USPAS Accelerator Physics 2024
Hampton VA / Northern Illinois University

Lattice Examples
(or putting much of the week together)
(or Stupid Lattice Tricks)

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http://www.toddsatogata.net/2024-USPAS

Happy Birthday to Polykarp Kusch (Nobel 1955; EDM), Paul Newman, and Wayne Gretzky!

Happy National Peanut Brittle Day, National Spouses Day, and Dental Drill Appreciation Day!

Entire Courses Also Taught On Lattice Design

- https://casa.jlab.org/publications/USPAS_Jan_2018.html

Practical Lattice Design

Alex Bogacz (Jefferson Lab) and Dario Pellegrini (CERN) with Randika Gamage (ODU)

January 15 - 19, 2018 Old Dominion University - Norfolk, VA

Timeline

Course Outline

Lecture 1:	Introduction to Transverse Optics	Dario Pellegrini
Lecture 2:	Introduction to OptiM, FODO Cell	Alex Bogacz
Lecture 3:	Dispersion Suppressors	Alex Bogacz
Lecture 4:	Arc-to-Straight Design	Alex Bogacz
Lecture 5:	Low Beta Optics	Dario Pellegrini
Lecture 6:	Lattice Imperfections	Dario Pellegrini
Lecture 7:	Radiation Damping	Alex Bogacz
Lecture 8:	Low Emittance Lattices, DBA Cell	Dario Pellegrini

ASSIGNMENTS

Day 1:	Example	Homework	Solutions
Day 2:	Example 1 Example 2	Homework	Solutions
Day 3:	Example 1 Example 2 Example 3	Homework	Solutions
Day 4:	Example	Homework	Solutions
January 19, 2018	Final Exam	Solutions	

Overview (Morning): Mostly 1D and 1D+

- Review: Linear optics, matrices, Twiss parameters
- Easing in: Two-bumps and three-bumps (homework)
- Dipole-Free Transverse Lattices
  - Review: FODO cell, without dipoles
  - Periodic triplet cell
  - $\pi/2$ and imaging insertions
  - Coupling (Mobius) insertion
  - Low-beta insertions (collision point, ion stripping, ...)
- Review: Dispersion
- Bending Transverse Lattices (FODO)
  - Review: FODO cell, *with dipoles*
  - FODO cell dispersion suppressors

Overview (Afternoon): 1D+ and 2-3D+

- Localizing Dispersion: Achromats
  - Achromatic doglegs/chicanes
  - Bunch compressors
  - Double bend achromat
  - Triple bend achromat
  - (MAX-IV multi-bend achromat (HMBA))
  - Lead in to Tuesday lecture on 3G light source lattices

- 2D/3D manipulation:
  - Longitudinal/transverse emittance exchange

Review: General Linear Transport Matrix

- We can parameterize a general non-periodic transport matrix from s_1 to s_2 using lattice parameters and $\Delta\phi \equiv \phi(s_2) - \phi(s_1)$

$$M_{s_1 \to s_2} = \begin{pmatrix} \sqrt{\frac{\beta(s_2)}{\beta(s_1}}[\cos\Delta\phi + \alpha(s_1)\sin\Delta\phi & \boxed{\sqrt{\beta(s_1)\beta(s_2)}\sin\Delta\phi} \\ -\frac{[\alpha(s_2)-\alpha(s_1)]\cos\Delta\phi + [1+\alpha(s_1)\alpha(s_2)]\sin\Delta\phi}{\sqrt{\beta(s_1)\beta(s_2)}} & \sqrt{\frac{\beta(s_1)}{\beta(s_2)}}[\cos\Delta\phi - \alpha(s_2)\sin\Delta\phi] \end{pmatrix}$$

- This does not have a pretty form like the periodic matrix

However both can be expressed as

$$\boxed{M = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}}$$

where the C and S terms are cosine-like and sine-like; the second row is the s-derivative of the first row!

A common use of this matrix is the m_{12} term:

$$\boxed{\Delta x(s_2) = \sqrt{\beta(s_1)\beta(s_2)}\sin(\Delta\phi)\,\Delta x'(s_1)}$$

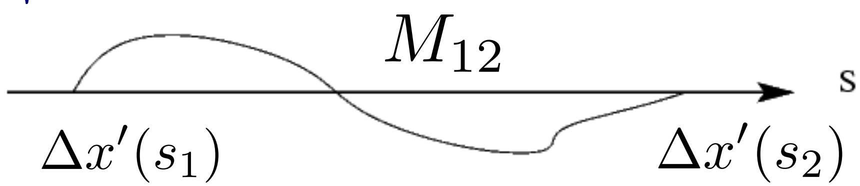
Effect of angle kick on downstream position

