Lecture 8:

Linear Errors & Corrections

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"... it turns out that in the presence of nonlinear perturbations only rational frequencies $Q = p/q$ with q = 2 or 3, or sometimes 4, lead to instability. Therefore, it is possible to design machines with stable orbits by ensuring that the tunes avoid these values."

E. Courant, "Accelerators, Colliders, and Snakes".

A) TUNE PLANE (INTRO.)

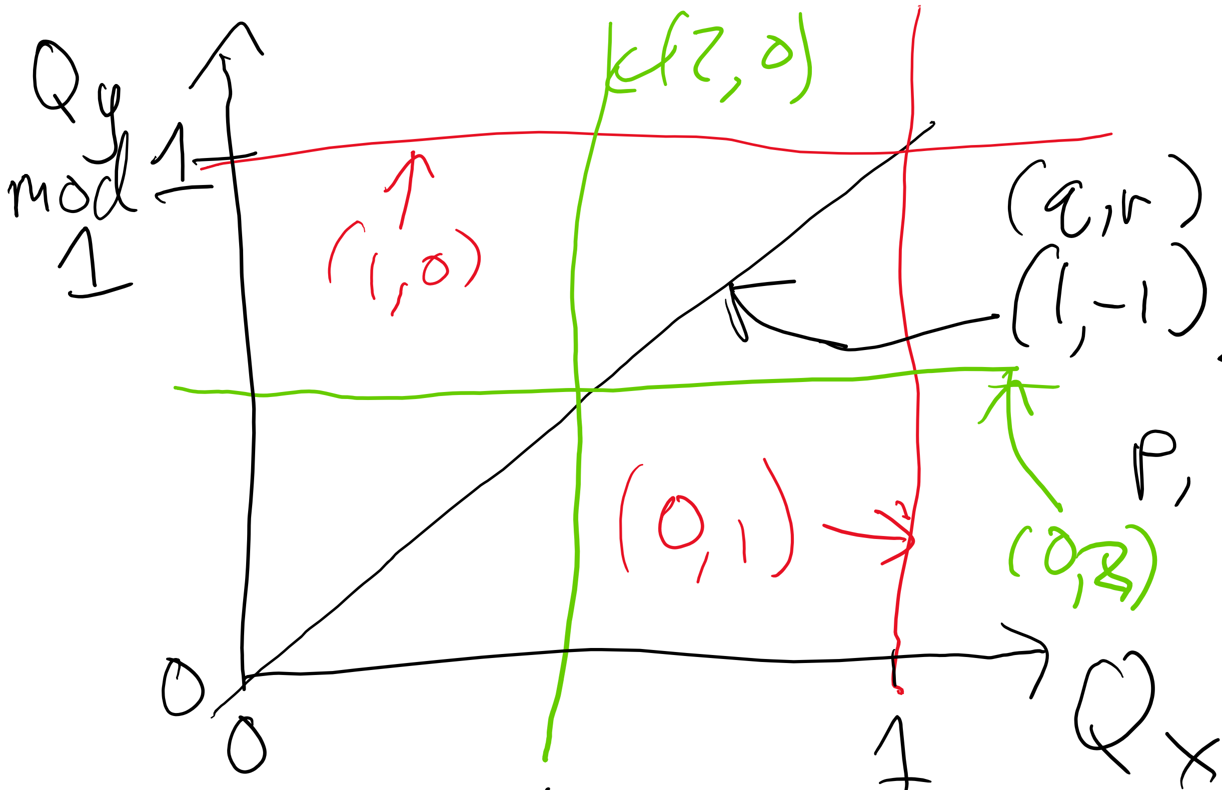
B) TRAJECTORY + CLOSED ORBIT

C) H-V COUPLING

D) QUAD STRENGTH ERRORS

HIDDEN AGENDA : INTRO TO RESONANCES

A) TUNE PLANE INTRO.



$$\boxed{p = q\,Q_x + r\,Q_y}$$

p, q, r integers

– p can be very large .... but often just the integer parts of $Q_x + Q_y$

B) TRAJECTORY + CLOSED ORBIT

Take a multipole magnet

$$B_y + iB_x = C_n (x+iy)^n$$

C_0 : dipole
C_1 : quad
C_2 : sext
$\vdots$

DISPLACE $x \to x + \Delta x$, $y \to y + \Delta y$

$$\boxed{B_y + iB_x = C_n \left[(x+iy)^n - \underbrace{n(x+iy)^{n-1}(\Delta x + i\Delta y)}_{\text{FEED DOWN}} + \ldots \right]}$$

QUADRUPOLE

$$\Delta x' = \frac{x}{f} + \underbrace{\frac{\Delta x}{f}}$$

CONSTANT TERM: DIPOLE!!

