# USPAS Accelerator Physics 2024
# Hampton VA / Northern Illinois University

# Chapter 6+: Magnets and Magnet Technology

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Happy Birthday to Michio Kaku, Dan Shechtman (Nobel 2011, quasicrystals), and John Belushi!
Happy Beer Can Appreciation Day, National Compliment Day, and National Peanut Butter Day!

# Overview

- Back to Maxwell
  - Parameterizing fields in accelerator magnets
  - Symmetries, comments about magnet construction
- Relating currents and fields
  - Equipotentials and contours, dipoles and quadrupoles
  - Thin magnet kicks and that ubiquitous rigidity
  - Complications: hysteresis, end fields
- More details about dipoles
  - Sector and rectangular bends; edge focusing
- Extras: Superconducting magnets
  - RHIC, LHC, etc

# Other References

- Magnet design and a construction is a specialized field all its own
  - Electric, Magnetic, Electromagnetic modeling
    - 2D, 3D, static vs dynamic
  - Materials science
    - Conductors, superconductors, ferrites, superferrites
  - Measurements and mapping
    - e.g. g-2 experiment: 1 PPM field uniformity, 14m SC dipole

- Entire CERN accelerator school courses have been given on just magnet design
  - https://indico.cern.ch/event/1227234/ (Nov-Dec 2023)

# g-2 magnet


- Magnet moved from Brookhaven to Fermilab in 2013
  - 17 tons, 44m circumference, 18 cm gap
  - 35 days, over 3200 miles
  - http://muon-g-2.fnal.gov/bigmove/


# EM/Maxwell Review I

- Recall our relativistic Lorentz force

$$\boxed{\frac{d(\gamma m\vec{v})}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)}$$

  - For large $\gamma$ common in accelerators, transverse magnetic fields are **much** more effective for changing particle momenta
  - Can mostly separate E (RF, septa, DC sources) and B (DC or adiabatically-ramping magnets)
    - Some exceptions, e.g. plasma wakefields, betatrons, RFQs
- Easiest/simplest: magnets with constant B field
  - Constant-strength optics
    - Most varying B field accelerator magnets change field so slowly that E fields are negligible
    - Consistent with our constant-field assumptions

# EM/Maxwell Review II

- Maxwell's Equations for $\vec{B}, \vec{H}$ and magnetization $\vec{M}$ are

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \vec{\nabla} \times \vec{H} = \vec{j} + \cancel{\epsilon_0 \frac{\partial \vec{E}}{\partial t}} = \vec{j} \qquad \vec{H} \equiv \vec{B}/\mu - \vec{M}$$

- A magnetic vector potential $\vec{A}$ exists

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \text{since } \vec{\nabla} \cdot \vec{\nabla} \times \vec{A} = 0$$

- Transverse 2D ($B_z$=$H_z$=0), paraxial approx ($p_{x,y}$<<$p_0$)
- Away from magnet coils ( $\vec{j} = 0, \ \vec{M} = 0$ )
  - Simple homogeneous differential equations for fields

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \qquad \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = 0$$

# Parameterizing Solutions

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \qquad \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = 0$$

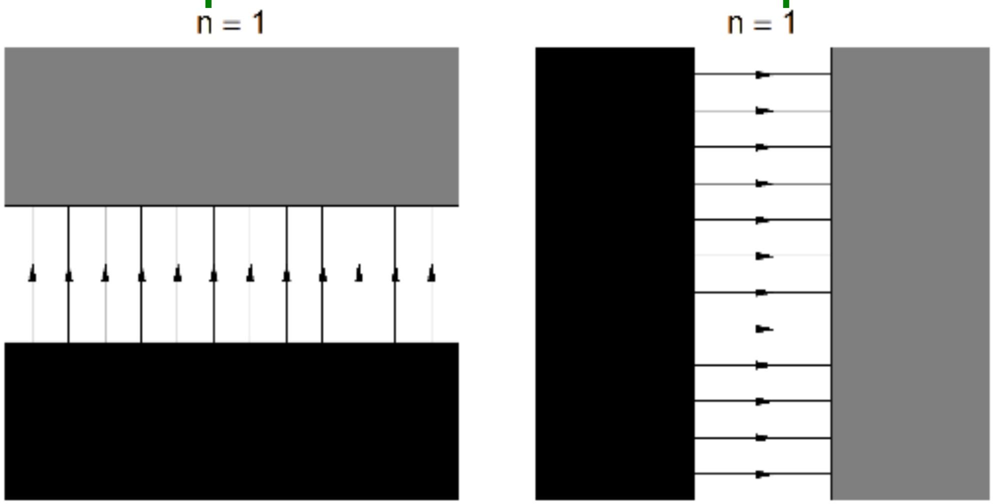
- What are solutions to these equations?
  - Constant field: $\vec{B} = B_x x^0 \hat{x} + B_y y^0 \hat{y}$
    - Dipole fields, usually either only $B_x$ or $B_y$
    - 360 degree (2π) rotational "symmetry"
  - First order field: $\vec{B} = (B_{xx}x + B_{xy}y)\hat{x} + (B_{yx}x + B_{yy}y)\hat{y}$
    - Maxwell reduces 4 vars to 2: $B_s=B_{xx}=-B_{yy}$ and $B_n=B_{xy}=B_{yx}$

$$\vec{B} = B_n(x\hat{y} + y\hat{x}) + B_s(x\hat{x} - y\hat{y})$$

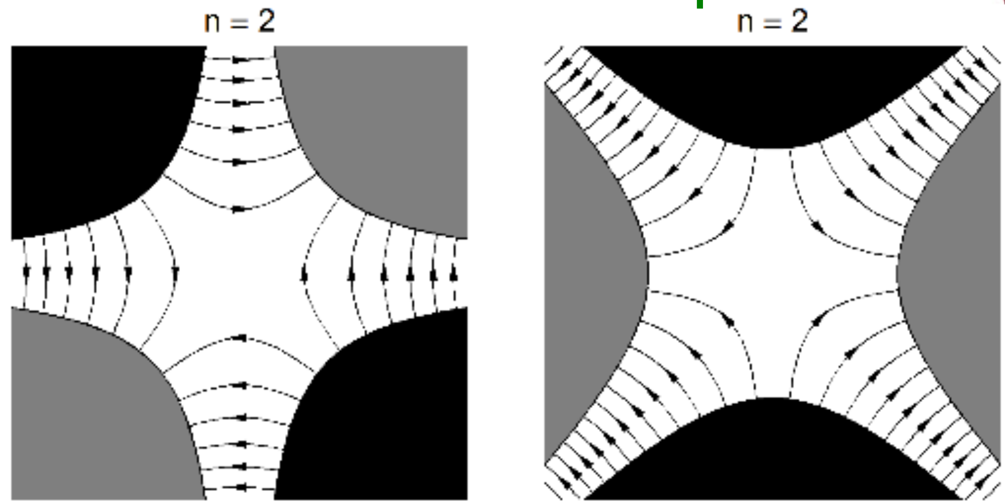
    - Quadrupole fields, either normal $B_n$ or skew $B_s$
    - 180 degree (π) rotational symmetry
    - 90 degree rotation interchanges normal/skew
  - Higher order…

# Visualizing Fields I

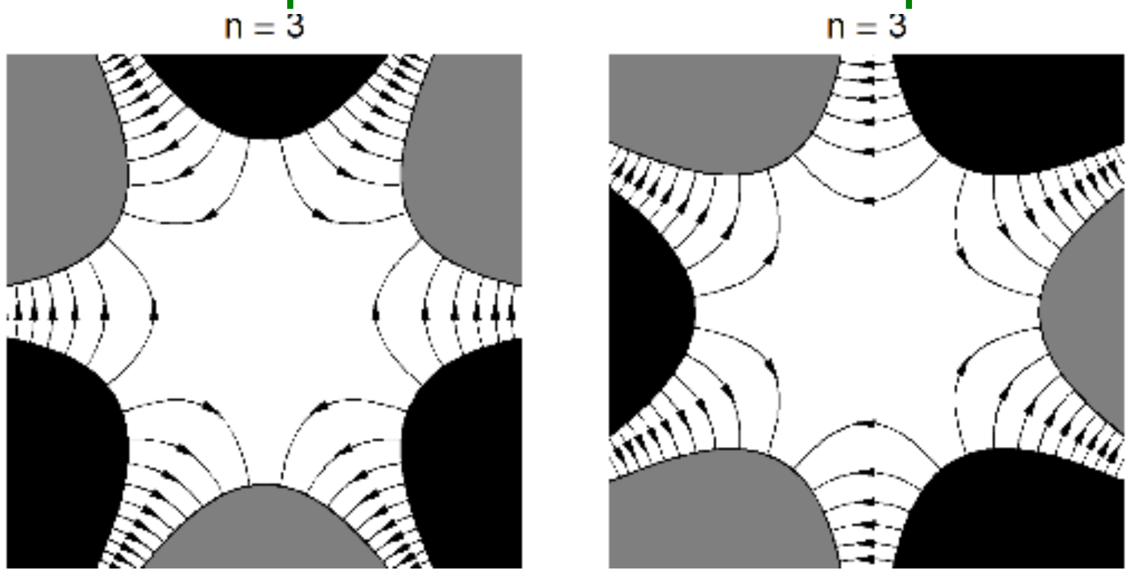
## Dipole and "skew" dipole




## Quad and skew quad

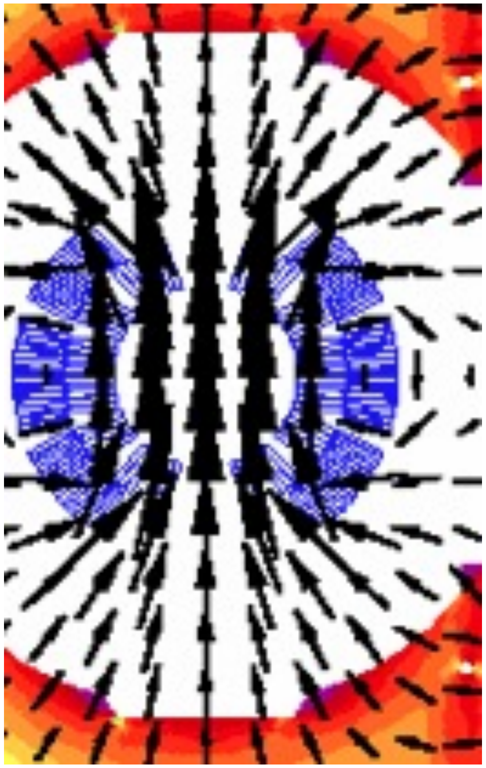


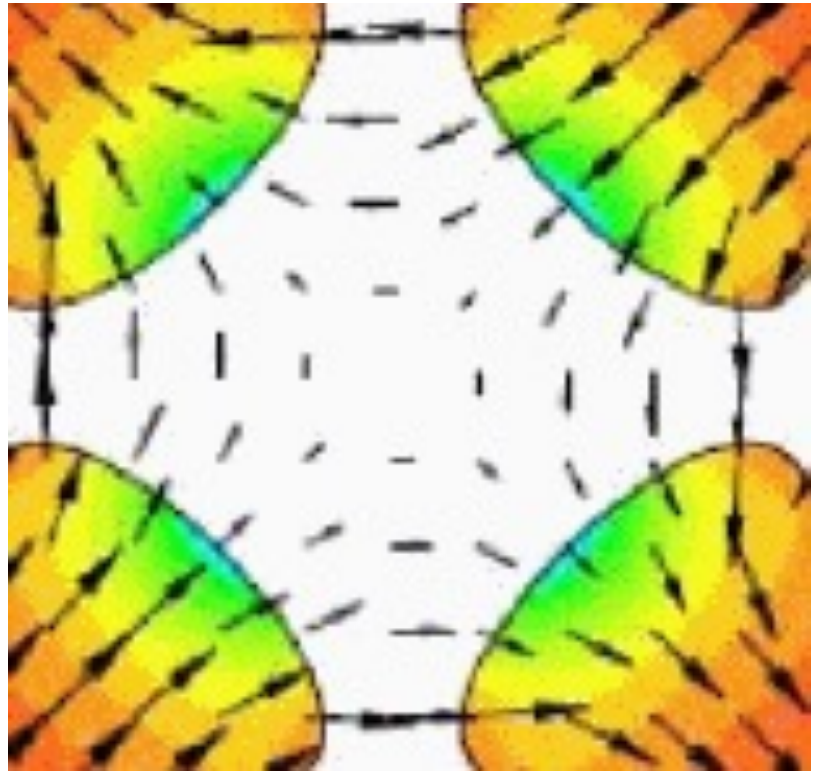

## Sextupole and skew sextupole



# Visualizing Dipole and Quadrupole Fields II

LHC dipole field





LEP quadrupole field

- LHC dipole: $B_y$ gives horizontal bending
- LEP quadrupole: $B_y$ on x axis, $B_x$ on y axis
  - Horizontal focusing=vertical defocusing or vice-versa
  - No coupling between horizontal/vertical motion
    - Note the nice “harmonic” field symmetries

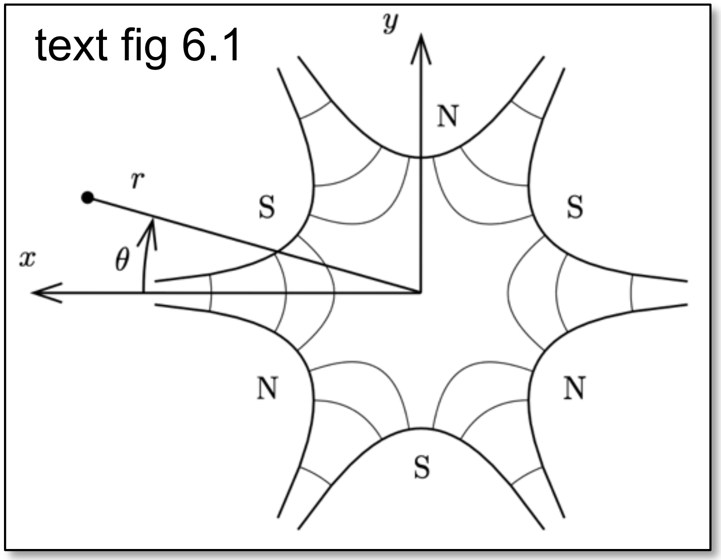
# General Multipole Field Expansions

- Rotational symmetries, cylindrical coordinates
  - Power series in radius r with angular harmonics in θ

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$B_y = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{a}\right)^n (b_n \cos n\theta - a_n \sin n\theta)$$

$$B_x = B_0 \sum_{n=0}^{\infty} \left(\frac{r}{a}\right)^n (a_n \cos n\theta + b_n \sin n\theta)$$




  - Need "reference radius" a (to get units right)
  - ($b_n$,$a_n$) are called (normal,skew) **multipole coefficients**
  - We can also write this succinctly using de Moivre as

$$B_x - iB_y = B_0 \sum_{n=0}^{\infty} (a_n - ib_n) \left(\frac{x+iy}{a}\right)^n$$

# (But Do These Equations Solve Maxwell?)

- Yes ☺ Convert Maxwell's eqns to cylindrical coords

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \qquad \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} = 0$$

$$\frac{\partial(\rho B_\rho)}{\partial \rho} + \frac{\partial B_\theta}{\partial \theta} = 0 \qquad \frac{\partial(\rho B_\theta)}{\partial \rho} - \frac{\partial B_\rho}{\partial \theta} = 0$$

  - Aligning r along the x-axis it's easy enough to see

$$\frac{\partial}{\partial x} \Rightarrow \frac{\partial}{\partial r} \qquad \frac{\partial}{\partial y} \Rightarrow \frac{1}{r}\frac{\partial}{\partial \theta}$$

  - In general it's (much, much) more tedious but it works

$$\frac{\partial r}{\partial x} = \frac{1}{\cos\theta}, \quad \frac{\partial \theta}{\partial x} = \frac{-1}{r\sin\theta}, \quad \frac{\partial r}{\partial y} = \frac{1}{\sin\theta}, \quad \frac{\partial \theta}{\partial y} = \frac{1}{r\cos\theta}$$