Orbit Control: Two-Bump

$$\Delta x(s_2) = \Delta x'(s_1)\sqrt{\beta(s_1)\beta(s_2)}\sin\Delta\phi$$

$$\Delta x'(s_2) = \Delta x'(s_1)\sqrt{\frac{\beta(s_1)}{\beta(s_2)}}[\cos\Delta\phi - \alpha(s_2)\sin\Delta\phi]$$

$$M_{12} = \begin{pmatrix} C_{12} & S_{12} \\ C'_{12} & S'_{12} \end{pmatrix}$$

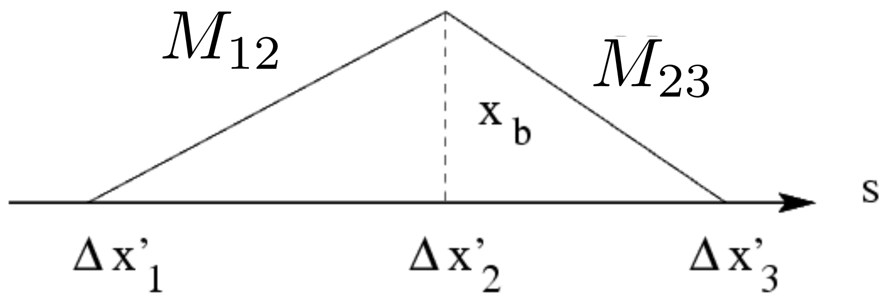


- A single orbit error changes all later positions and angles
  - Add another dipole corrector at a location where $\Delta\phi = k\pi$ At this point the distortion from the original dipole corrector is all x' that we can cancel with the second dipole corrector.

$$\Delta x'(s_2) = \Delta x'(s_1)\sqrt{\frac{\beta(s_1)}{\beta(s_2)}} + \text{angle from } \mathrm{s}_2 \text{ dipole}$$

  - Called a **two-bump**: localized orbit distortion from two correctors
  - But requires $\Delta\phi = k\pi$ between correctors

Orbit Control: Three-Bump (another view of homework)



- A general local orbit distortion from three dipole correctors
  - Constraint is that net orbit change from sum of all three kicks must be zero

$$\begin{pmatrix} C_{23} & S_{23} \\ C'_{23} & S'_{23} \end{pmatrix} \left[\begin{pmatrix} C_{12} & S_{12} \\ C'_{12} & S'_{12} \end{pmatrix} \begin{pmatrix} 0 \\ \Delta x'_1 \end{pmatrix} + \begin{pmatrix} 0 \\ \Delta x'_2 \end{pmatrix} \right] + \begin{pmatrix} 0 \\ \Delta x'_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Delta x'_1 = \frac{x_b}{S_{12}} \qquad \Delta x'_2 = -\left(\frac{C_{23} S_{12} + S_{23} S'_{12}}{S_{12} S_{23}} \right) x_b \qquad \Delta x'_3 = \frac{S_{23}}{S_{12}^2} x_b$$

  - Bump amplitude $x_{\mathrm{b}} = S_{12} \Delta x'_1$
  - Only **three-bump** requirement is that S_{12}, $S_{23} \neq 0$
    - Need a transverse position change between points 1,2 and 2,3

Review: Matrices of Magnetic Elements

- For our purposes this morning:
  - All motion is linearized
  $$\begin{pmatrix} x \\ x' \end{pmatrix}_2 = M \begin{pmatrix} x \\ x' \end{pmatrix}_1$$
  $$x' \equiv \frac{p_x}{p_0}$$
  paraxial approximation
  - Linear transport matrices:
  $$M_{\mathrm{drift}} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$
  $$M_{\mathrm{quad}} \approx \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$
  thin quads
  - (Sector) dipole includes constant fractional momentum offset

$$\begin{pmatrix} x \\ x' \\ \delta \end{pmatrix}_2 = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ \frac{1}{\rho}\sin\theta & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ \delta \end{pmatrix}_1$$