$\boxed{\text{E.G}}$ $f = 10\ \text{m}$, $\Delta x = 1\ \text{mm}$

$\Rightarrow \Delta x / f = 100\ \mu\text{rad}$

DIPOLE ROLL (Displacements do nothing)

E.G. A dipole of BEND angle θ is rolled about the longitudinal axis by α

$$\Delta y' = \alpha\theta \quad [\text{CONSTANT}]$$

E.G. $\theta = 40$ mrad, $\alpha = 1$ mrad

$$\Rightarrow \Delta y' = 40 \ \mu\text{rad}$$

So: a single misaligned magnet (dipole or quad) generates angle kicks ~ 100 μrad, +/- eV

Q1: How bad is this?

Q2: How to correct it?

Q3: Free wave or closed orbit (with PBC)?

FREE WAVE In a transfer line or linac

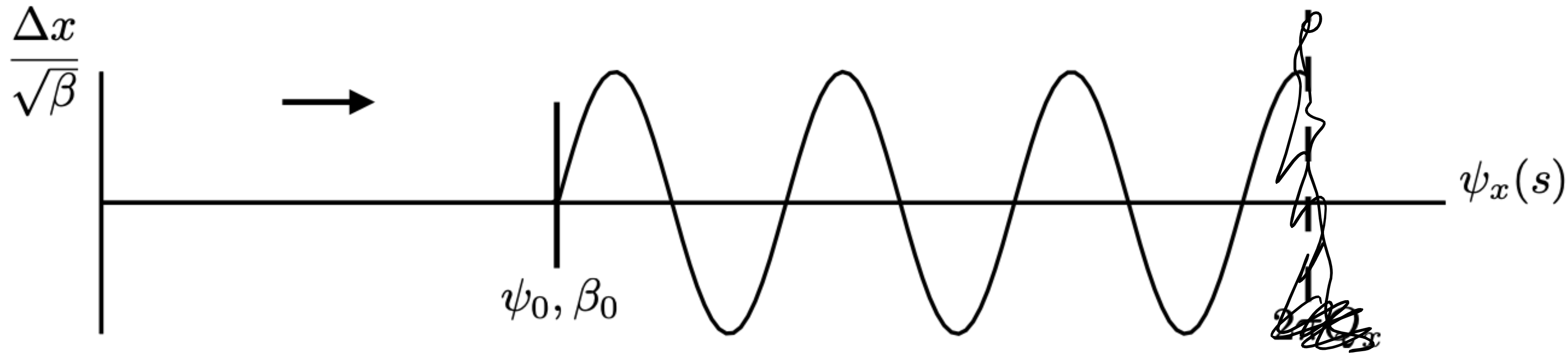
$$\boxed{\frac{x}{\sqrt{\beta}} = \Delta x' \sqrt{\beta_0} \cdot \sin\left(\psi(s) - \psi_0\right)} \quad s > s_0$$

E.G. $\Delta x' = 10^{-4}$ radians, $\beta = 50$ m (large..?)

$$\langle x^2 \rangle^{\frac{1}{2}} \simeq 5 \text{ mm}$$

Can't take many such (uncorrected) errors before beam hits beam pipe!!

8.1 Downstream free wave response to an angular kick error.



Q: What happens in a storage ring with PBC?