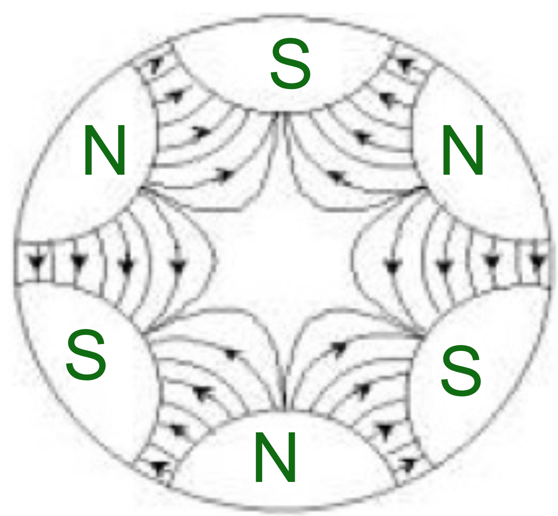
$$\frac{\partial B_x}{\partial x} = \frac{\partial B_x}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial B_x}{\partial \theta}\frac{\partial \theta}{\partial x} \ldots$$

# Multipoles

$(b,a)_n$ "unit" is $10^{-4}$ (natural scale)    $(b,a)_n$ (US) = $(b,a)_{n+1}$

| coefficient | multipole | field | notes |
|---|---|---|---|
| $b_0$ | normal dipole | $B_y = B_0 b_0$ | horz. bending |
| $a_0$ | skew dipole | $B_x = B_0 a_0$ | vert. bending |
| $b_1$ | normal quadrupole | $B_x = B_0 \left(\frac{r}{a}\right) b_1 \sin\theta = B_0 \left(\frac{y}{a}\right) b_1$ <br> $B_y = B_0 \left(\frac{r}{a}\right) b_1 \cos\theta = B_0 \left(\frac{x}{a}\right) b_1$ | focusing <br> defocusing |
| $a_1$ | skew quadrupole | $B_x = B_0 \left(\frac{r}{a}\right) a_1 \cos\theta = B_0 \left(\frac{x}{a}\right) a_1$ <br> $B_y = -B_0 \left(\frac{r}{a}\right) a_1 \sin\theta = -B_0 \left(\frac{y}{a}\right) a_1$ | coupling |
| $b_2$ | normal sextupole | $B_x = B_0 \left(\frac{r}{a}\right)^2 b_2 \sin(2\theta)$ <br> $B_y = B_0 \left(\frac{r}{a}\right)^2 b_1 \cos(2\theta)$ | nonlinear! |

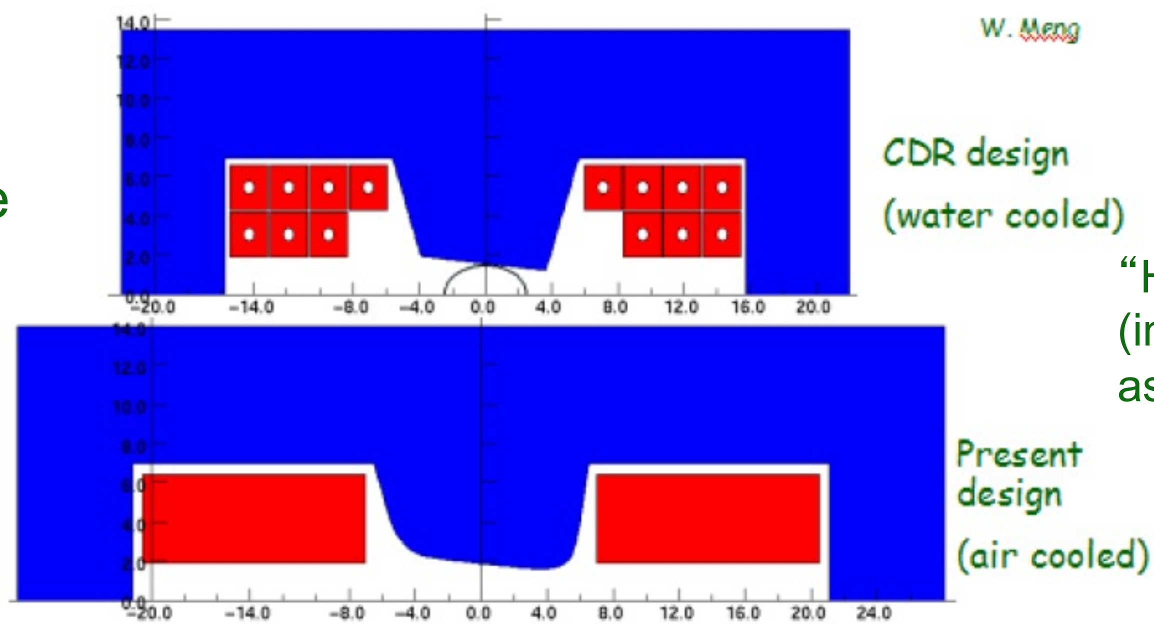

Elettra magnets



# Multipole Symmetries

- Dipole has $2\pi$ rotation symmetry (or $\pi$ upon current reversal)
- Quad has $\pi$ rotation symmetry (or $\pi/2$ upon current reversal)
- k-pole has $2\pi/k$ rotation symmetry upon current reversal
- We try to enforce symmetries in design/construction
  - Limits permissible magnet errors
  - Higher order fields that obey main field symmetry are called allowed multipoles

RCMS half-dipole laminations (W. Meng, BNL)

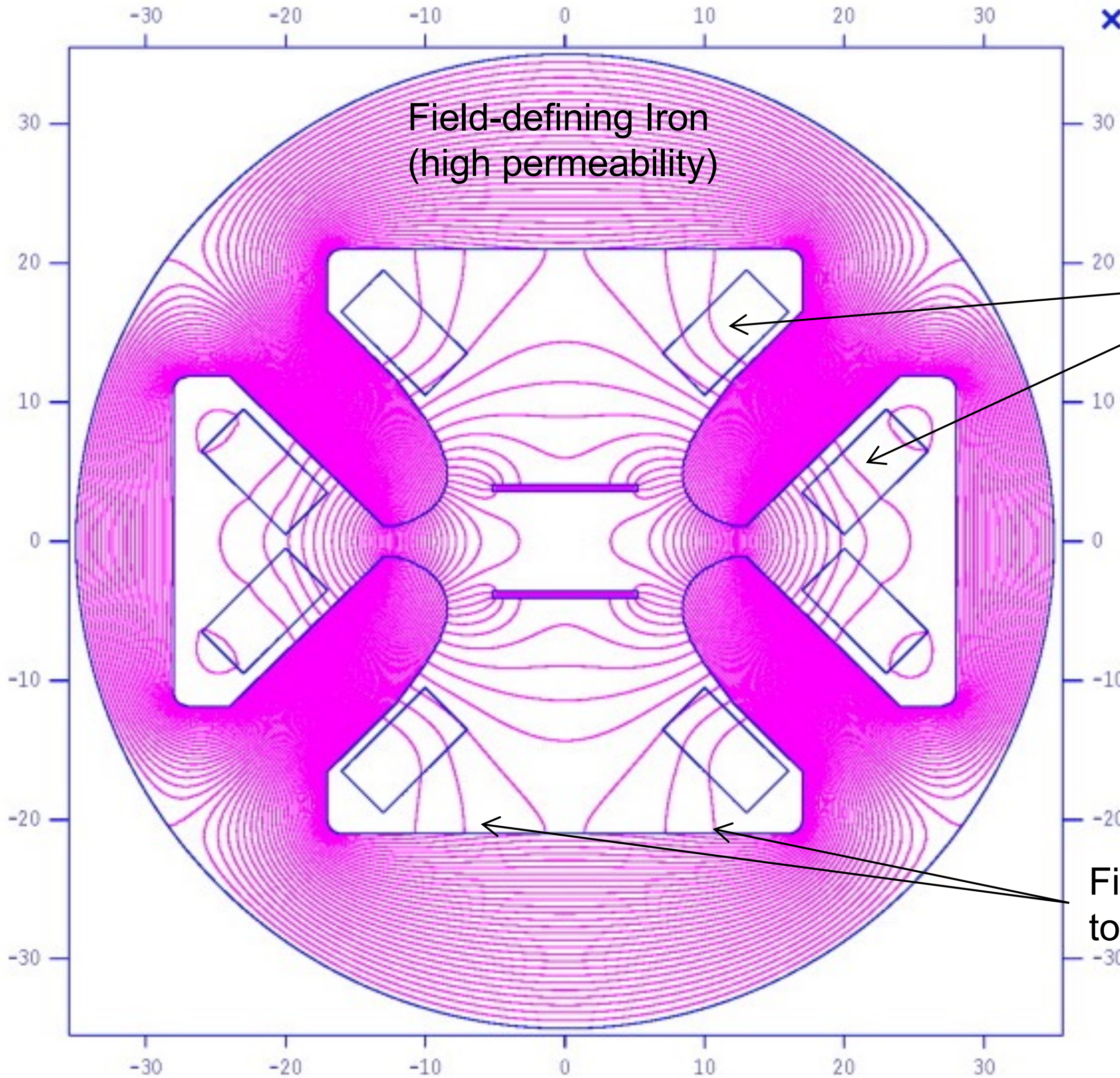



"H style dipoles" (include focusing as well as bending)

# Multipole Symmetries II

- So a dipole (n=0, 2 poles) has allowed multipoles:
  - Sextupole (n=2, 6 poles), Decapole (n=4, 10 poles)...
- A quadrupole (n=1, 4 poles) has allowed multipoles:
  - Dodecapole (n=5, 12 poles), Twenty-pole (n=9, 20 poles)…
- General allowed multipoles: (2k+1)(n+1)-1
  - Or, more conceptually, (3,5,7,…) times number of poles
- Other multipoles are forbidden by symmetries
  - Smaller than allowed multipoles, but no magnets are perfect
    - Large measured forbidden multipoles mean fabrication or fundamental design problems!
- Better magnet pole face quality with punched laminations
- Dynamics are usually dominated by lower-order multipoles

# Equipotentials and Contours




y position [mm]

x position [mm]

Current carrying conductors (coils)

Field lines perpendicular to iron surface

Z. Guo et al, NIM:A 691, pp. 97-108, 1 Nov 2012. A novel structure of multipole field magnets and their applications in uniformizing beam spot at target

# Equipotentials and Contours

- Let's get around to designing some magnets
  - Conductors on outside, field on inside
  - Use high-permeability iron to shape fields: **iron-dominated**
    - Pole faces are very nearly equipotentials
    - We work with a magnetostatic *scalar* potential $\Psi$
    - B, H field lines are perpendicular to equipotential lines of $\Psi$

$$\vec{H} = \vec{\nabla}\Psi$$

$$\Psi = \sum_{n=0}^{\infty} \frac{a}{n+1}\left(\frac{r}{a}\right)^{n+1}\left[F_n\cos((n+1)\theta) + G_n\sin((n+1)\theta)\right]$$

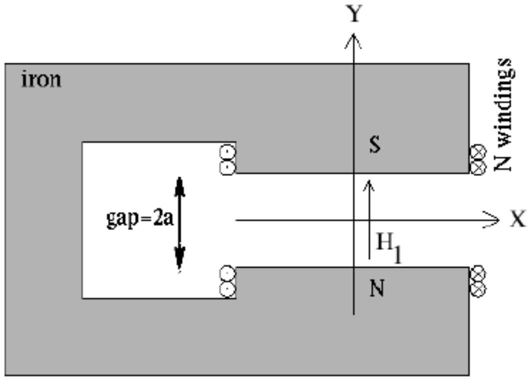
$$\text{where } G_n \equiv B_0 b_n/\mu_0,\, F_n \equiv B_0 a_n/\mu_0$$

This comes from integrating our B field expansion.
Let's look at normal multipoles $G_n$ and pole faces…

# Equipotentials and Contours II

- For general $G_n$ normal multipoles (i.e. for $b_n$)

$$\Psi(\text{equipotential for}\, b_n) \propto r^{n+1} \sin[(n+1)\theta] = \text{constant}$$

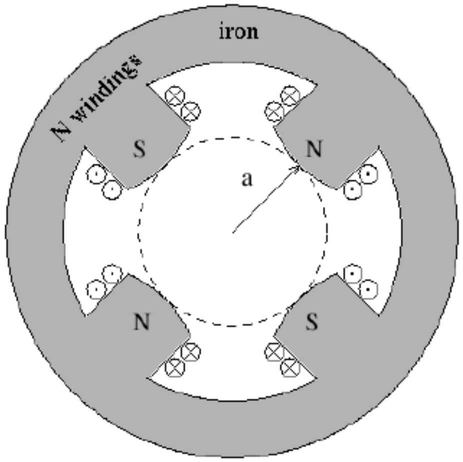



- Dipole (n=0): $\Psi(\text{dipole}) \propto r \sin\theta = y$
  - Normal dipole pole faces are y=constant
- Quadrupole (n=1):

$$\Psi(\text{quadrupole}) \propto r^2 \sin(2\theta) = 2xy$$

  - Normal quadrupole pole faces are xy=constant (hyperbolic)

- So what conductors and currents are needed to generate these fields?



# E&M Tools and Reminders (Wikipedia)

The magnetic $\mathbf{H}$ field is defined:[10]:269[11]:192[1]:ch36

**Definition of the $\mathbf{H}$ field** *(vector form, SI units)*

$$\mathbf{H} \equiv \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$$

where $\mu_0$ is the vacuum permeability, and $\mathbf{M}$ is the magnetization vector. In a vacuum, $\mathbf{B}$ and $\mathbf{H}$ are proportional to each other. Inside a material they are different (see H and B inside and outside magnetic materials). The SI unit of the $\mathbf{H}$-field is the ampere per metre (A/m),[14] and the CGS unit is the oersted (Oe).[12][9]:286

**Alternative names for H**[7]

- Magnetic field intensity[8]
- Magnetic field strength[4]:139
- Magnetic field
- Magnetizing field
- Auxiliary magnetic field

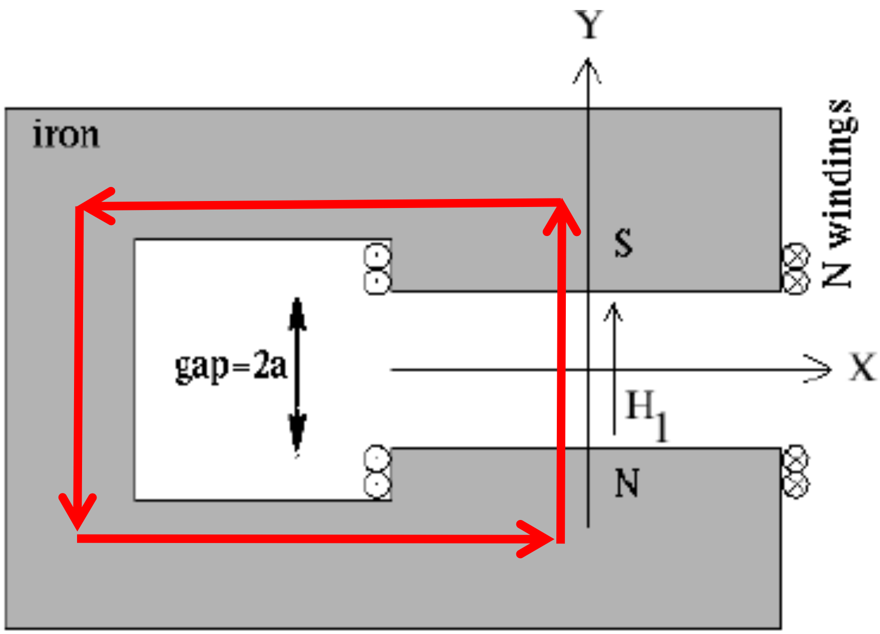
- *Extremely* common to call **B** the magnetic field (really flux density)
- Fields that (our) particles experience in vacuum chambers $\vec{\mathbf{B}} = \mu_0 \vec{\mathbf{H}}$
- Ampere's law relates currents and fields

$$I_{\text{enclosed}} = \oint \vec{\mathbf{H}} \cdot d\vec{l} \qquad \mu_0 I_{\text{enclosed}} = \oint \vec{\mathbf{B}} \cdot d\vec{l}$$

- Permanent magnets get more complicated: magnetization $\vec{\mathbf{M}}$

# Dipole Field/Current

- Use Ampere's law to calculate field in gap
  - N "turns" of conductor around each pole
  - Each turn of conductor carries current I




(C-style dipole)

- Field integral is through N-S poles and (highly permeable) iron (including return path)

$$2NI = \oint \vec{H} \cdot d\vec{l} = 2aH \quad \Rightarrow \quad H = \frac{NI}{a}, \quad B = \frac{\mu_0 NI}{a}$$

$$\Delta x' = \frac{BL}{(B\rho)}$$

- NI is in "Amp-turns", $\mu_0$~1.257 cm-G/A
  - So a=2cm, B=600G requires NI~955 Amp-turns

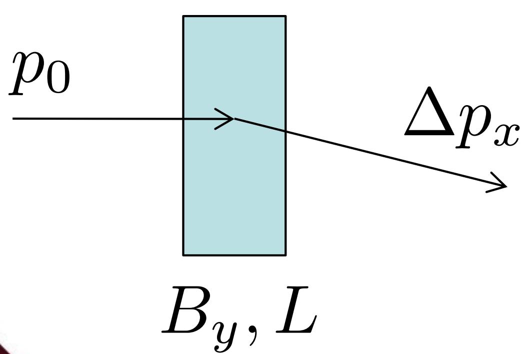
# Wait, What's That Δx' Equation?

$$\Delta x' = \frac{BL}{(B\rho)}$$

⟵ Field, length: Properties of **magnet**

⟵ **Rigidity**: property of **beam** (really p/q!)