$$\delta \equiv \frac{\Delta p}{p_0}$$

Review: Periodic Transport Matrix Parameterization

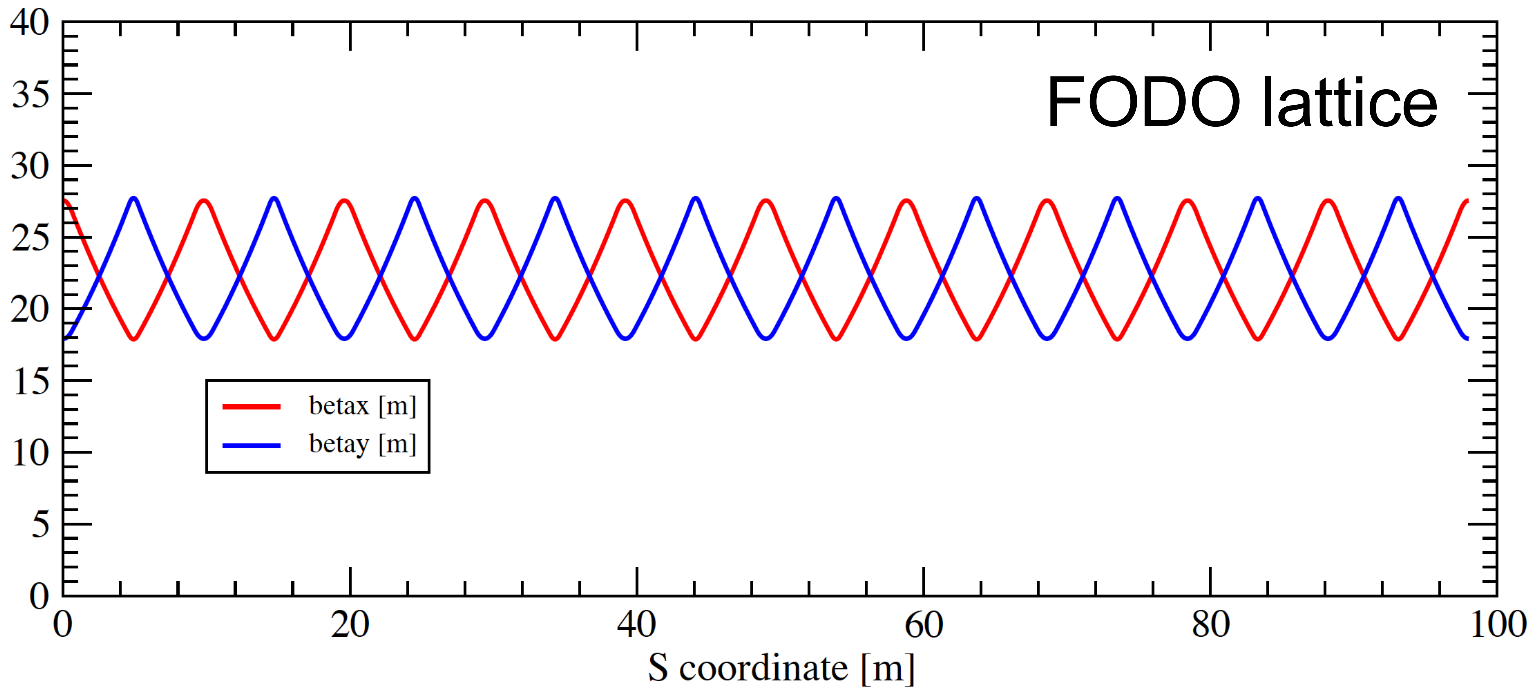
- Periodic transport matrices can be parameterized as

$$M = I\cos\mu + J\sin\mu = \mathrm{e}^{J\mu} \qquad J \equiv \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} \qquad J^2 = -I$$

Twiss or C-S parameters

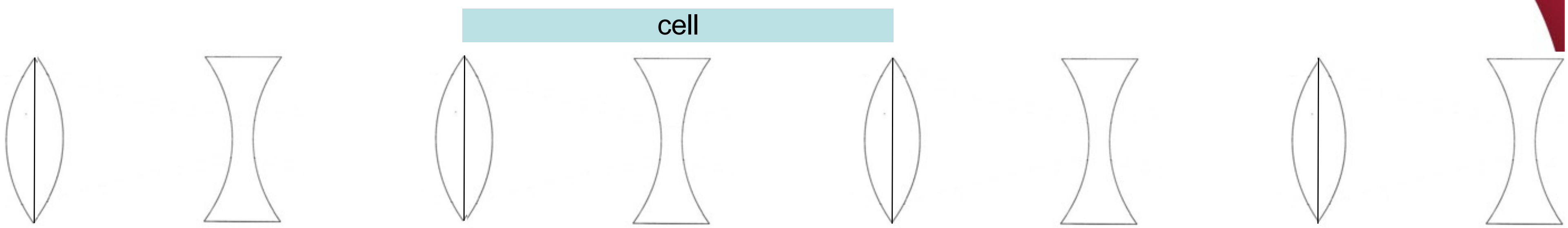
$(\beta, \alpha, \gamma \equiv (1+\alpha^2)/\beta)$ all depend on s location

all have the periodicity of the system



Dipole-Free Transverse Lattices: FODO Review

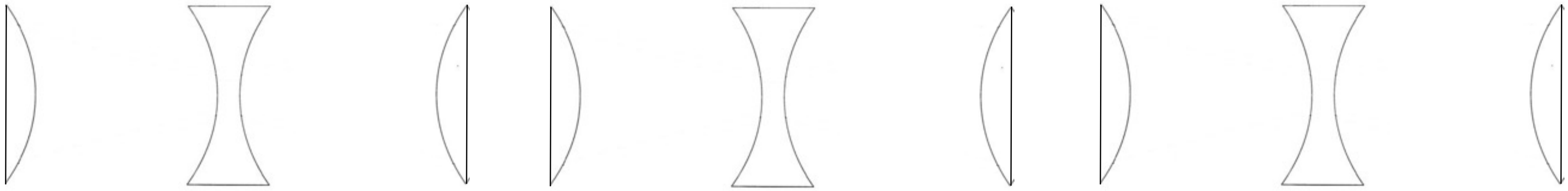
(Be very careful in comparisons to other references!)