CLOSED ORBIT ERRORS

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{co} = M \begin{pmatrix} x \\ x' \end{pmatrix}_{co} + \begin{pmatrix} 0 \\ \Delta x' \end{pmatrix}$$

so $$\boxed{\begin{pmatrix} x \\ x' \end{pmatrix}_{co} = (I-M)^{-1}\begin{pmatrix} 0 \\ \Delta x' \end{pmatrix}}$$

I : Identity 2×2

WATCH OUT if $\det(I-M)\to 0$!!

$$\boxed{(I-M)^{-1} = \frac{1}{2(1-C)}\begin{pmatrix} 1-C & \beta S \\ -\frac{S}{\beta} & 1-C \end{pmatrix}}$$

$C = \cos(2\pi Q_x)$

$S = \sin(2\pi Q_x)$

watch out for

$C = 1$: $Q_x = 0, 1, 2 \ldots$

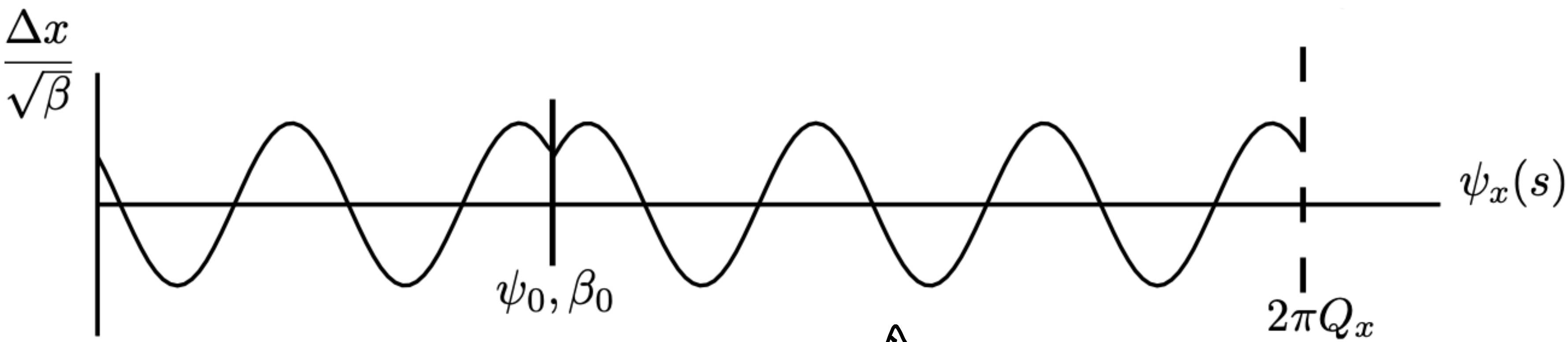
AT KICK LOCATION

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{CO} = \frac{\Delta x'}{2\sin(2\pi Q_x)} \begin{pmatrix} \beta_0 \cos(\pi Q_x) \\ \sin(\pi Q_x) - \alpha_0 \cos(\pi Q_x) \end{pmatrix}$$

for any α_0

resonance denominator

8.2 Closed orbit wave response to an angular kick.



At ANY location

$$\boxed{\frac{x_{co}}{\sqrt{\beta}} = \frac{\Delta x' \sqrt{\beta_0}}{2\sin(\pi Q_x)} \cdot \cos\left(|\psi - \psi_0| - \pi Q_x\right)}$$