- This is the angular transverse kick from a thin hard-edge dipole, like a dipole corrector
  - Really a change in $p_x$ but paraxial approximation applies
  - The B in (Bρ) is not necessarily the main dipole B
  - The ρ in (Bρ) is not necessarily the ring circumference/2π
  - And neither is related to this particular dipole kick!



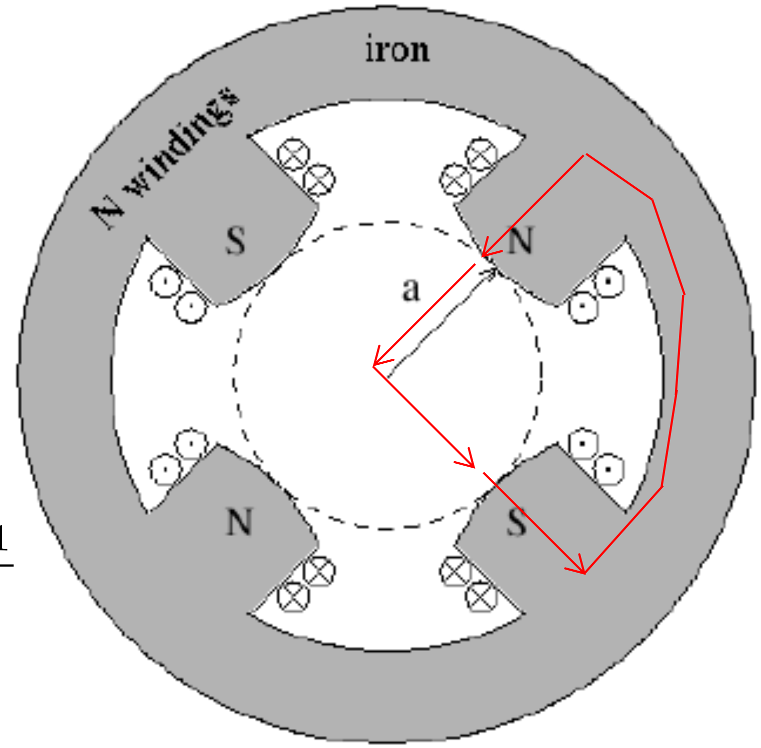

$$F_x = \frac{\Delta p_x}{\Delta t} = q(\beta c)B_y \quad \Delta t = L/(\beta c)$$

$$\Delta p_x = qLB_y$$

$$\Delta x' \approx \frac{\Delta p_x}{p} = \frac{q}{p}LB_y = \frac{B_y L}{(B\rho)}$$

# Quadrupole Field/Current

- Use Ampere's law again
  - Easiest to do with magnetic potential Ψ, encloses 2NI




$$\Psi(a,\theta) = \frac{a}{2}\frac{B_0 b_1}{\mu_0}\sin(2\theta)$$

$$2NI = \oint \vec{H}\cdot d\vec{l} = \Psi(a,\pi/4) - \Psi(a,-\pi/4) = \frac{aB_0 b_1}{\mu_0}$$

$$\Psi = NI\sin(2\theta) = \frac{2NI}{a^2}xy$$

$$\vec{H} = \nabla\Psi = \frac{2NI}{a^2}(y\hat{x} + x\hat{y}) \qquad \vec{B} = \frac{2\mu_0 NI}{a^2}(y\hat{x} + x\hat{y})$$

- Quadrupole strengths are expressed as transverse gradients

$$B' \equiv \frac{\partial B_y}{\partial x}\Big|_{y=0} = \frac{\partial B_x}{\partial y} = \frac{2\mu_0 NI}{a^2} \qquad \Delta x' = \frac{B'L}{(B\rho)}x$$

(NB: Be careful, ' has different meaning in B', B'', B'''…)

# Quadrupole Transport Matrix

- Paraxial equations of motion for constant quadrupole field

$$\frac{d^2x}{ds^2} + kx = 0 \qquad \frac{d^2y}{ds^2} - ky = 0 \qquad s \equiv \beta ct$$

$$k \equiv \frac{B'}{(B\rho)} = \frac{2\mu_0 NI}{a^2}\left(\frac{q}{p}\right)$$

- Integrating over a magnet of length L gives (exactly)

Focusing Quadrupole

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos(L\sqrt{k}) & \frac{1}{\sqrt{k}}\sin(L\sqrt{k}) \\ -\sqrt{k}\sin(L\sqrt{k}) & \cos(L\sqrt{k}) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} = \mathbf{M}_F \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

Defocusing Quadrupole

$$\begin{pmatrix} y \\ y' \end{pmatrix} = \begin{pmatrix} \cosh(L\sqrt{k}) & \frac{1}{\sqrt{k}}\sinh(L\sqrt{k}) \\ \sqrt{k}\sinh(L\sqrt{k}) & \cosh(L\sqrt{k}) \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix} = \mathbf{M}_D \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

# Thin Quadrupole Transport Matrix

Focusing Quadrupole
$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \cos(L\sqrt{k}) & \frac{1}{\sqrt{k}}\sin(L\sqrt{k}) \\ -\sqrt{k}\sin(L\sqrt{k}) & \cos(L\sqrt{k}) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} = \mathbf{M}_F \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

Defocusing Quadrupole
$$\begin{pmatrix} y \\ y' \end{pmatrix} = \begin{pmatrix} \cosh(L\sqrt{k}) & \frac{1}{\sqrt{k}}\sinh(L\sqrt{k}) \\ \sqrt{k}\sinh(L\sqrt{k}) & \cosh(L\sqrt{k}) \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix} = \mathbf{M}_D \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

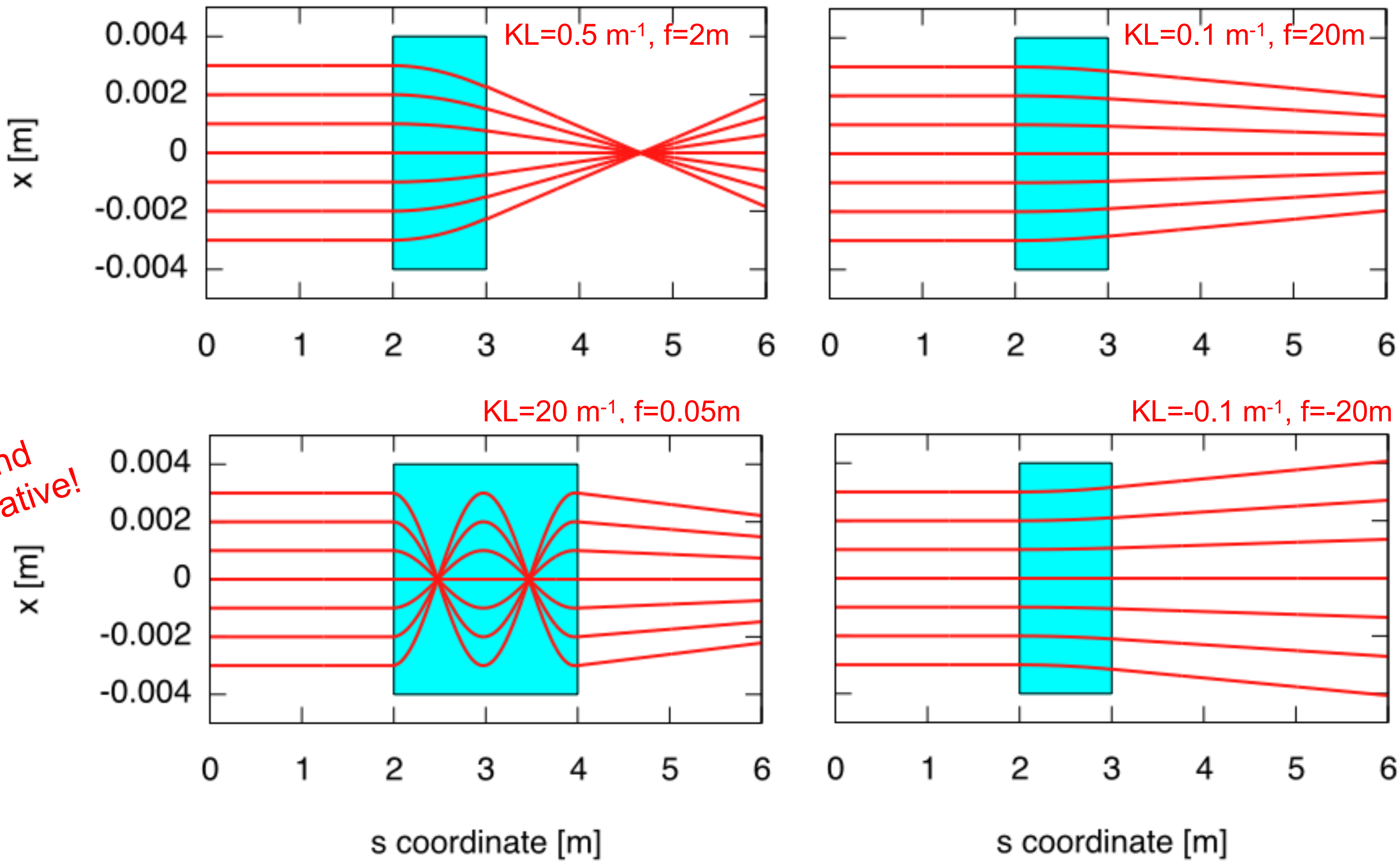
- Quadrupoles are often "thin"
  - Focal length is much longer than magnet length
- Then we can use the thin-lens approximation $L\sqrt{k} \ll 1$

Thin quadrupole approximation
$$\mathbf{M}_{F,D} = \begin{pmatrix} 1 & 0 \\ \mp kL & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \mp\frac{1}{f} & 1 \end{pmatrix}$$

where f=1/(kL) is the quadrupole focal length

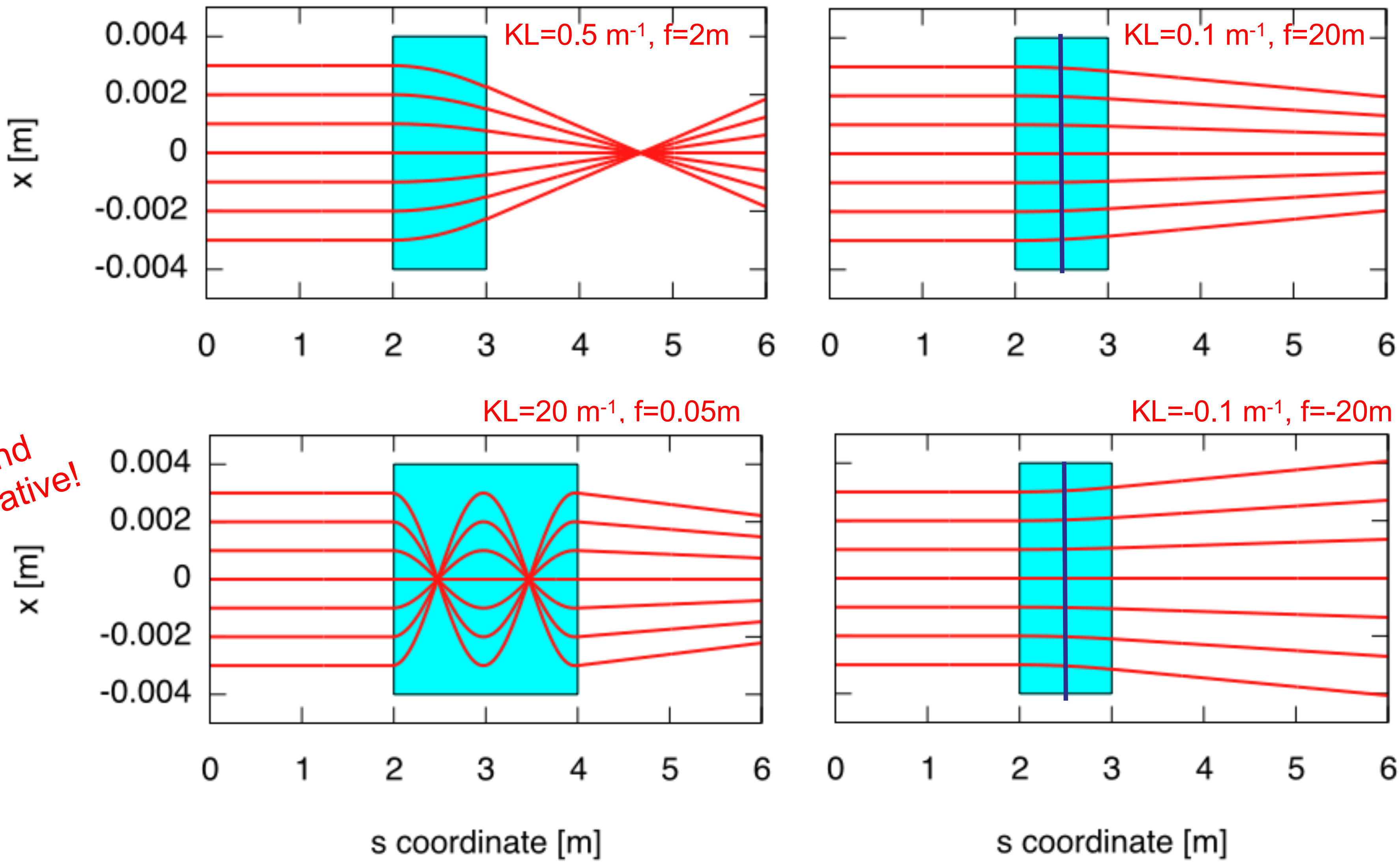
$$\Delta x' = \frac{B'L}{(B\rho)} x$$

# Picturing Drift and Quadrupole Motion




Fun and illustrative!