- Most accelerator lattices are designed in modular ways
  - Design and operational clarity, separation of functions
- One of the most common modules is a FODO module
  - Alternating focusing and defocusing “strong” quadrupoles
  - Spaces between are combinations of drifts and dipoles
  - Strong quadrupoles dominate the focusing
  - Periodicity is one FODO “cell” so we’ll investigate that motion
  - Horizontal beam size largest at centers of focusing quads
  - Vertical beam size largest at centers of defocusing quads

Dipole-Free Transverse Lattices: FODO Review

cell



$$-2f \qquad L \qquad f \qquad L \qquad -2f$$

cell length 2L

Book $q = 1/(2f)$ $\qquad -q \qquad 2q \qquad -q$

- Select periodicity between centers of **focusing** quads
  - A natural periodicity if we want to calculate maximum β(s)

$$M = \begin{pmatrix} 1 & 0 \\ -q & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2q & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -q & 1 \end{pmatrix}$$

book 3.47

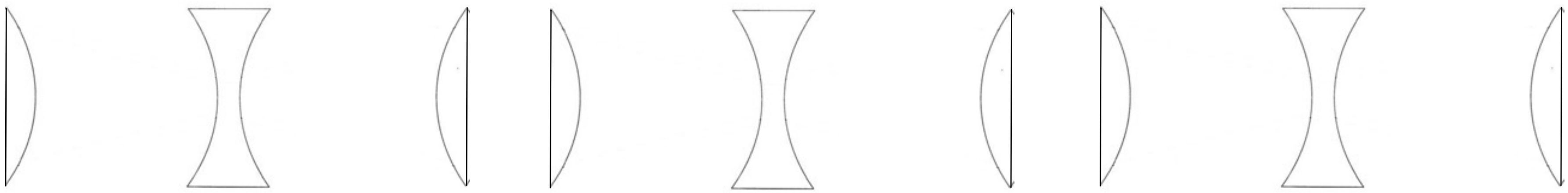
$$M = \begin{pmatrix} 1-2(qL)^2 & 2L(1+(qL)) \\ -2q(qL)(1-qL) & 1-2(qL)^2 \end{pmatrix} \qquad \mathrm{Tr} M = 2\cos\mu = 2 - 4(qL)^2$$

$$1 - 2(qL)^2 = \cos\mu = 1 - 2\sin^2\frac{\mu}{2} \quad \Rightarrow \quad \sin\frac{\mu}{2} = \pm qL = \pm\frac{L}{2f}$$

- μ only has real solutions (stability) if $\dfrac{L}{2} < f$ book 3.49

Dipole-Free Transverse Lattices: FODO Review

cell



$$-2f \quad L \quad f \quad L \quad -2f$$

cell length 2L

Book $q = 1/(2f)$ $\qquad -q \qquad 2q \qquad -q$

- What is the maximum beta function, $\hat{\beta}$?
  - A natural periodicity if we want to calculate maximum β(s)

$$M = \begin{pmatrix} 1-2(qL)^2 & 2L(1+(qL)) \\ -2q(qL)(1-qL) & 1-2(qL)^2 \end{pmatrix} \Leftarrow \qquad m_{12} = \beta \sin\mu$$