Shows that $\sqrt{\beta}$ or $\sqrt{\beta_0}$, is a "sensitivity" measure

TUNES WITH $p \approx Q_x$ are dangerous

RESONANCE EXAMPLE 1 $(q,r)=(1,0)$

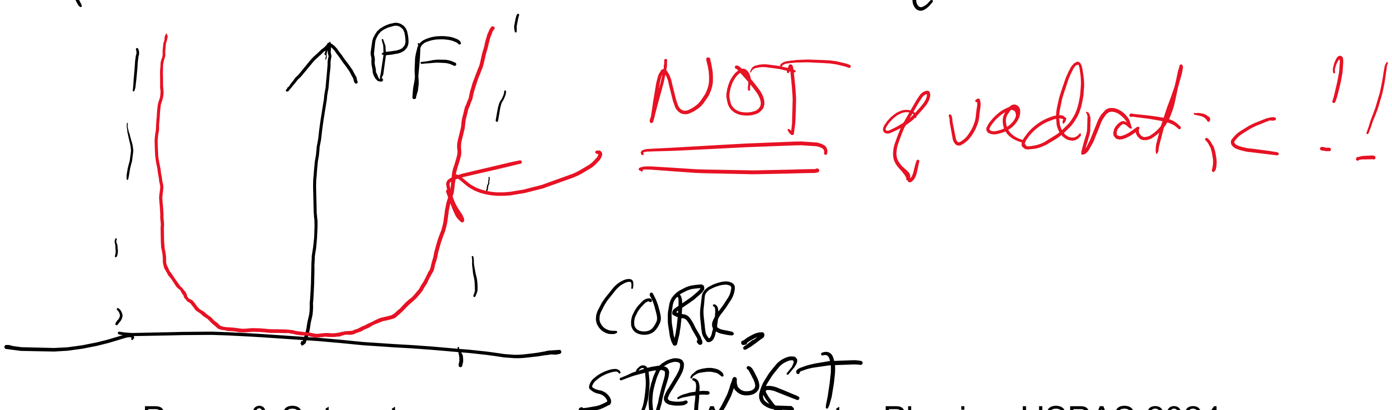
$(0,1)$

CLOSED ORBIT CORRECTION

1) Measure C.O. location at many BPMs
2) Correct by powering many weak dipole correctors

THERE ARE MANY ALGORITHMS to solve this linear problem.

Q1: What is the penalty function? PF
Q2: What about dipole corrector limits?
Q3: Must we use 19th century tools?



ONE OF MANY ALGORITHMS is the "SLIDING 3-BUMP" which allows any ΦF

$\boxed{\text{Downstream}}$ of 3 dipole correctors 1,2,3

$$\boxed{\frac{x(\psi)}{\sqrt{\beta}} = \sum_{1}^{3} \Delta x_i' \sqrt{\beta_i} \cdot \sin(\psi - \psi_i)}$$

IF this sum is zero, no need to apply PBC!

$\Longrightarrow$ displacements are localised to

$$\psi_1 < \psi < \psi_3$$

3-BUMP localization is guaranteed if

$$\boxed{\frac{\Delta x_i \sqrt{\beta_i}}{\sin(\psi_j - \psi_k)} = K}$$

Kappa

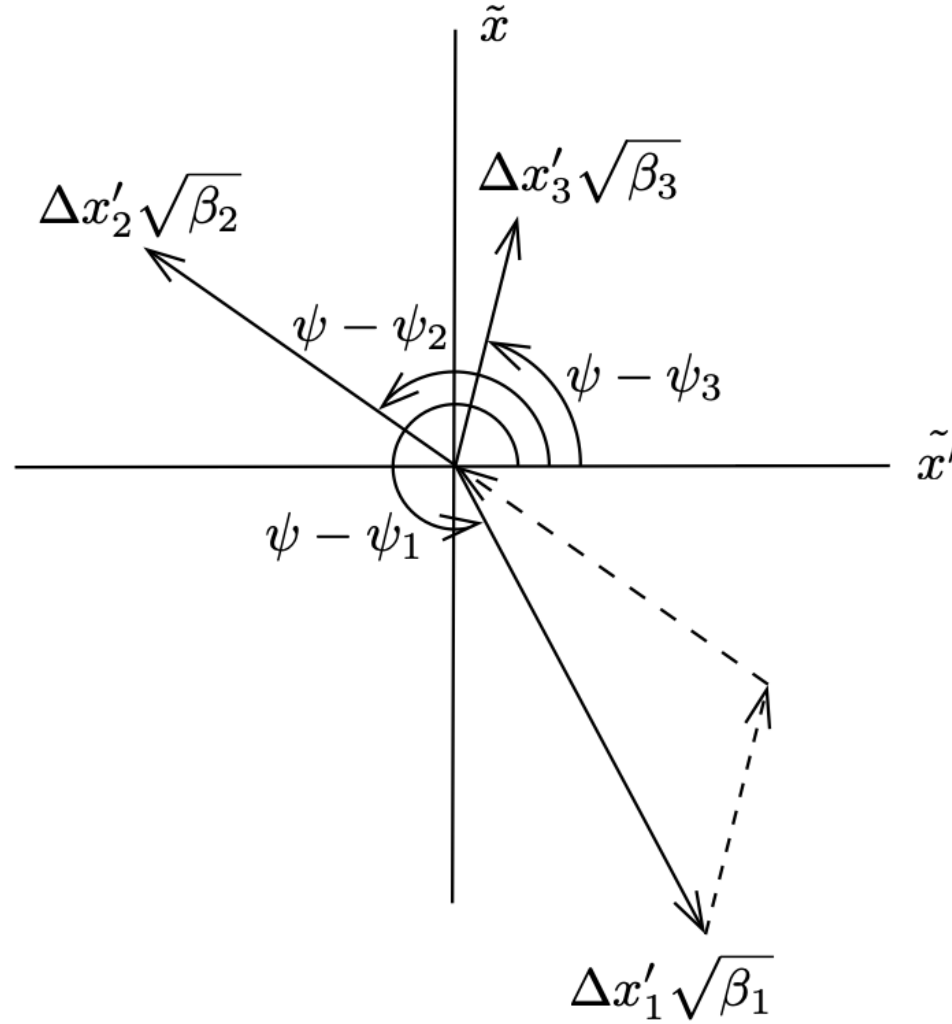
for (i, j, k) cyclic combination of $(1, 2, 3)$