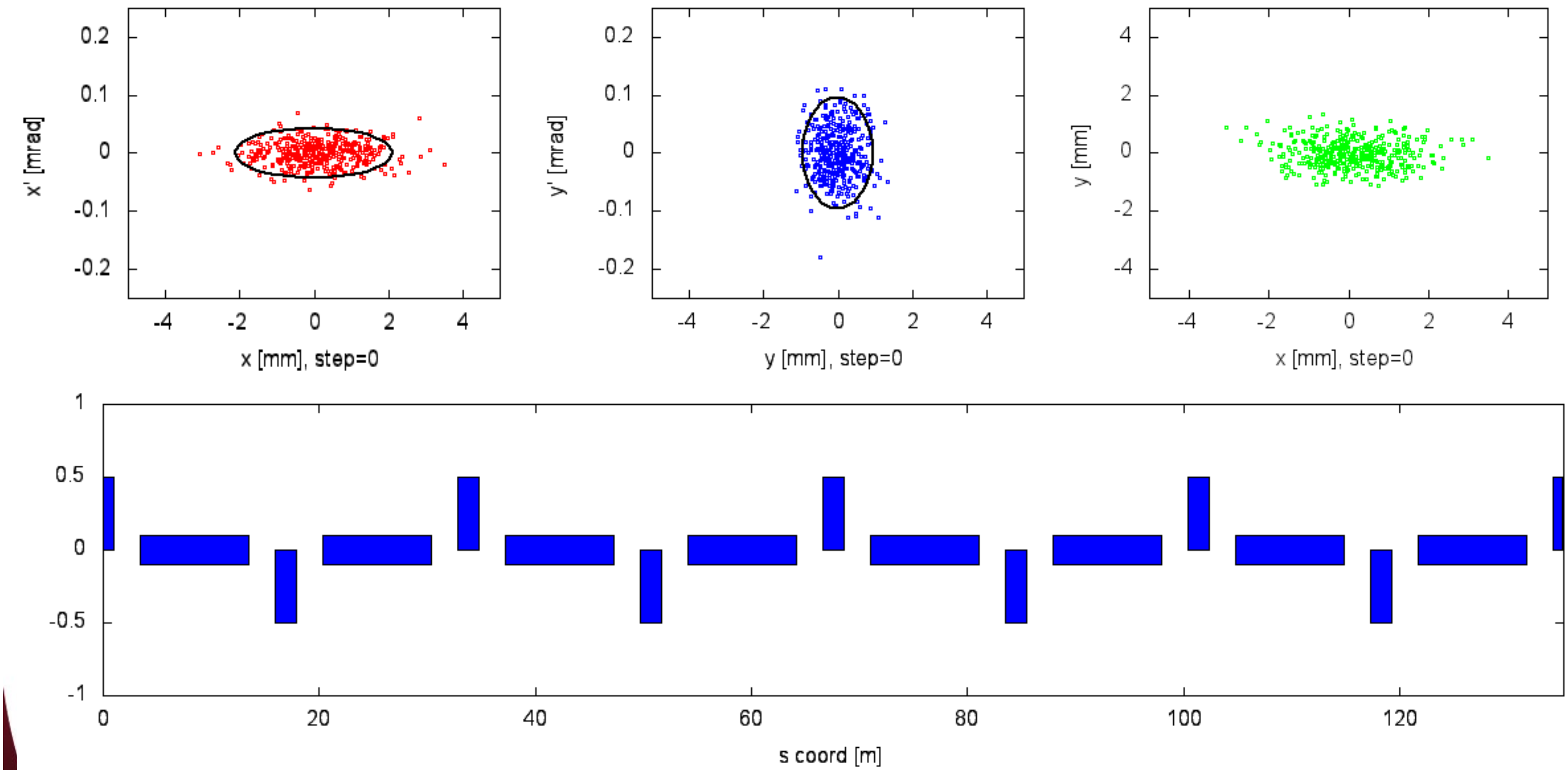
# Picturing Drift and Quadrupole Motion




Fun and illustrative!

Thin Quadrupole Approximations

# Break




Watch the quadrupole motion carefully
Large focusing effect; small defocusing effect

# Higher Orders

- We can follow the full expansion for 2(n+1)-pole:

$$\Psi_n = NI \left(\frac{r}{a}\right)^{n+1} \sin((n+1)\theta)$$

$$H_x = (n+1)\frac{NI}{a}\left(\frac{r}{a}\right)^n \sin n\theta \qquad H_y = (n+1)\frac{NI}{a}\left(\frac{r}{a}\right)^n \cos n\theta$$

- For the sextupole (n=2) we find the nonlinear field as

$$\vec{B} = \frac{3\mu_0 NI}{a^3}[2xy\hat{x} + (x^2+y^2)\hat{y}]$$

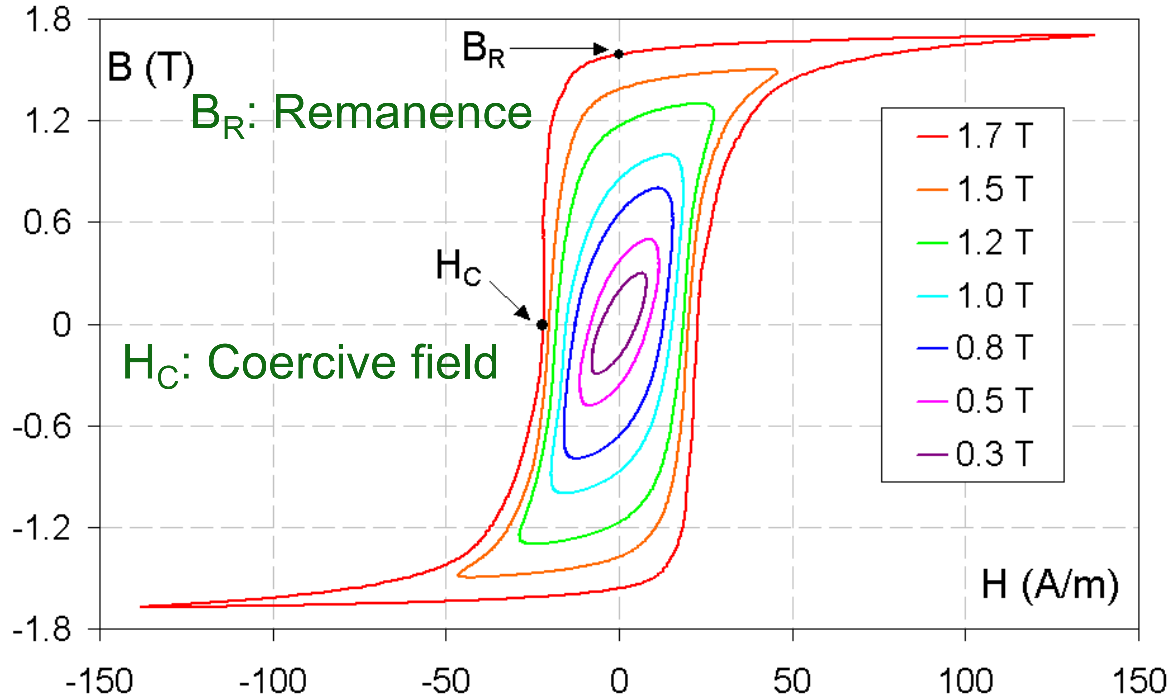
- Now define a strength as an n[th] derivative

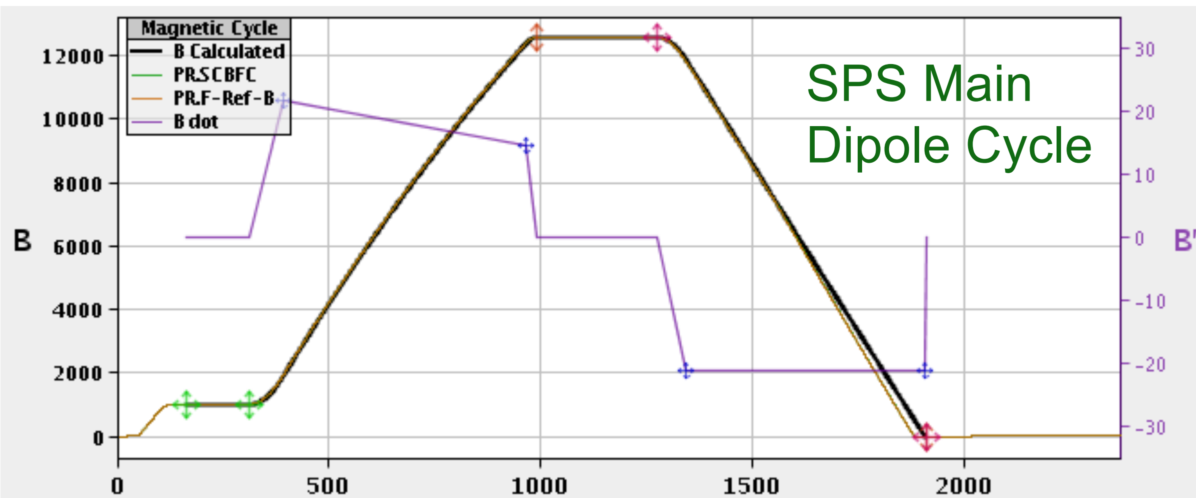
$$B'' \equiv \frac{\partial^2 B_y}{\partial x^2}\Big|_{y=0} = \frac{6\mu_0 NI}{a^3} \qquad \Delta x' = \frac{1}{2}\frac{B''L}{(B\rho)}(x^2+y^2)$$

(NB: Be careful, ‘ has different meaning in B’, B’’, B’’’…)

# Hysteresis

- Magnets with variable current carry "memory"
  - Hysteresis is quite important in iron-dominated magnets
- Usually try to run magnets "on hysteresis"
  - e.g. always on one side of hysteresis loop
  - Large spread at large field (1+ T): saturation
  - Degaussing



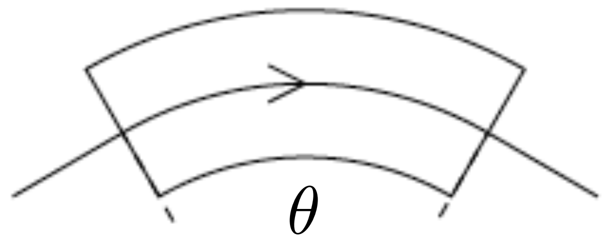
# End Fields

- Magnets are not infinitely long: ends are important!
  - Conductors: where coils usually come in and turn around
  - Longitudinal symmetries break down
  - Sharp corners on iron are first areas to saturate
  - Usually a concern over distances of $\pm$1-2 times magnet gap
    - A big deal for short, large-aperture magnets; ends dominate!
- Solution: simulate… a lot
  - Test prototypes too
  - Quadratic end chamfer eases sextupoles from ends (first allowed harmonic of dipole)
- More on dipole end focusing…

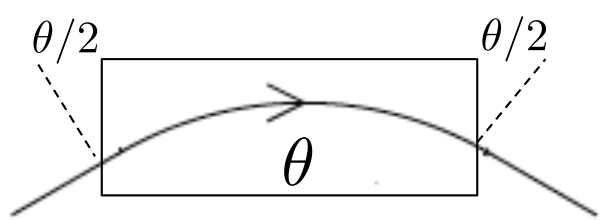
PEFP prototype magnet (Korea)
9 cm gap,1.4m long

# Dipoles, Sector and Rectangular Bends

- Sector bend (sbend)
  - Beam design entry/exit angles are perpendicular to end faces

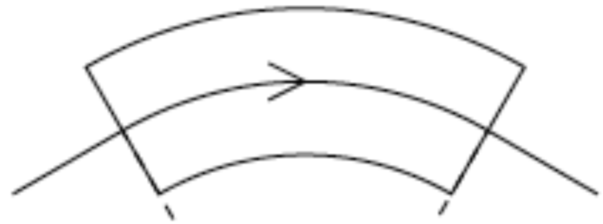



  - Simpler to conceptualize, but harder to build
- Rectangular bend (rbend)
  - Beam design entry/exit angles are half of bend angle




  - Easier to build, but must include effects of edge focusing

# Sector Bend Transport Matrix



- The dipole "rotation" that we see in phase space movies

$$M_{\text{sector dipole}} = \begin{pmatrix} \cos\theta & \rho\sin\theta & 0 & 0 & 0 & \rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & 0 & 0 & \sin\theta & \\ 0 & 0 & 1 & \rho\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin\theta & -\rho(1-\cos(\theta)) & 0 & 0 & 1 & -\rho(\theta-\sin\theta) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

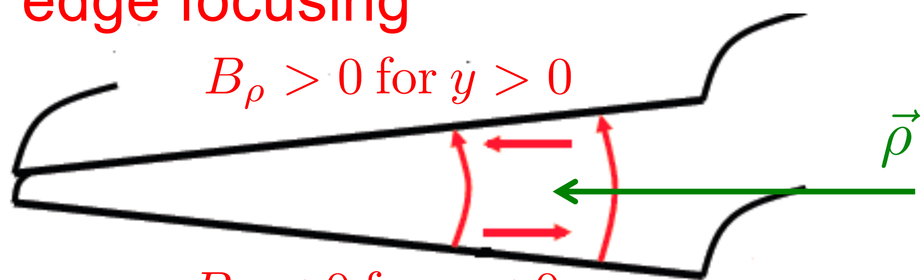
- Has all the "right" behaviors
  - But what about edge focusing and "rectangular" bends?

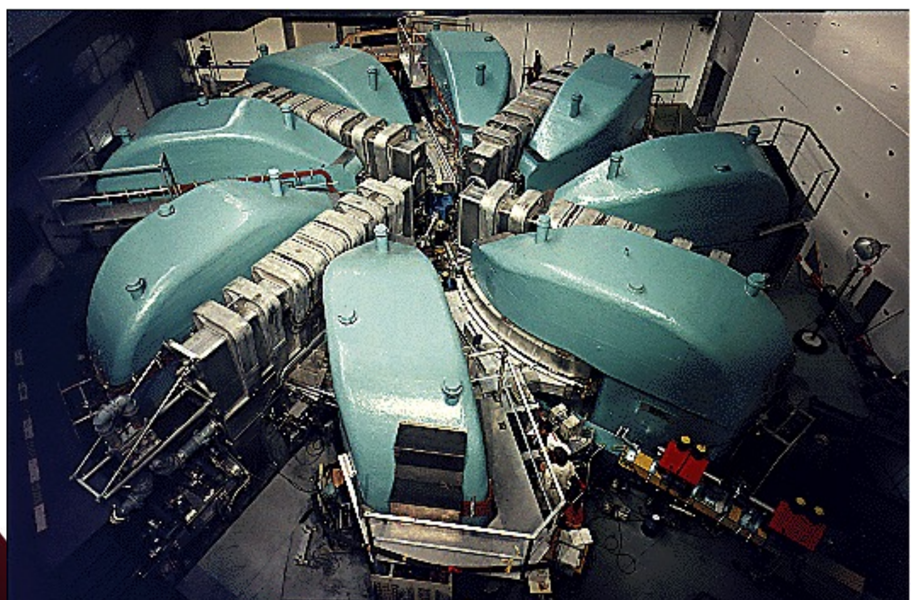
# Flashback: Modern Isochronous Cyclotrons

- Higher bending field at higher energies
  - But also introduces vertical defocusing
  - Use bending magnet "edge focusing"