$$\hat{\beta}\sin\mu = 2L\left(1+\sin\frac{\mu}{2}\right)$$

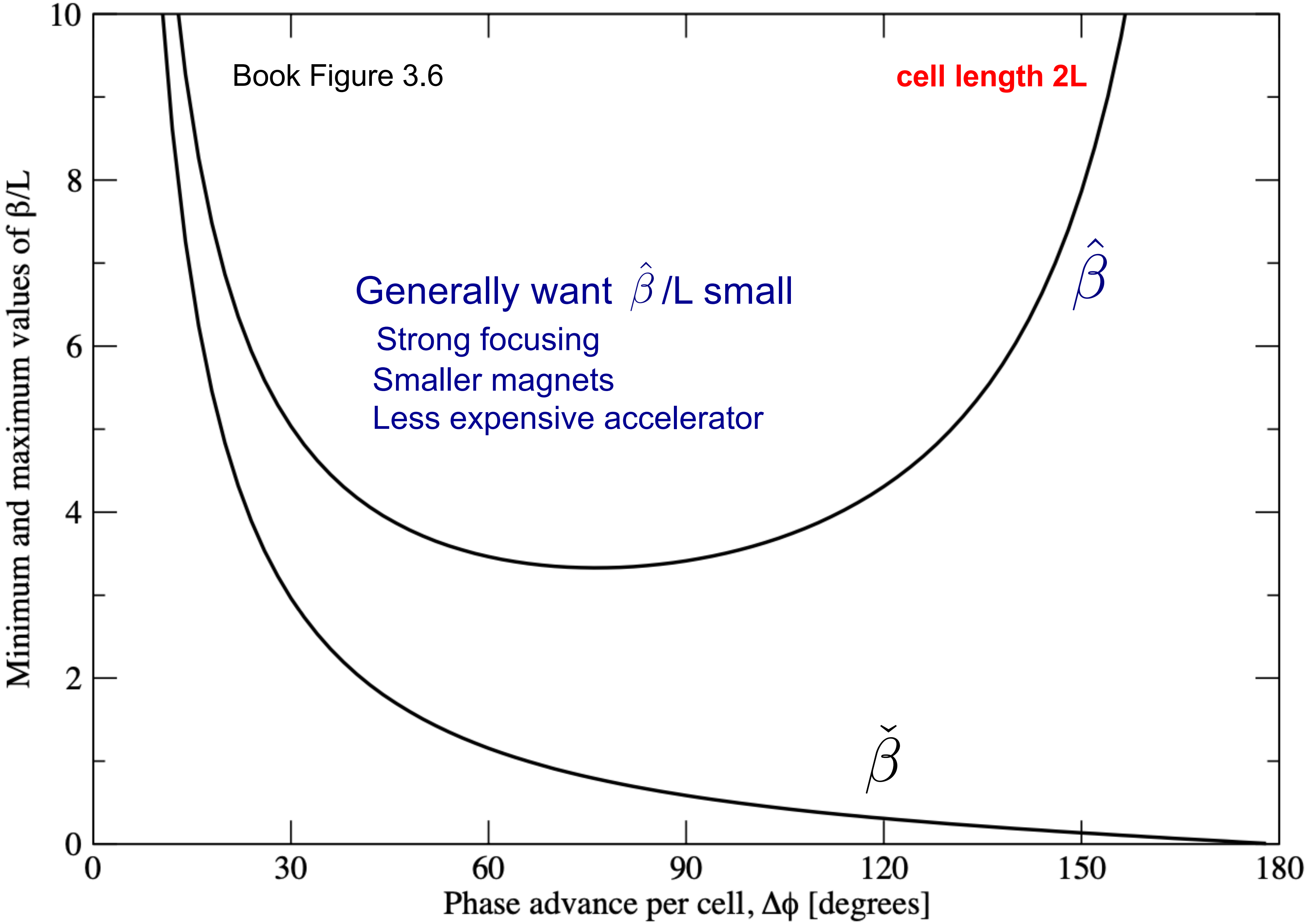
$$\boxed{\hat{\beta} = \frac{2L}{\sin\mu}\left(1+\sin\frac{\mu}{2}\right)}$$

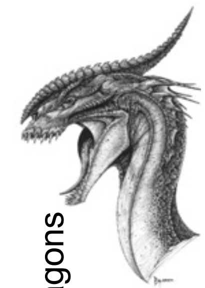
  - Follow a similar strategy reversing F/D quadrupoles to find the minimum β(s) within a FODO cell (center of D quad)

