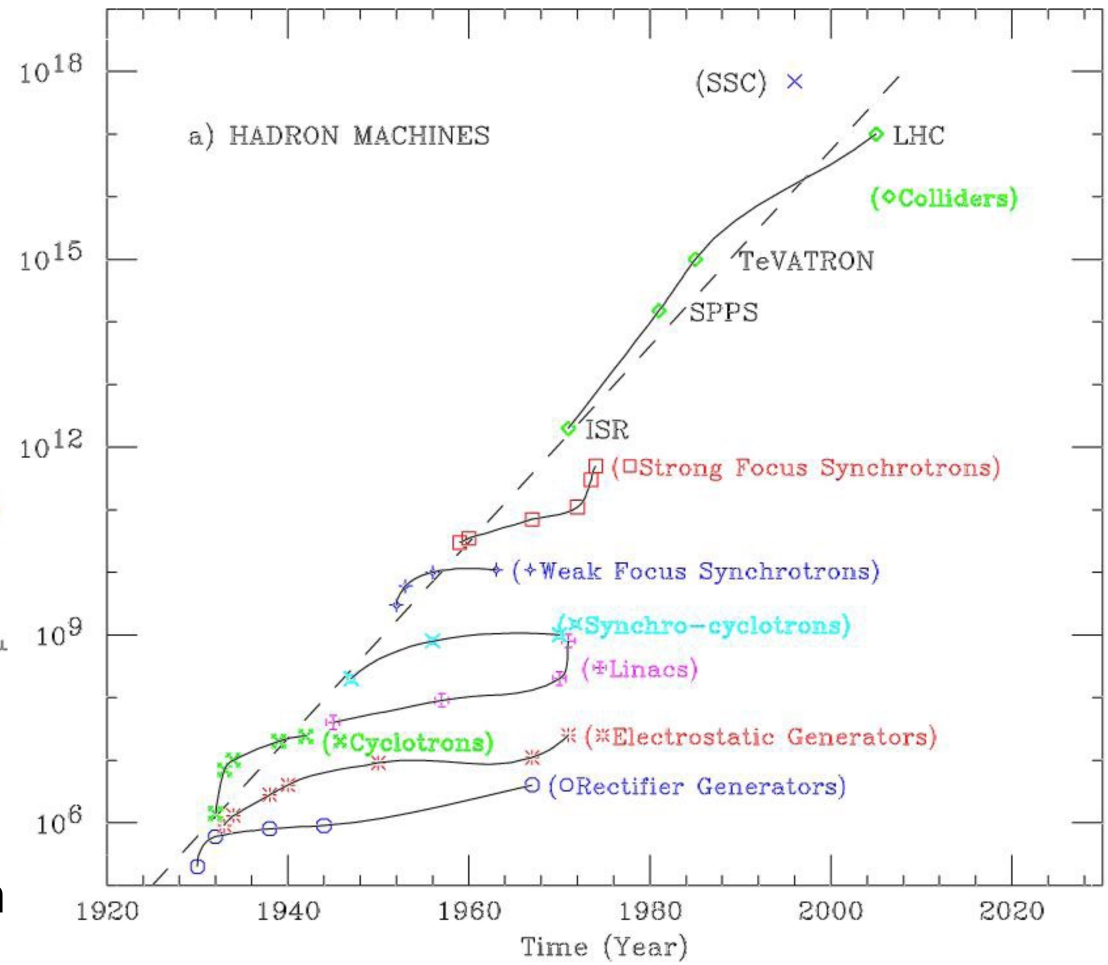
Note: For ion collisions the nucleon-pair luminosity is shown. The nucleon-pair luminosity is defined as $L_{NN} = A_1 A_2 L$, where L is the luminosity, and A_1 and A_2 are the number of nucleons of the ions in the two beam respectively. The highest energies for the machines are: ISR 31 GeV, SPS 315 GeV, Tevatron 980 GeV, HERA 920 GeV (p) 27.5 GeV (e), RHIC 250 GeV, LHC 3.5 TeV.

W. Fischer, http://www.rhichome.bnl.gov/RHIC/Runs

Livingston Plots



Livingston observed that accelerator energy was growing exponentially (in 1950!)



Still holds true over 60 years (!!!) later

Technologies tend to saturate then new technologies are introduced

(G. Hoffstaetter, Cornell)

========= Extra Slides =========

Lorentz Lie Group Generators I

- Lorentz transformations can be described by a Lie group where a general Lorentz transformation is

$$A = e^{L} \qquad \det A = e^{\mathrm{Tr}\, L} = +1$$

where L is 4x4, real, and traceless. With metric g, the matrix gL is also antisymmetric, so L has the general six-parameter form

$$L = \begin{pmatrix} 0 & L_{01} & L_{02} & L_{03} \\ L_{01} & 0 & L_{12} & L_{13} \\ L_{02} & -L_{12} & 0 & L_{23} \\ L_{03} & -L_{13} & -L_{23} & 0 \end{pmatrix}$$

Deep and profound connection to EM tensor $F^{\alpha\beta}$

J.D. Jackson, Classical Electrodynamics 2nd Ed, Section 11.7

Lorentz Lie Group Generators II

- A reasonable basis is provided by six generators
  - Three generate rotations in three dimensions

$$S_{1,2,3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

  - Three generate boosts in three dimensions

$$K_{1,2,3} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Lorentz Lie Group Generators III

- $(S_{1,2,3})^2$ and $(K_{1,2,3})^2$ are diagonal.
- $(\epsilon \cdot S)^3 = -\epsilon \cdot S$ and $(\epsilon \cdot K)^3 = \epsilon \cdot K$ for any unit 3-vector ϵ
- Nice commutation relations:

$$[S_i, S_j] = \epsilon_{ijk} S_k \quad [S_i, K_j] = \epsilon_{ijk} K_k \quad [K_i, K_j] = -\epsilon_{ijk} S_k$$

- We can then write the Lorentz transformation in terms of two three-vectors (6 parameters) ω, ζ as

$$L = -\omega \cdot S - \zeta \cdot K \qquad A = e^{-\omega \cdot S - \zeta \cdot K}$$

  - Electric fields correspond to boosts
  - Magnetic fields correspond to rotations
  - Deep beauty in Poincare, Lorentz, Einstein connections