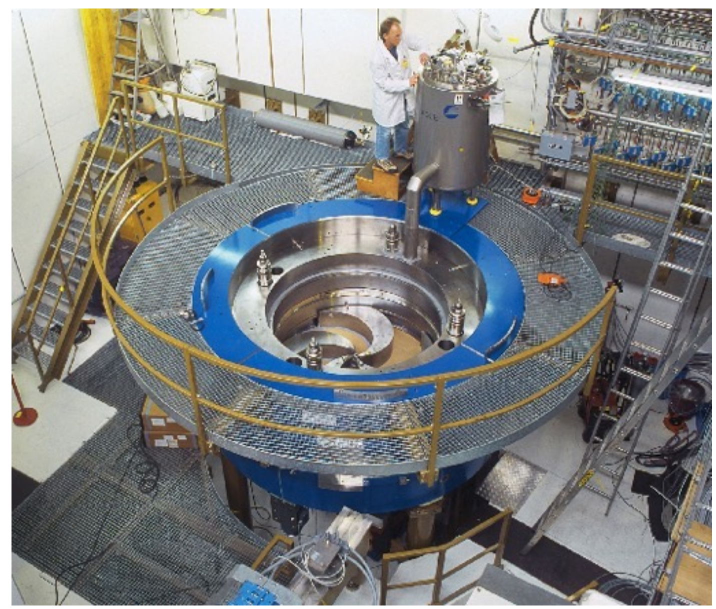
    (Weds magnet lecture)

$$f_{\rm rf} = \frac{qB(\rho)}{2\pi\gamma(\rho)m}$$



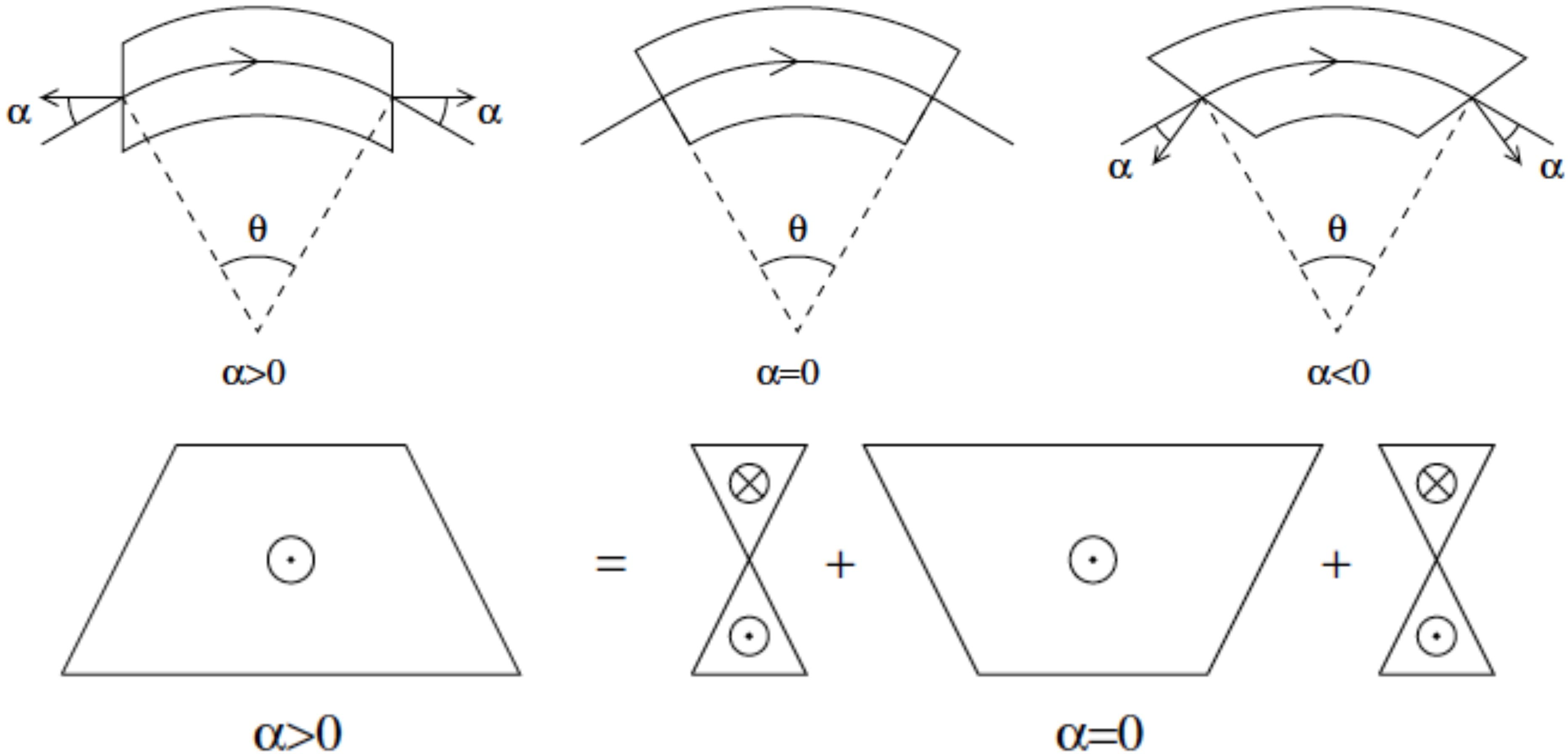



590 MeV PSI Isochronous Cyclotron (1974)



250 MeV PSI Isochronous Cyclotron (2004)

# Dipole End Angles



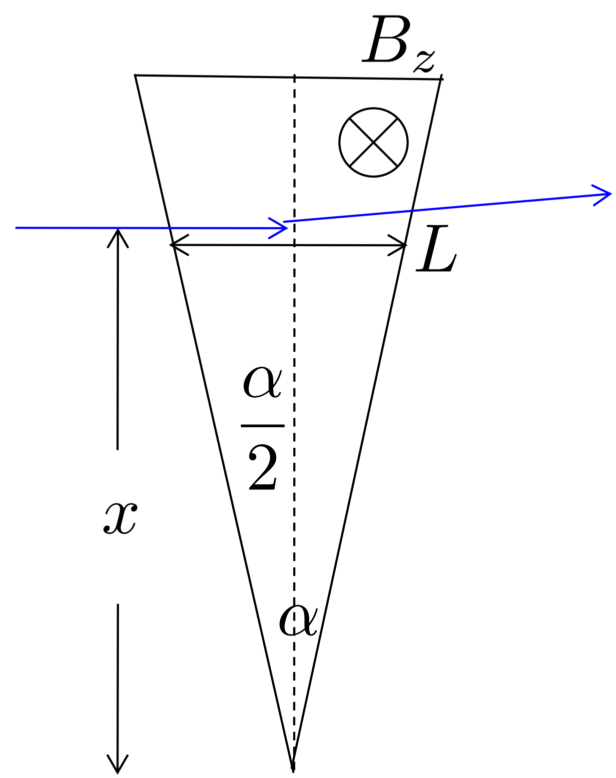

- We treat general case of symmetric dipole end angles
  - Superposition: looks like wedges on end of sector dipole
  - Rectangular bends are a special case

# Kick from a Thin Wedge

- The edge focusing calculation requires the kick from a thin wedge

$$\Delta x' = \frac{B_z L}{(B\rho)}$$

What is L? (distance in wedge)




$$\tan\left(\frac{\alpha}{2}\right) = \frac{L/2}{x}$$

$$L = 2x\tan\left(\frac{\alpha}{2}\right) \approx x\tan\alpha$$

$$\text{So}\;\; \Delta x' = \frac{B_z \tan\alpha}{(B\rho)} x = \frac{\tan\alpha}{\rho} x$$

Here ρ is the curvature for a particle of this momentum!!

# Dipole Matrix with Ends

- The matrix of a dipole with thick ends is then

$$M_{\text{sector dipole}}(x, x', \delta) = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix}$$

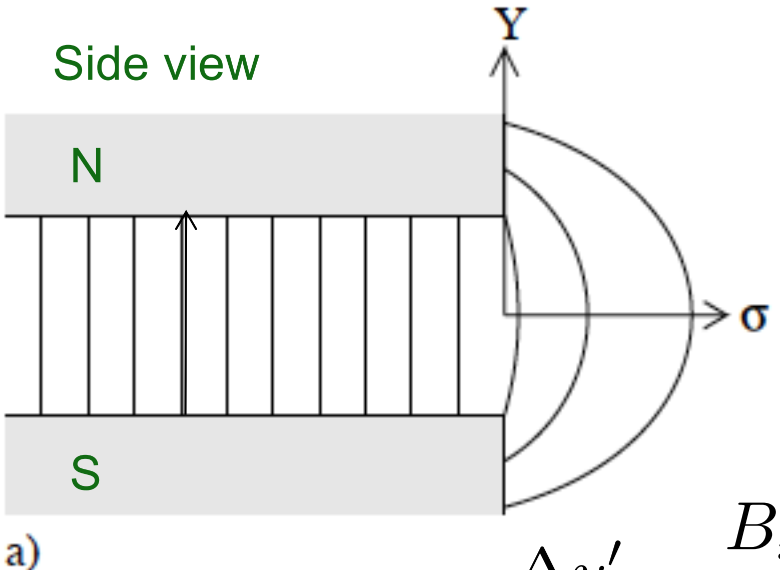
$$M_{\text{end lens}} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{\tan\alpha}{\rho} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

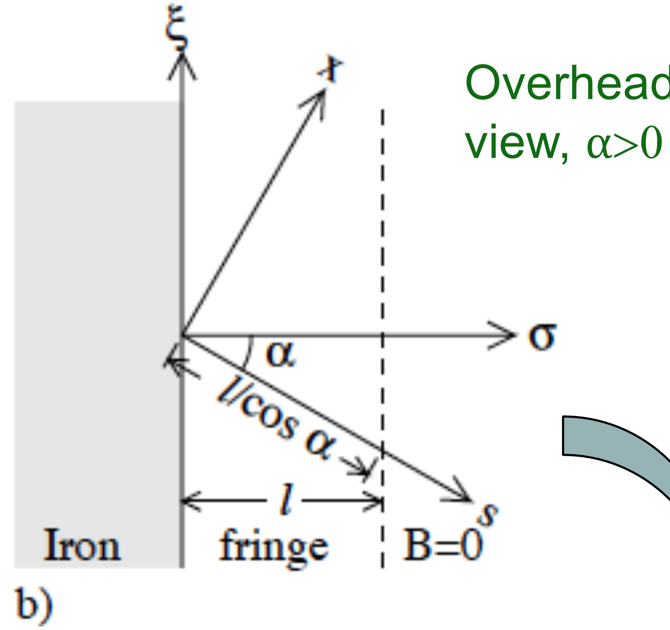
$$M_{\text{edge}-\text{focused dipole}} = M_{\text{end lens}} M_{\text{sector dipole}} M_{\text{end lens}}$$

$$M_{\text{edge}-\text{focused dipole}} = \begin{pmatrix} \frac{\cos(\theta-\alpha)}{\cos\alpha} & \rho\sin\theta & \rho(1-\cos\theta) \\ -\frac{\sin(\theta-2\alpha)}{\rho\cos^2\alpha} & \frac{cos(\theta-\alpha)}{\cos\alpha} & \frac{\sin(\theta-\alpha)+\sin\alpha}{\cos\alpha} \\ 0 & 0 & 1 \end{pmatrix}$$

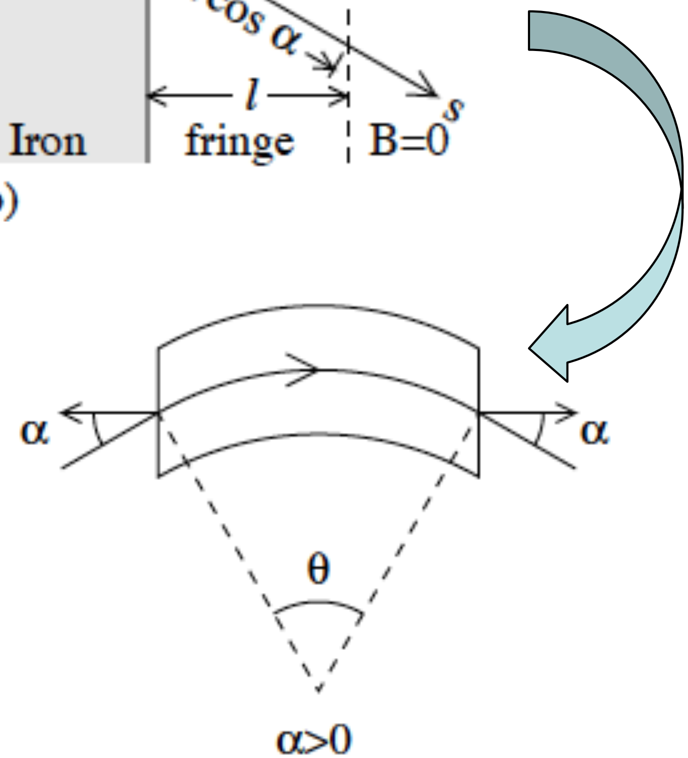
- Rectangular bend is special case where $\alpha=\theta/2$

# (End Fields)




$$\Delta y' = \frac{B_x l_{\mathrm{fringe}}}{(B\rho)}$$



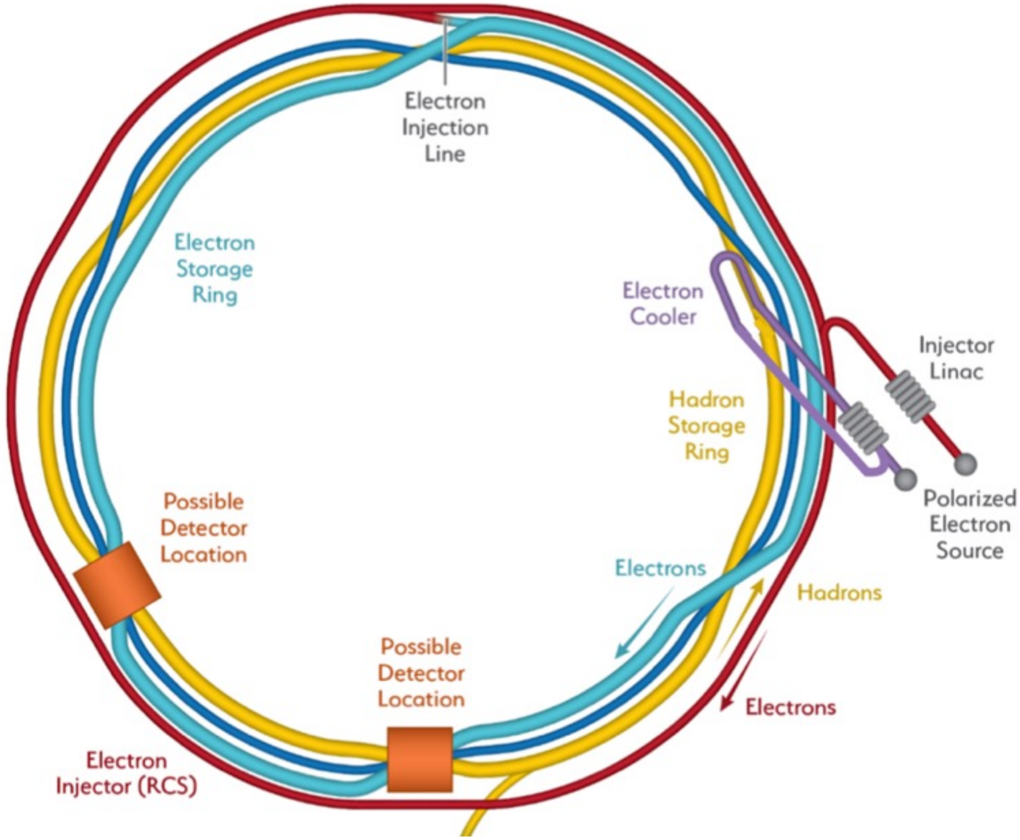

- Field lines go from –y to +y for a positively charged particle
  - $B_x < 0$ for $y > 0$; $B_x > 0$ for $y < 0$
    - Net focusing!
  - Field goes like $\sin(\alpha)$
    - get $\cos(\alpha)$ from integral length

# Application: EIC Rapid Cycling Synchrotron

- EIC RCS design:
  - e- 400 MeV -18 GeV
  - Circumference 3840 m
  - 384 dipoles with ρ~300m

$$B[\mathrm{T}] = \frac{p[\mathrm{GeV/c}]}{0.3\,\rho[\mathrm{m}]} = 0.004 - 0.2\,\mathrm{T}$$

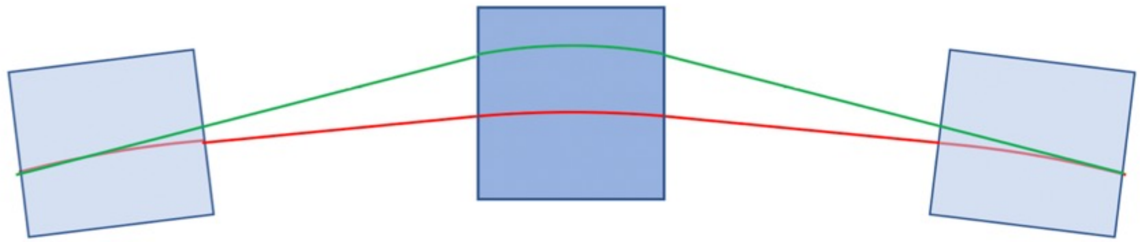
  - 0.004 T = 40 Gauss
  - Earth field is ~1 Gauss!