$$\boxed{\check{\beta} = \frac{2L}{\sin\mu}\left(1-\sin\frac{\mu}{2}\right)}$$

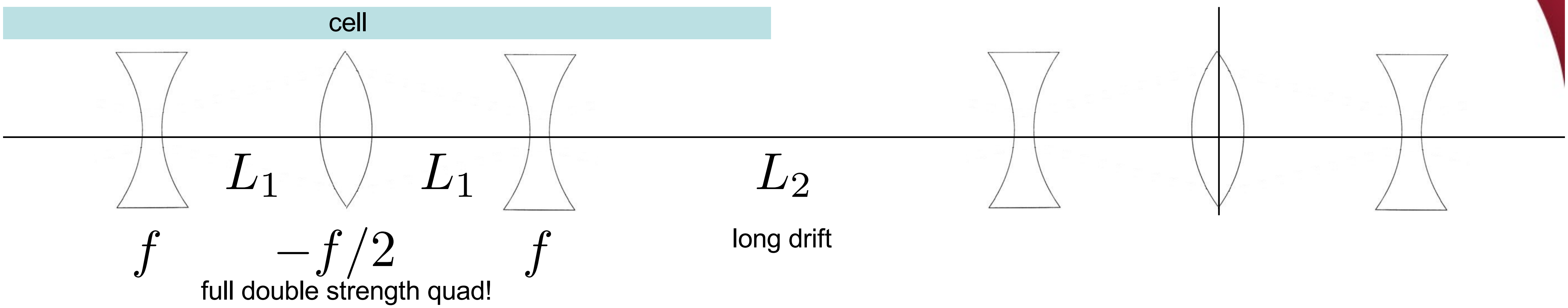
equivalent to book 3.50

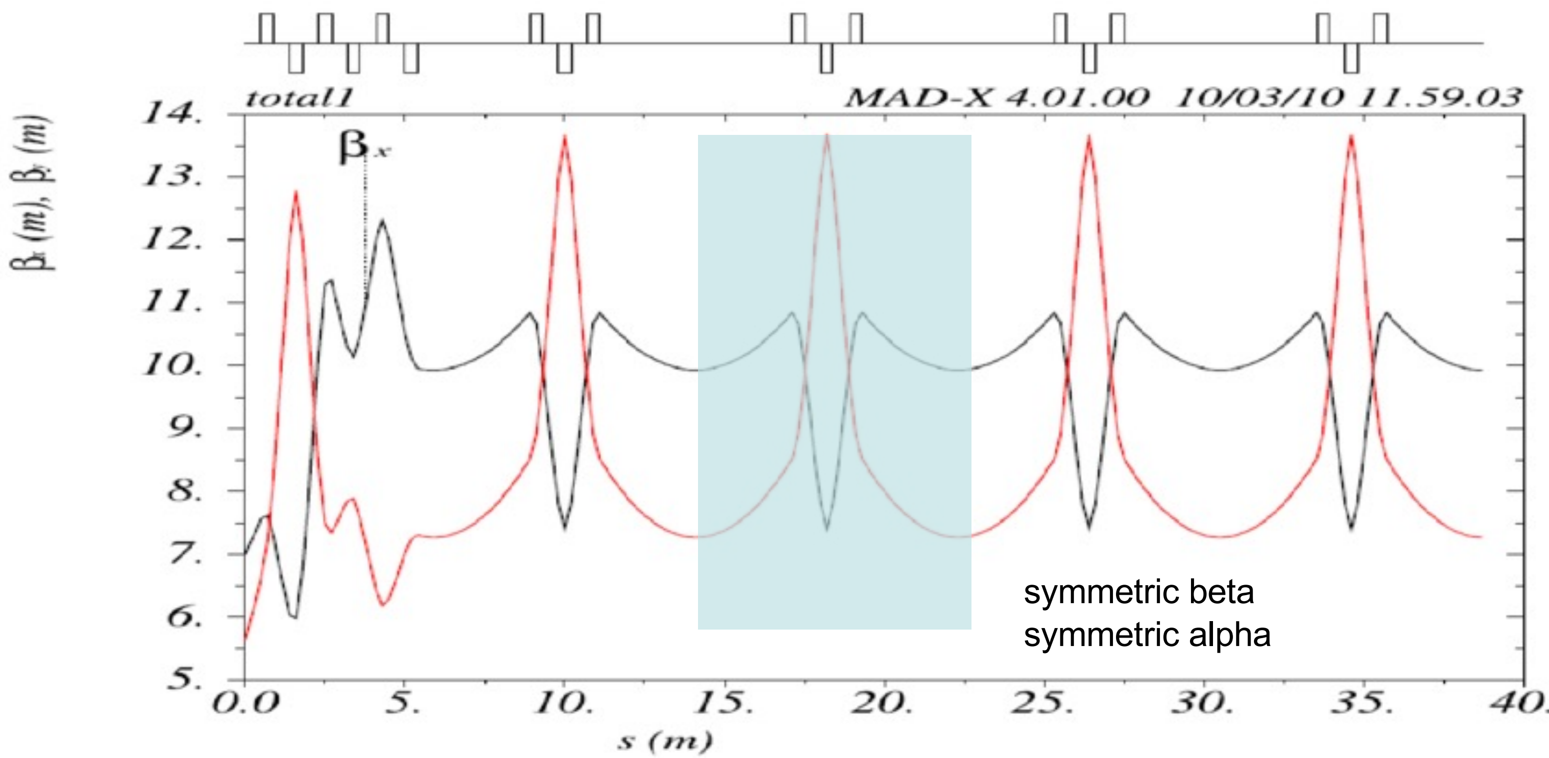
Review: FODO Betatron Functions vs Phase Advance