Adjust bump strength K for $(1,2,3)$ to minimize local PF, then $(2,3,4)$, then $(3,4,5)$ et cetera.

- REPEAT until global PF is minimized

- Correction repetition is often necessary in the control room, as well as in calculation.

8.4 Three-bump vectors rotating in normalized phase space, always adding to zero.



c) <u>H-V COUPLING</u>

ROLLED QUADS

LINEAR COUPLING - ROLLED QUADS

Roll ANY magnet with 4X4 matrix M by angle α

$$\boxed{M_{ROLLED} = R(-\alpha)\, M\, R(\alpha)}$$

where $R \begin{pmatrix} c & 0 & s & 0 \\ 0 & c & 0 & s \\ -s & 0 & c & 0 \\ 0 & -s & 0 & c \end{pmatrix}$ $\quad c = \cos(\alpha) \simeq 1$, $\quad s = \sin(\alpha) \simeq \alpha$

so a Rolled Thin Quad becomes

$$M_{RTQ} \simeq M_{TQ} + \alpha \left(\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ 0 & 0 & -2q & 0 \\ \hline 0 & 0 & 0 & 0 \\ -2q & 0 & 0 & 0 \end{array}\right)$$

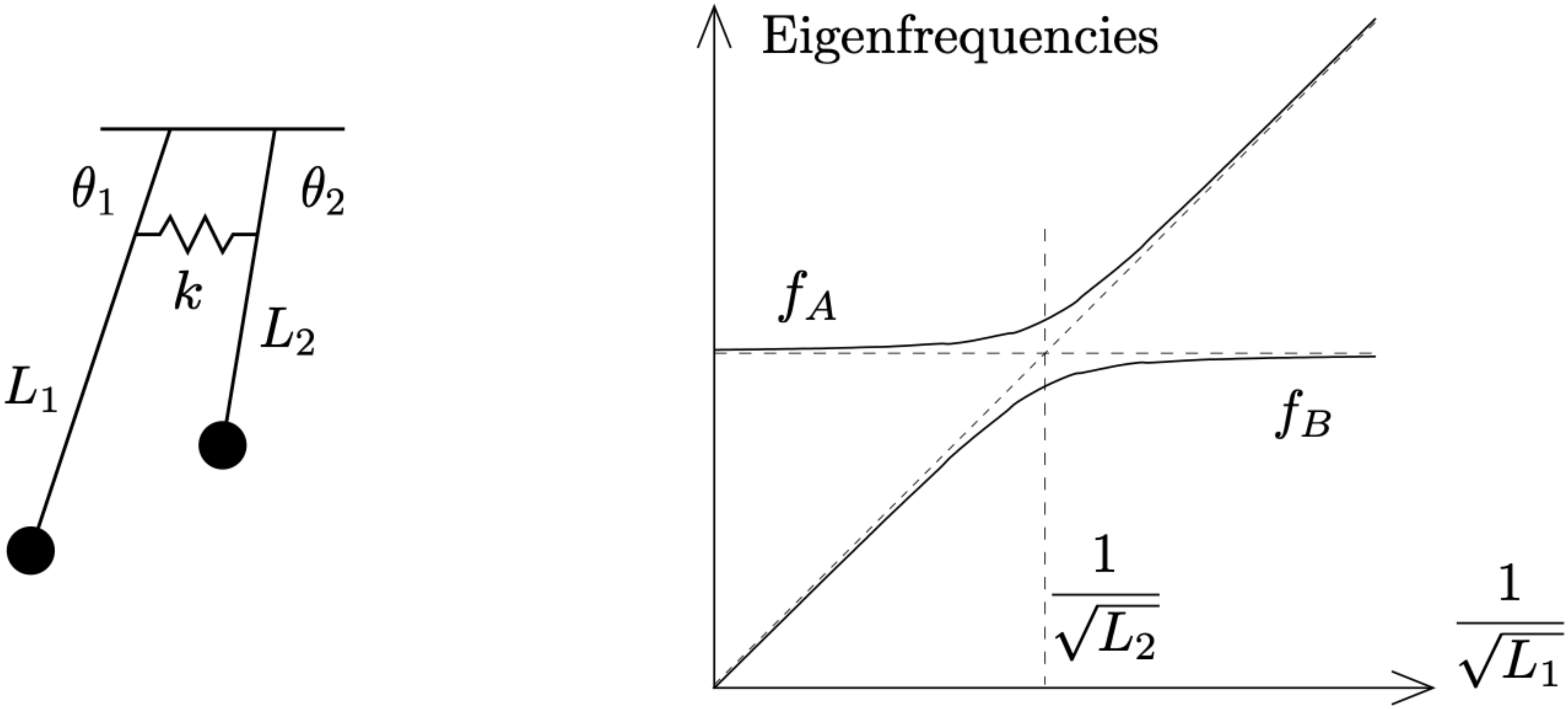
COUPLING H & V motion!!

ANALOGY: 2 weakly coupled pendula

No coupling: $f_1 \sim 1/\sqrt{L_1}$, $f_2 \sim 1/\sqrt{L_2}$