- Two problems:
  - Injection field too low (should be at least 3x higher)
  - Overall rigidity range of 50x is too aggressive (usually 10-20)
    - Good dipole field quality over this range is a ”technical risk”

# Application: EIC Rapid Cycling Synchrotron

- Idea: Split into three dipoles (two "families"/strengths)



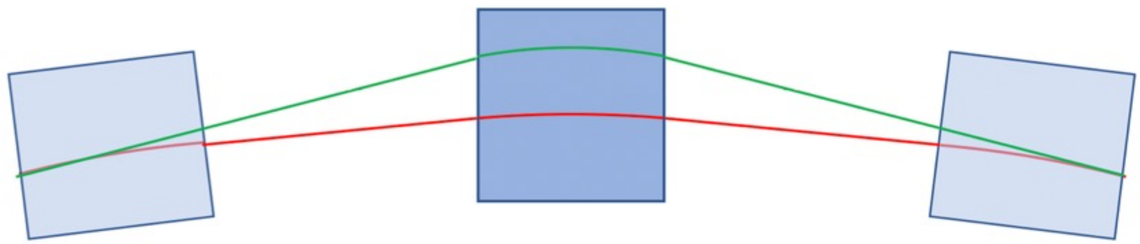
injection: only center dipole on; other dipoles off
top energy: all dipoles on equally
entrance/exit angles kept constant for all energies

- Raises field of center dipole by x3 at injection ☺
- Center dipole does not get too strong at top energy ☺

- Q: What are two challenges with this approach?

# Application: EIC Rapid Cycling Synchrotron

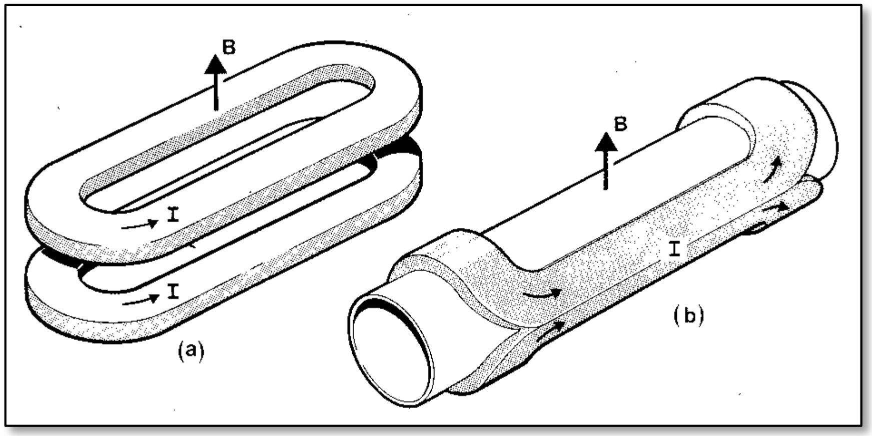
- Idea: Split into three dipoles (two “families”/strengths)

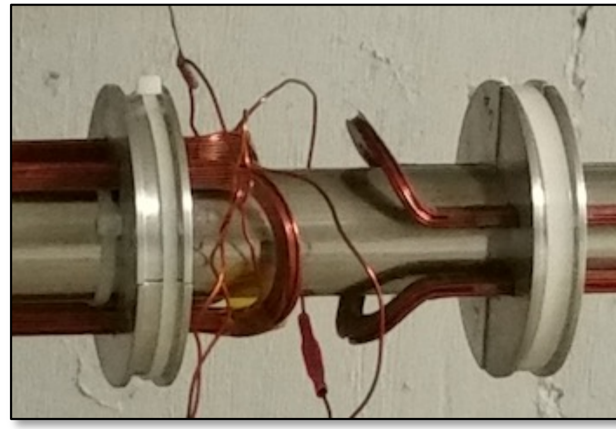


injection: only center dipole on; other dipoles off
top energy: all dipoles on equally
entrance/exit angles kept constant for all energies

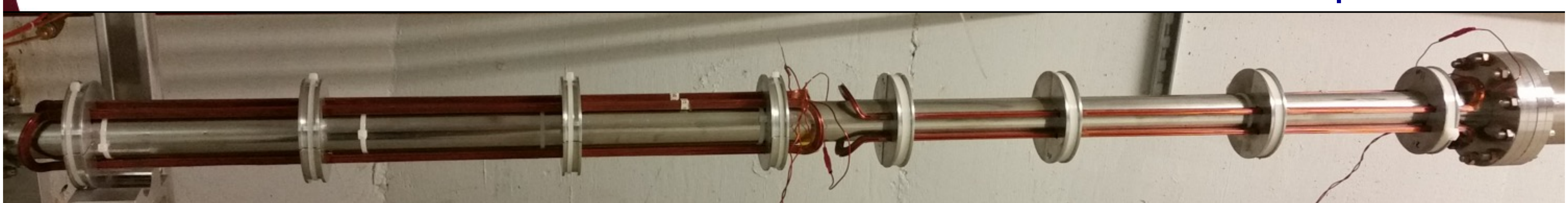
- Raises field of center dipole by x3 at injection ☺
- Center dipole does not get too strong at top energy ☺
- Pathlength (and $f_{rev}$) changes with energy ☹
- Dipole edge focusing/optics change with energy ☹ ☹

# Other Familiar Dipoles

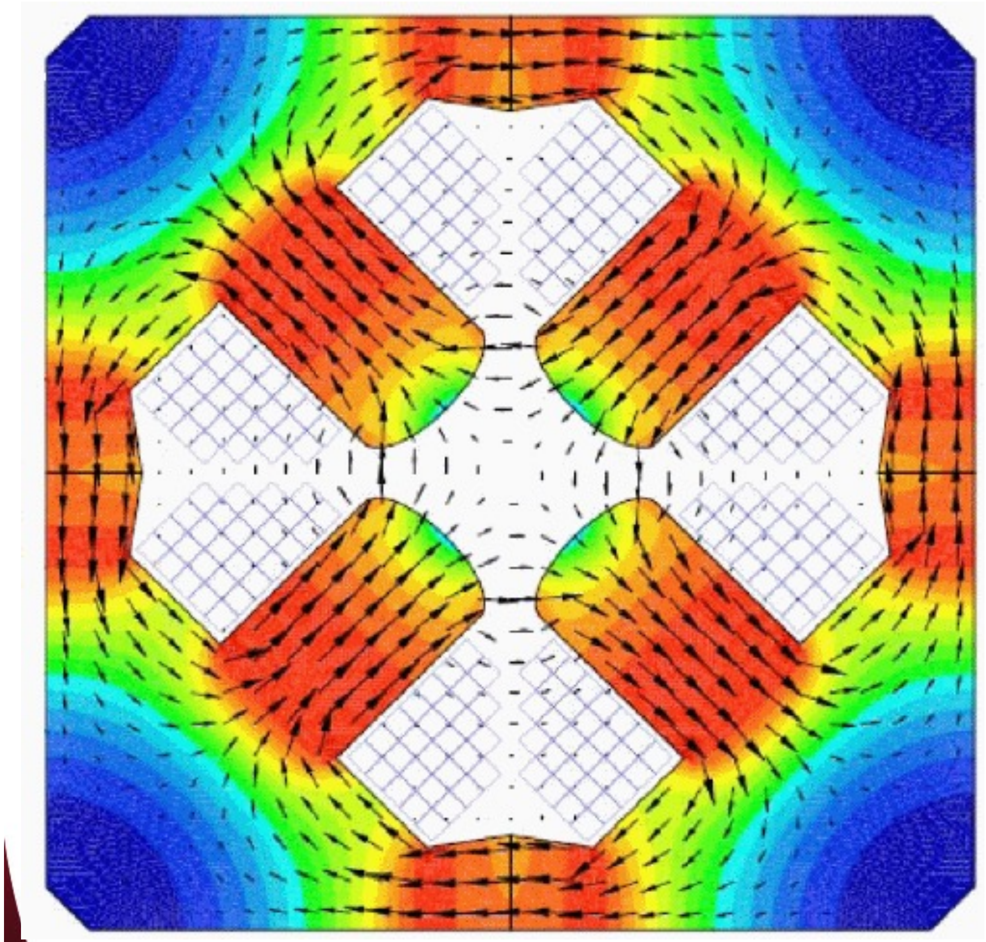


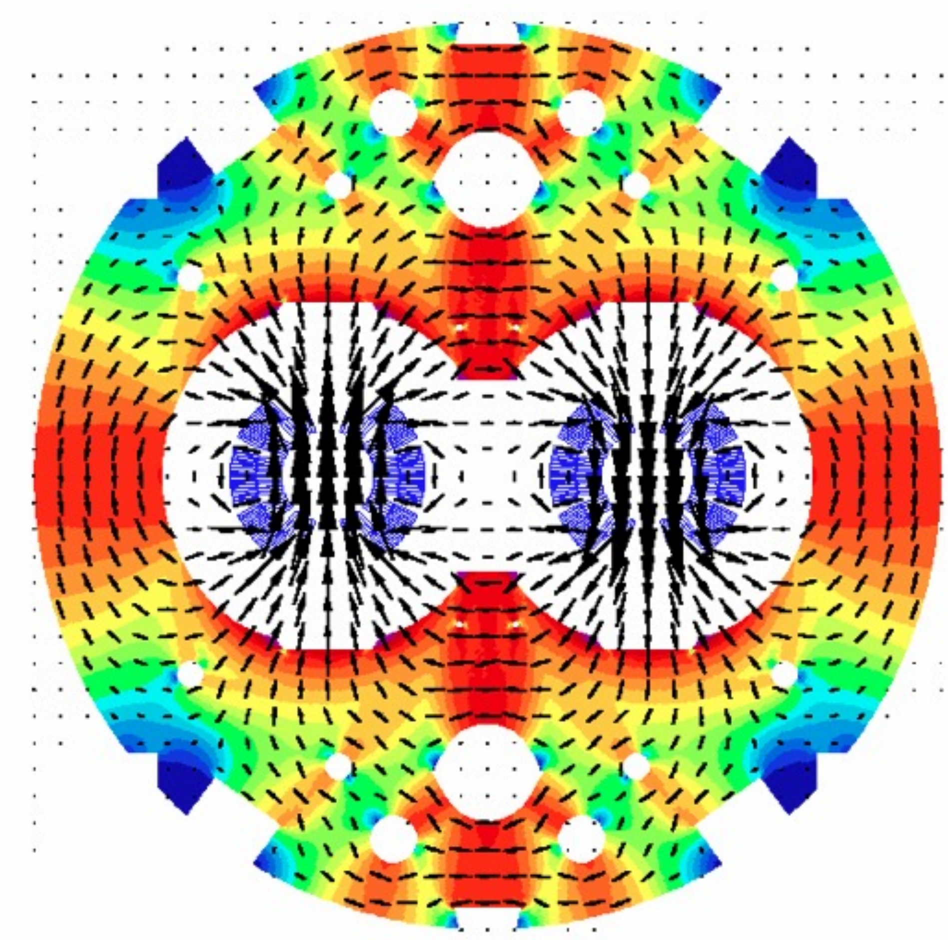
- Weaker, cheaper, faster dipoles can be made by conforming coils to a beam-pipe (no iron)
- Relatively inexpensive, but not very precise
  - Field quality on the order of ~1e-2 (vs ~1e-4 for iron)
- Often used for ”fast” response
  - ~1 kHz orbit “fast feedback” in CEBAF Hall D transport



# Normal vs Superconducting Magnets



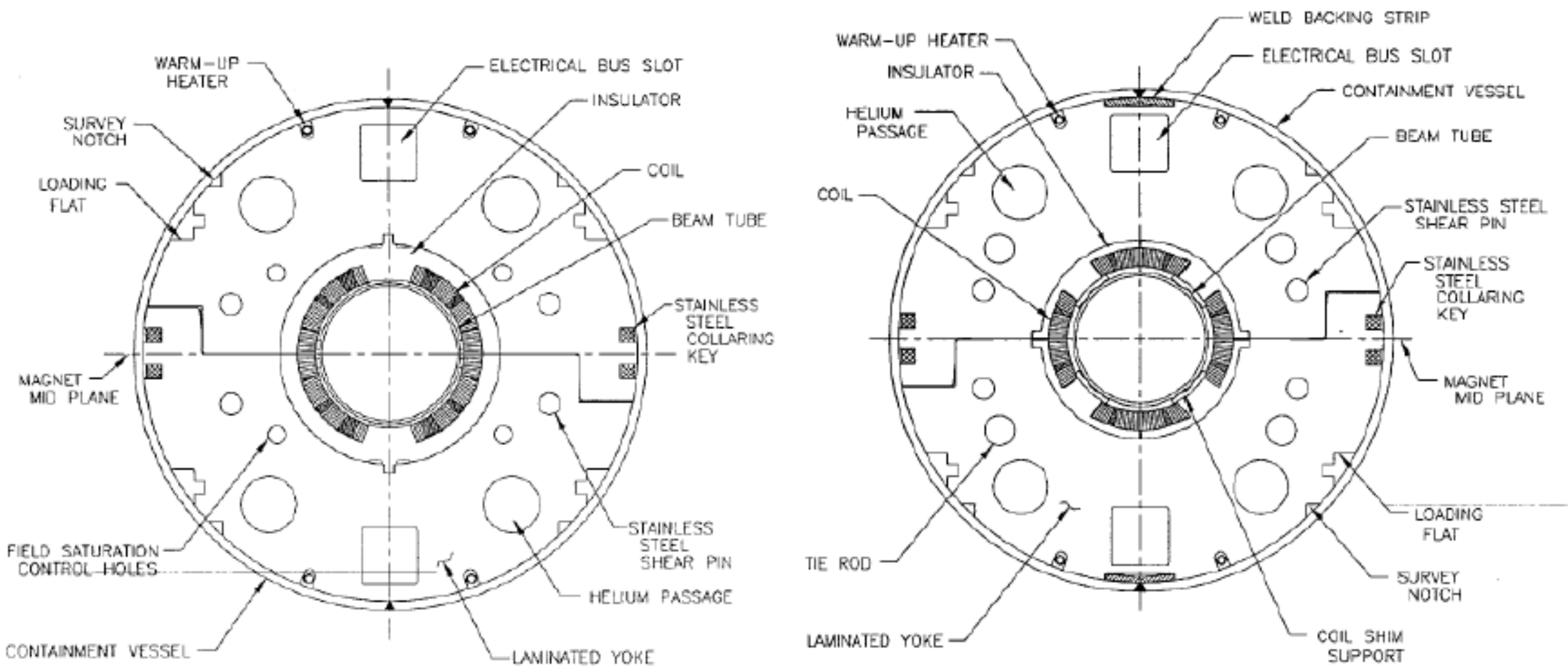
LEP quadrupole magnet (NC)



LHC dipole magnets (SC)