Triplet Optics: Extra Straight Space





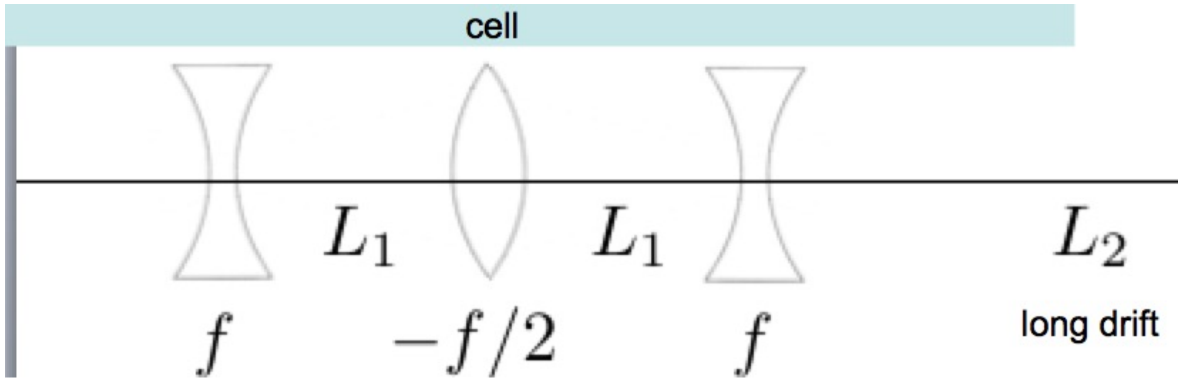
More on Friday

It's Friday!

α=0 is a boundary condition

From R. Chehab et al., "The CLIC Positron Capture and Acceleration in the Injector Linac", 2010.

Triplet Cell Strategy: Not Exactly FODO

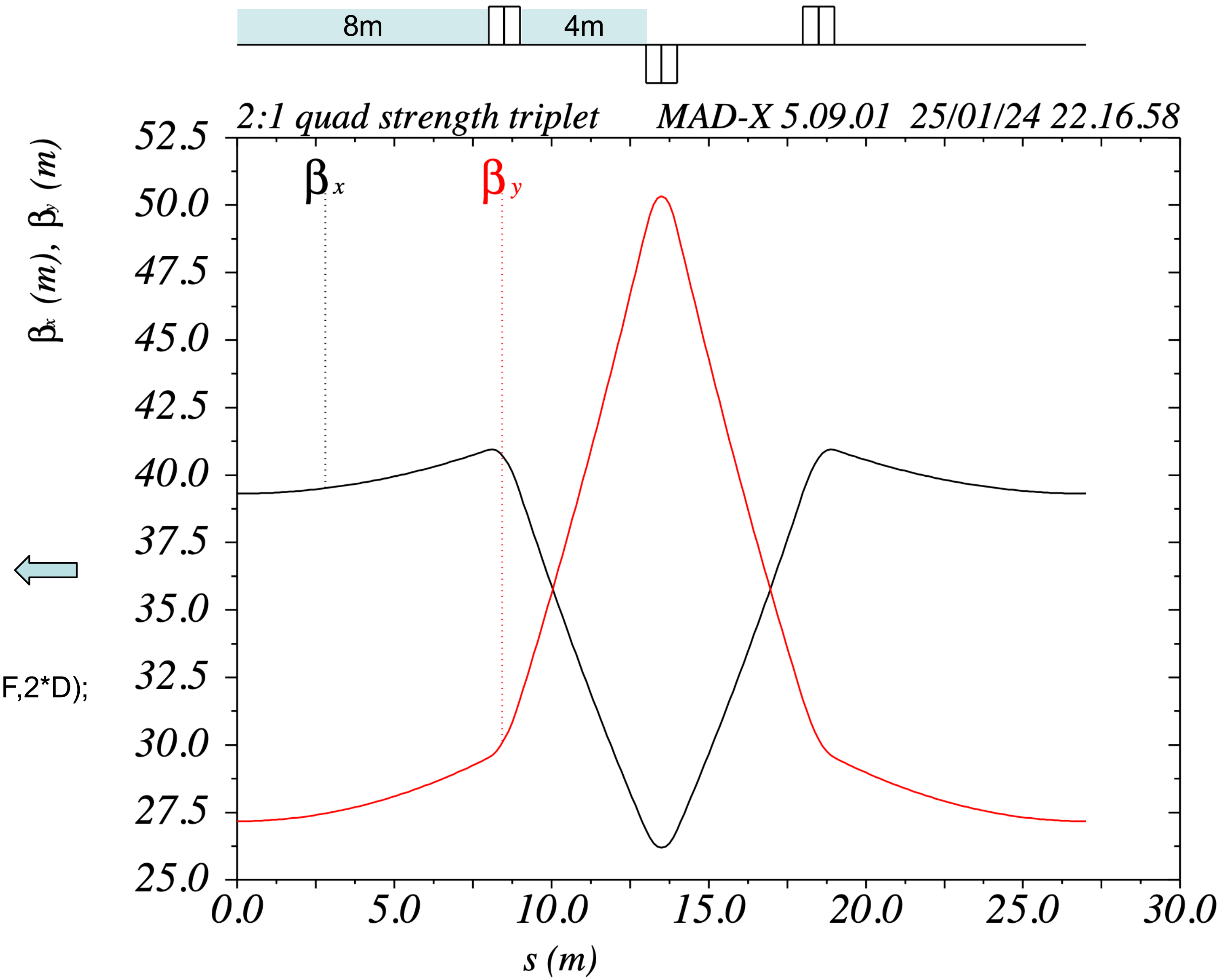


- Calculate transport matrix in terms of L_1, L_2, f
  - Three degrees of freedom
  - Use (3.34) from book/hw and find eigenvalues with α=0

$$\begin{pmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\ -m_{21}m_{11} & 1+2m_{12}m_{21} & -m_{12}m_{22} \\ m_{21}^2 & -2m_{22}m_{21} & m_{22}^2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{pmatrix}$$