WITH COUPLING .... vary L_1 across value of L_2

8.5 The eigenfrequencies of two weakly coupled pendula as the length L_1 is varied.



WITH coupling, still have 2 eigenfrequecies f_A + f_B

$$\boxed{\begin{aligned}\theta_1 &= a_{1A}\cos(f_A t + \phi_{1A}) + a_{1B}\cos(f_B t + \phi_{1B}) \\ \theta_2 &= a_{2A}\cos(f_A t + \phi_{2A}) + a_{2B}\cos(f_B t + \phi_{2B})\end{aligned}}$$

- f_A is always $> f_B$
- There is a closest approach of $|f_A - f_B|$, depending on spring constant k

Similarly, H & V accelerator motion is coupled & confused if

$$\boxed{|p + Q_x - Q_y| \lesssim Q_{MIN}}$$

$Q_{min} \approx .001$ well tuned

or .1 badly tuned

design tunes

RESONANCE EXAMPLE 2 : $(q, r) = (1, -1)$

CORRECT COUPLING WITH SKEW QUADS

IN CONTROL ROOM:

{
measure Q_{min}
tweak skew quad 1, 2
}

eg
CESR
SPS

D) QUAD STRENGTH ERRORS

TUNE SHIFTS & β-WAVES

Suppose a quad at s_0 has a strength error

$$\Delta q = \Delta\left(\frac{1}{f}\right)$$

Q1) Is the motion still stable? If yes...

Q2) How much do Q_x & Q_y change?

Q3) What happens to β-functions everywhere?