- Note high field strengths (red) where flux lines are densely packed together

# RHIC Dipole/Quadrupole Cross Sections



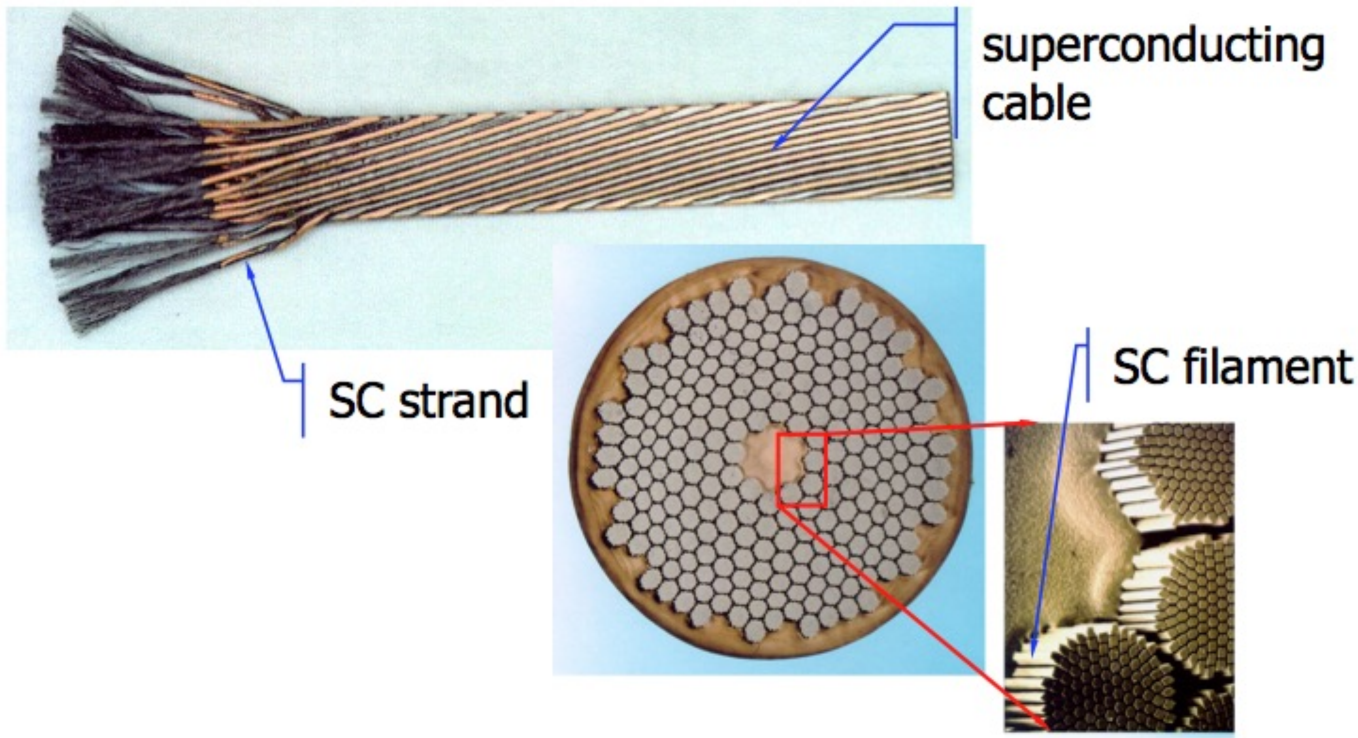

RHIC cos(θ)-style superconducting magnets and yokes

NbTi in Cu stabilizer, iron yokes, saturation holes

Full field design strength is up to 20 MPa (3 kpsi)

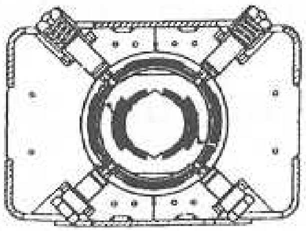
4.5 K, 3.45 Tesla

# Rutherford Cable

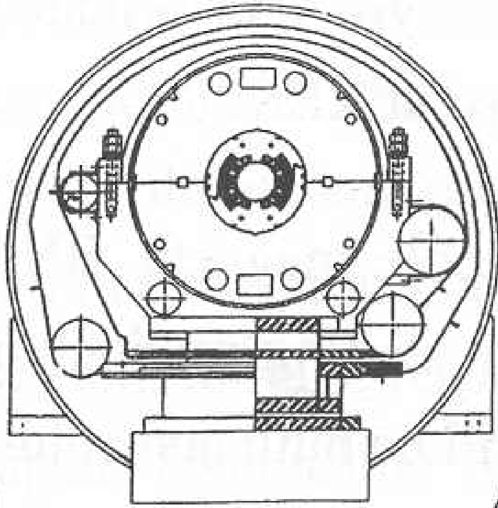



- Superconducting cables: NbTi in Cu matrix
  - Single 5 μm filament at 6T carries ~50 mA of current
  - Strand has 5-10k filaments, or carries 250-500 A
  - Magnet currents are often 5-10 kA: 10-40 strands in cable
    - Balance of stresses, compactable to stable high density

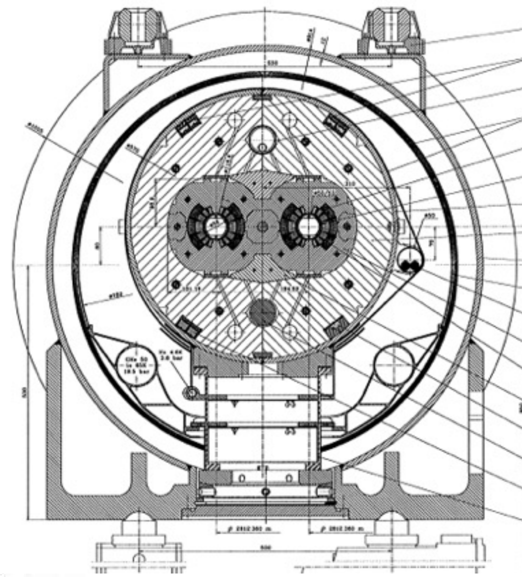
# Superconducting Dipole Magnet Comparison



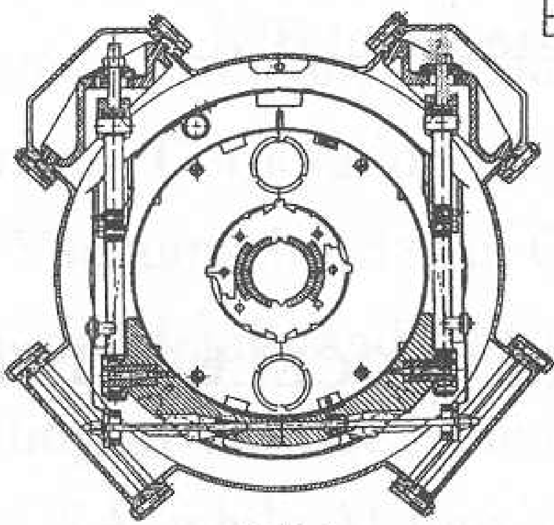
Tevatron
4T, 90mm



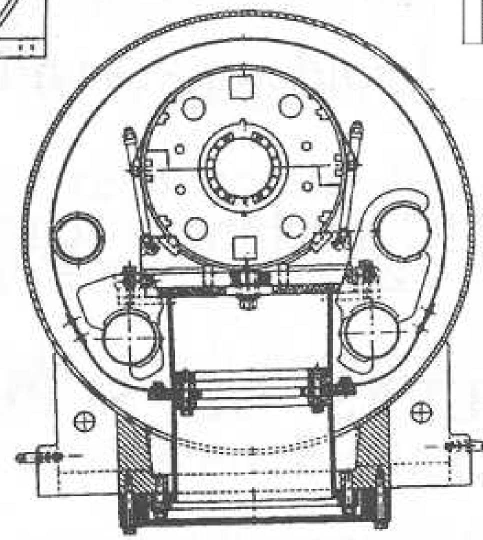
SSC
6.8 T, 50 mm



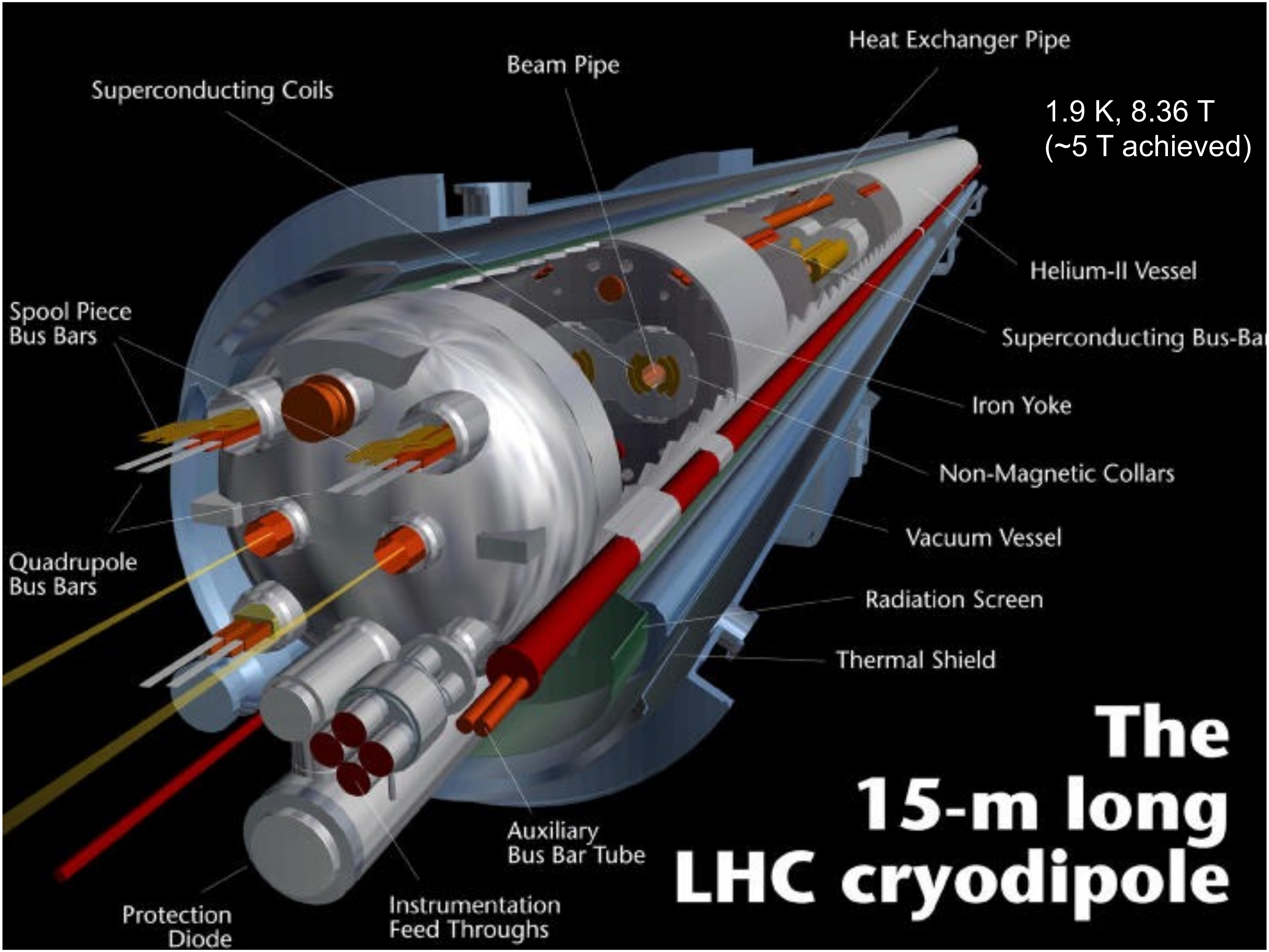
LHC
8.36T, 56mm



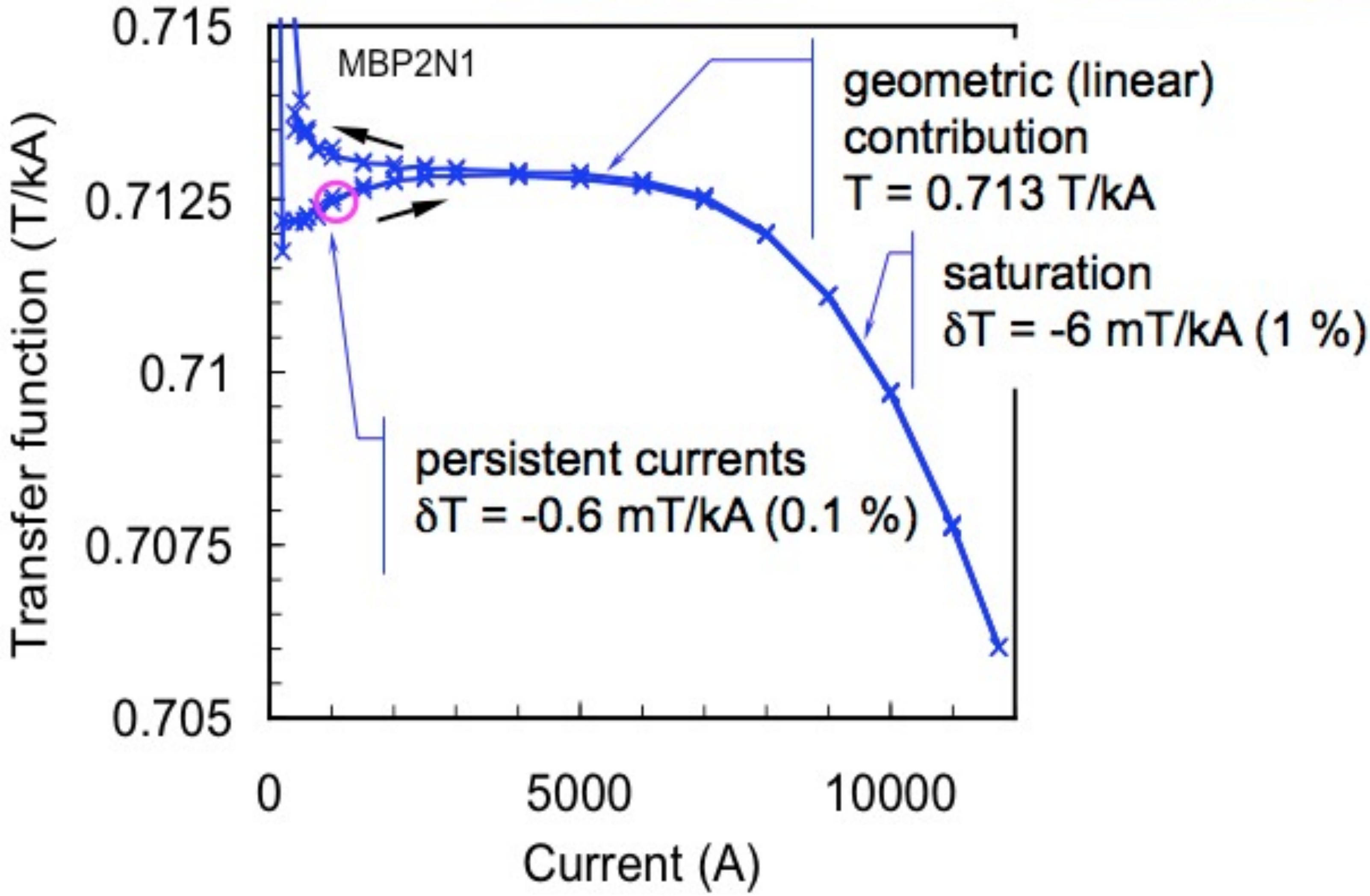
HERA
4.7T, 75 mm



RHIC
3.4T, 80mm


1.9 K, 8.36 T
(~5 T achieved)
Heat Exchanger Pipe
Beam Pipe
Superconducting Coils
Helium-II Vessel
Spool Piece
Bus Bars
Superconducting Bus-Ba
Iron Yoke
Non-Magnetic Collars
Vacuum Vessel
Quadrupole
Bus Bars
Radiation Screen
Thermal Shield
Auxiliary
Bus Bar Tube
Protection
Diode
Instrumentation
Feed Throughs
The
15-m long
LHC cryodipole

# Superconducting Magnet Transfer Function




- Transfer function: relationship between current/field
  - Persistent currents: surface currents during magnet ramping

Luca Bottura, CERN

# Quenching

- Magnetic stored energy

$$E = \frac{B^2}{2\mu_0}$$

$$B = 5\,\mathrm{T}, \quad E = 10^7\ \mathrm{J/m}^3$$

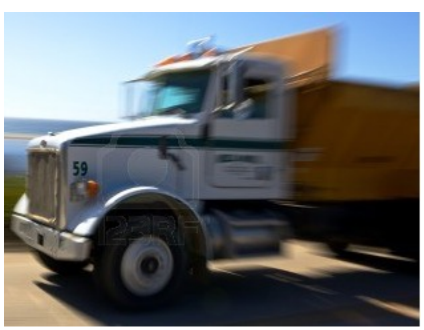
- LHC dipole

$$E = \frac{LI^2}{2} \quad L = 0.12\,\mathrm{H} \quad I = 11.5\,\mathrm{kA}$$

$$\Rightarrow E = 7.8 \times 10^6\ \mathrm{J}$$

$$22 \text{ ton magnet}$$

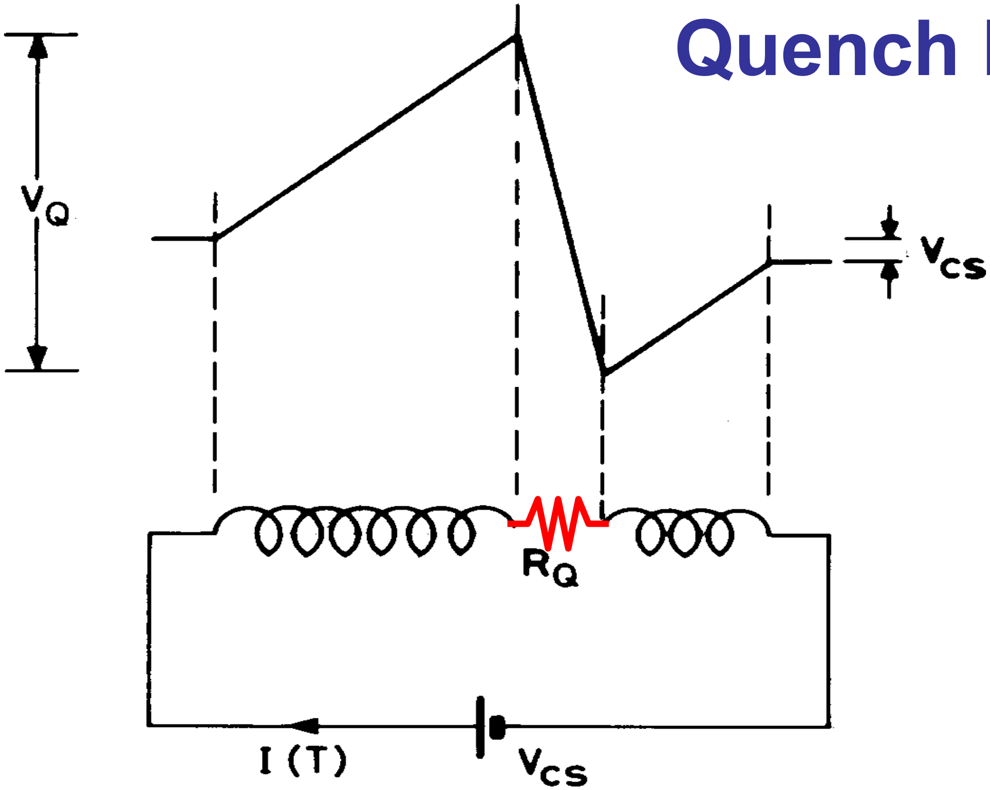
$$\Rightarrow \text{Energy of } 22 \text{ tons}, v = 92\ \mathrm{km/hr!}$$




*the most likely cause of **death** for a superconducting magnet*

Martin Wilson, JUAS Feb 2006

# Quench Process

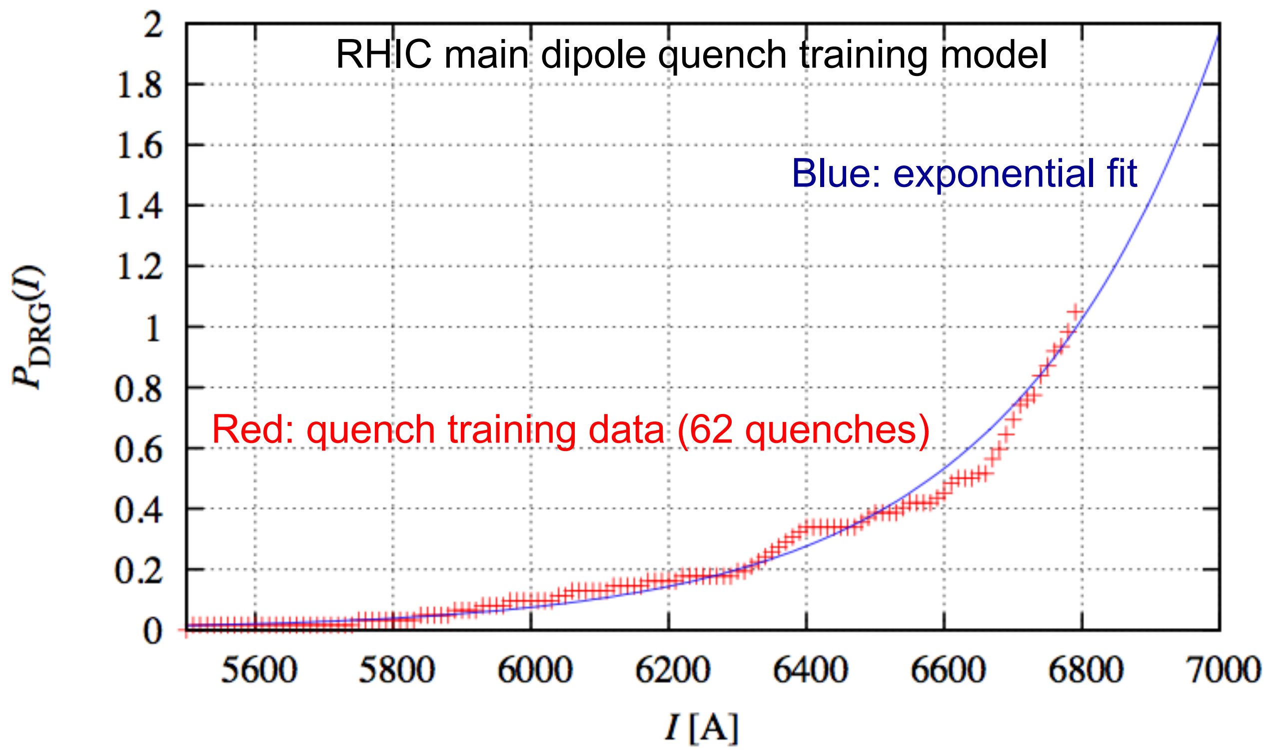



- Resistive region starts somewhere in the winding at a point: A problem!
  - Cable/insulation slipping
  - Inter-cable short; insulation failure
- Grows by thermal conduction
- Stored energy *½LI²* of the magnet is dissipated as heat
- Greatest integrated heat dissipation is at localized point where the quench starts
- Internal voltages **much** greater than terminal voltage (= $V_{cs}$ current supply)
  - Can profoundly damage magnet
  - Quench protection is important!

Martin Wilson, JUAS Feb 2006

# Quench Training

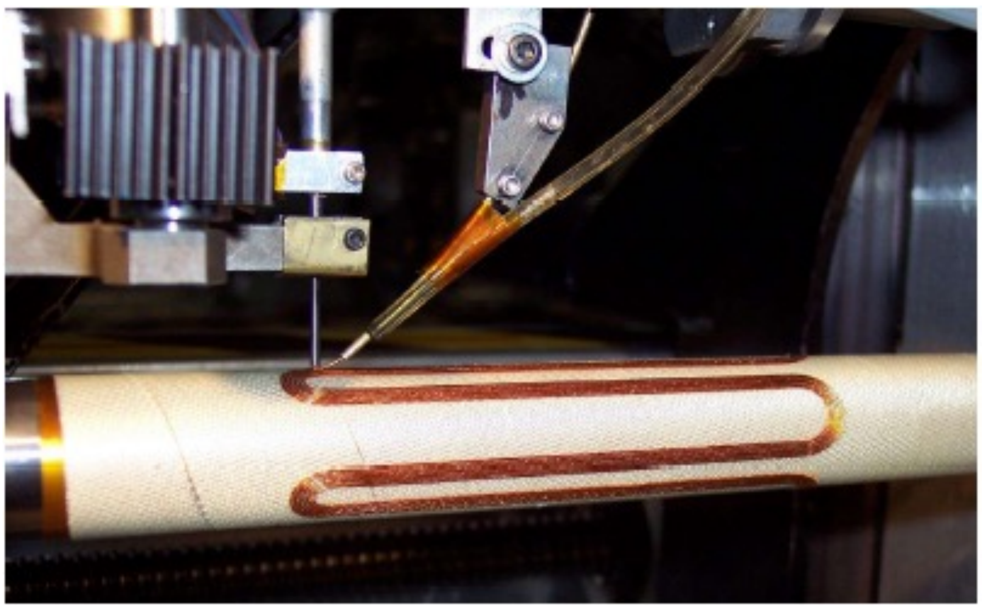
- Intentionally raising current until magnet quenches
  - Later quenches presumably occur at higher currents
    - Compacts conductors in cables, settles in stable position
  - Sometimes necessary to achieve operating current



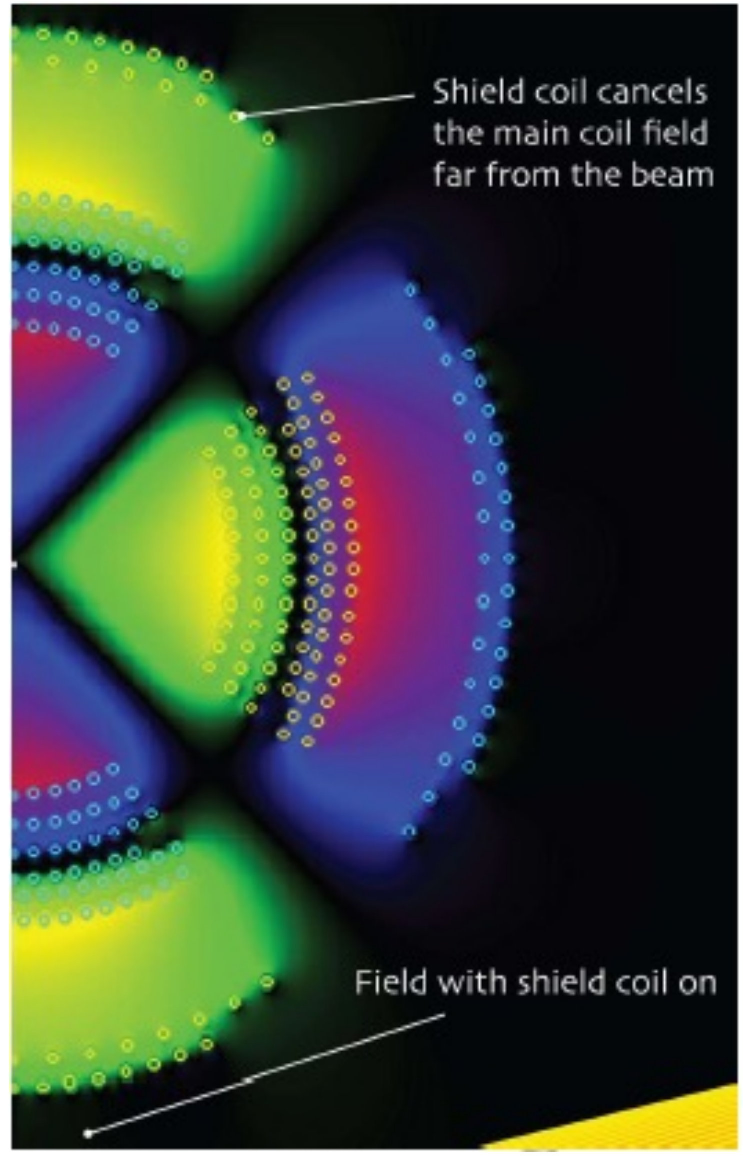

"Energy Upgrade as Regards Quench Performance", W.W. MacKay and S. Tepikian

# Direct-Wind Superconducting Magnets (BNL)

- 6T Iron-free (superconducting)
- Solid state coolers (no Helium)
- Field containment (LC magnet)
- "Direct-wind" construction

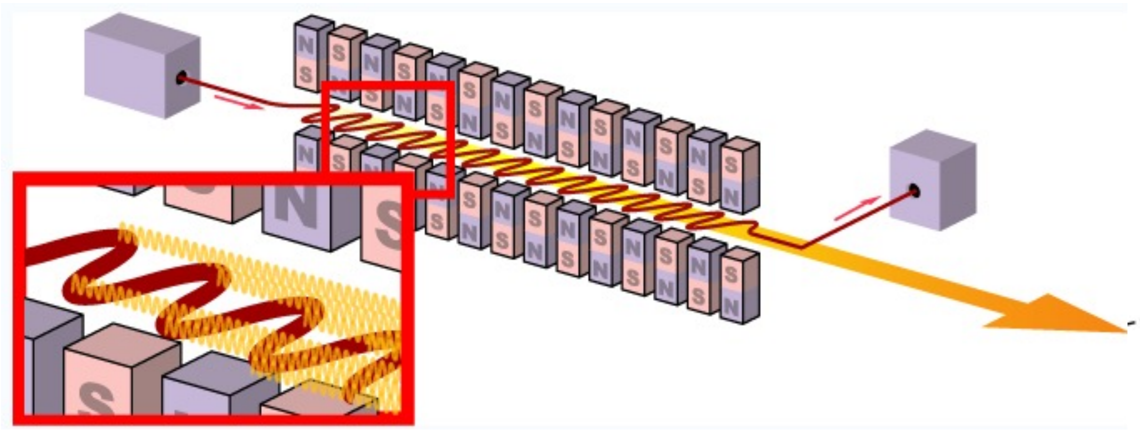


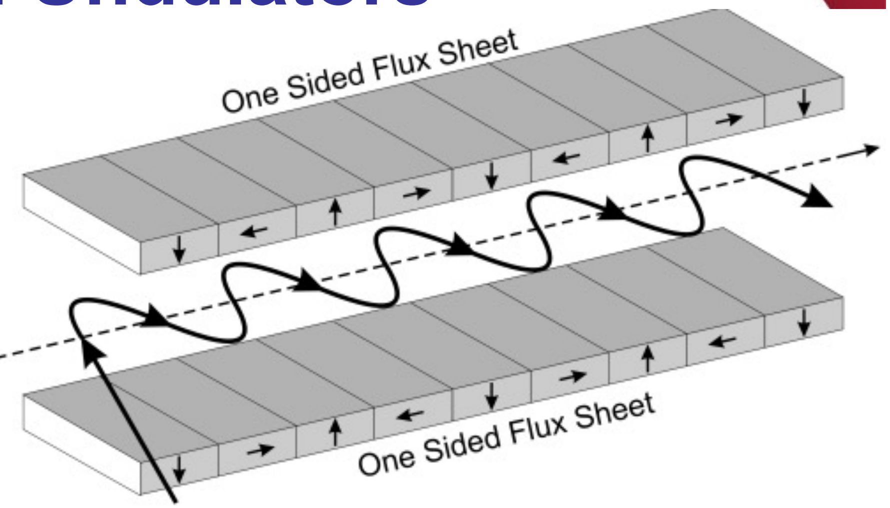
World's first "direct wind" coil machine at BNL




Linear Collider magnet

# FELs: Wigglers and Undulators






- Used to produce synchrotron radiation for FELs
  - Often rare earth permanent magnets in Halbach arrays
  - Adjust magnetic field intensity by moving array up/down
  - **Undulators**: produce nm wavelength FEL light from ~cm magnetic periods ($\gamma^2$ leverage in undulator equation)
    - Narrow band high spectral intensity
  - **Wigglers**: higher energy, lower flux, more like dipole synchrotron radiation
    - More about synchrotron light and FELs etc next week
    - LCLS: 130+m long undulator!

# Feedback to Magnet Builders

http://www.agsrhichome.bnl.gov/AP/ap_notes/RHIC_AP_80.pdf

**FEEDBACK BETWEEN ACCELERATOR PHYSICISTS AND MAGNET BUILDERS**

S. PEGGS

*Relativistic Heavy Ion Collider, Brookhaven National Laboratory, Upton, New York 11973, USA*

*Submitted to the proceedings of the LHC Single Particle Dynamics Workshop, Montreux, 1996.*

## 1 PHILOSOPHY

Our task is not to record history but to change it. *K. Marx (paraphrased)*

How should Accelerator Physicists set magnet error specifications? In a crude social model, they place tolerance limits on undesirable nonlinearities and errors (higher order harmonics, component alignments, et cetera). The Magnet Division then goes away for a suitably lengthy period of time, and comes back with a working magnet prototype that is reproduced in industry.