One turn matrix becomes

$$\tilde{M} = M\begin{pmatrix} 1 & 0 \\ -\Delta q & 1 \end{pmatrix}$$

and perturbed tune is found by solving

$$\mathrm{Tr}(\hat{M}) = 2\cos(2\pi\hat{Q})$$

or

$$\boxed{\cos(2\pi\hat{Q}) = \cos(2\pi Q) - \frac{\beta_0\Delta q}{2}\sin(2\pi Q)}$$

- Exact!
- β_0 is a sensitivity factor

A1: Motion is still stable if $|\mathrm{Tr}(\hat{M})| \le 2$

- If Δq is small, then

$$\boxed{\Delta Q = \hat{Q} - Q = \frac{\beta_0\Delta q}{4\pi}}$$

- Δq has different signs in H & V
  $\Rightarrow \Delta Q_x$ & ΔQ_y have different signs
- E.g. 1% error in $f = 10$ m quad
  with $(\beta_x, \beta_y) = (50, 10)$ m
  gives $(\Delta Q_x, \Delta Q_y) = (.0040, -0.0008)$
- Tune shifts are easily corrected by the families of main arc F & D quadrupoles
- β-function errors are TRICKIER!!

β- WAVES : FREE

JUST DOWNSTREAM of error quad

$$\begin{pmatrix} \tilde{\beta} \\ \tilde{\alpha} \\ \tilde{\gamma} \end{pmatrix}_{0+\epsilon} = \begin{pmatrix} 1 & 0 & 0 \\ \Delta q & 1 & 0 \\ (\Delta q)^2 & 2\Delta q & 1 \end{pmatrix} \begin{pmatrix} \beta \\ \alpha \\ \gamma \end{pmatrix}_0$$

launching a FREE WAVE

$$\boxed{\frac{\Delta\beta}{\beta} = -\Delta q \cdot \beta_0 \sin(2(\psi - \psi_0))}$$

TWICE the speed of phase advance

E.G. $\Delta q = 10^{-2}\left(\frac{1}{10}\right)$, $\beta_0 = 50$ m $\Rightarrow$ 5% error wave!

$\Rightarrow$ Dangerous! Control at 10^{-4} level if possible.

β-WAVES: PBC

In a circular accelerator with PBC

$$\boxed{\frac{\Delta\beta}{\beta} = \frac{-\Delta q\,\beta_0}{2\sin(2\pi Q)} \cdot \cos\left(2|\psi - \psi_0| - 2\pi Q\right)}$$

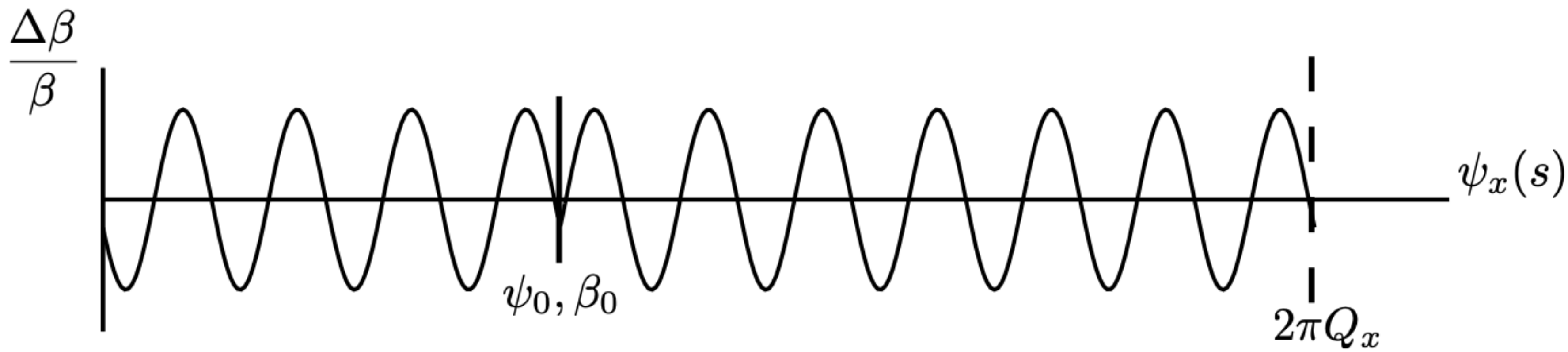
RESONANCE DENOMINATOR

RESONANCE EXAMPLE 3 !

Optics are especially vulnerable to errors if $p \simeq 2Q_x$, or $p \simeq 2Q_y$

that is: $(q, r) = (2, 0)$ or $(0, 2)$

8.3 Closed beta wave response due to a quad strength error.



Q: Why also include NONLINEAR correctors??

Q: Where do higher order (nonlinear) resonances come from? ...