Emphasis on periodic solutions for repeating cells
Can also design with madx (often know ~lengths $L_{1,}$ L_2)

Triplet Focusing: "Equal" strength quads



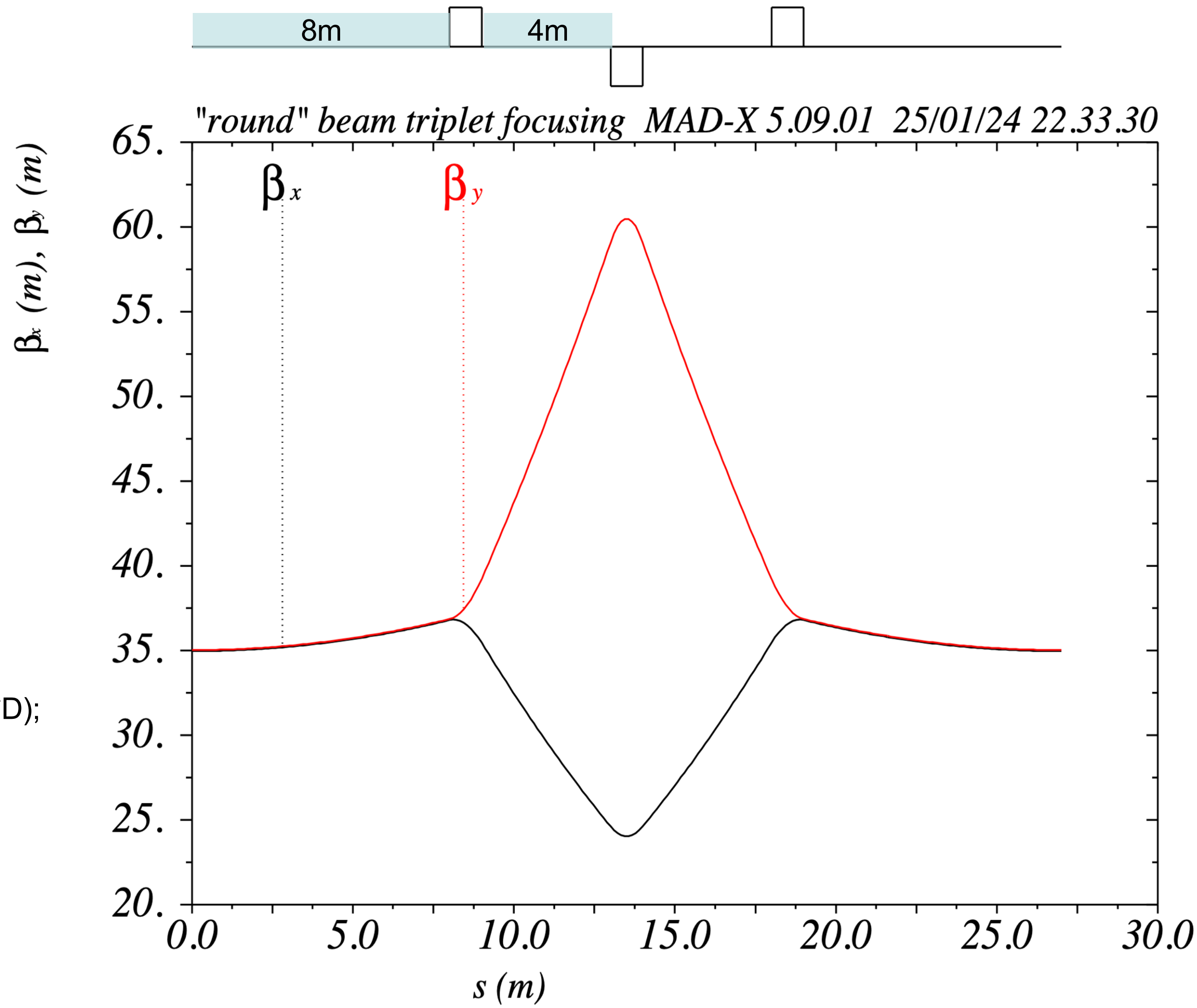
madx file link

```
// Element definitions
D: DRIFT, L=4.0;
QF: QUADRUPOLE, L=1,K1=0.05;
QD: QUADRUPOLE, L=1,K1=-0.1;

// Lattice definitions
TRIPLET: LINE=(2*D,QF,D,QD,D,QF,2*D);
```

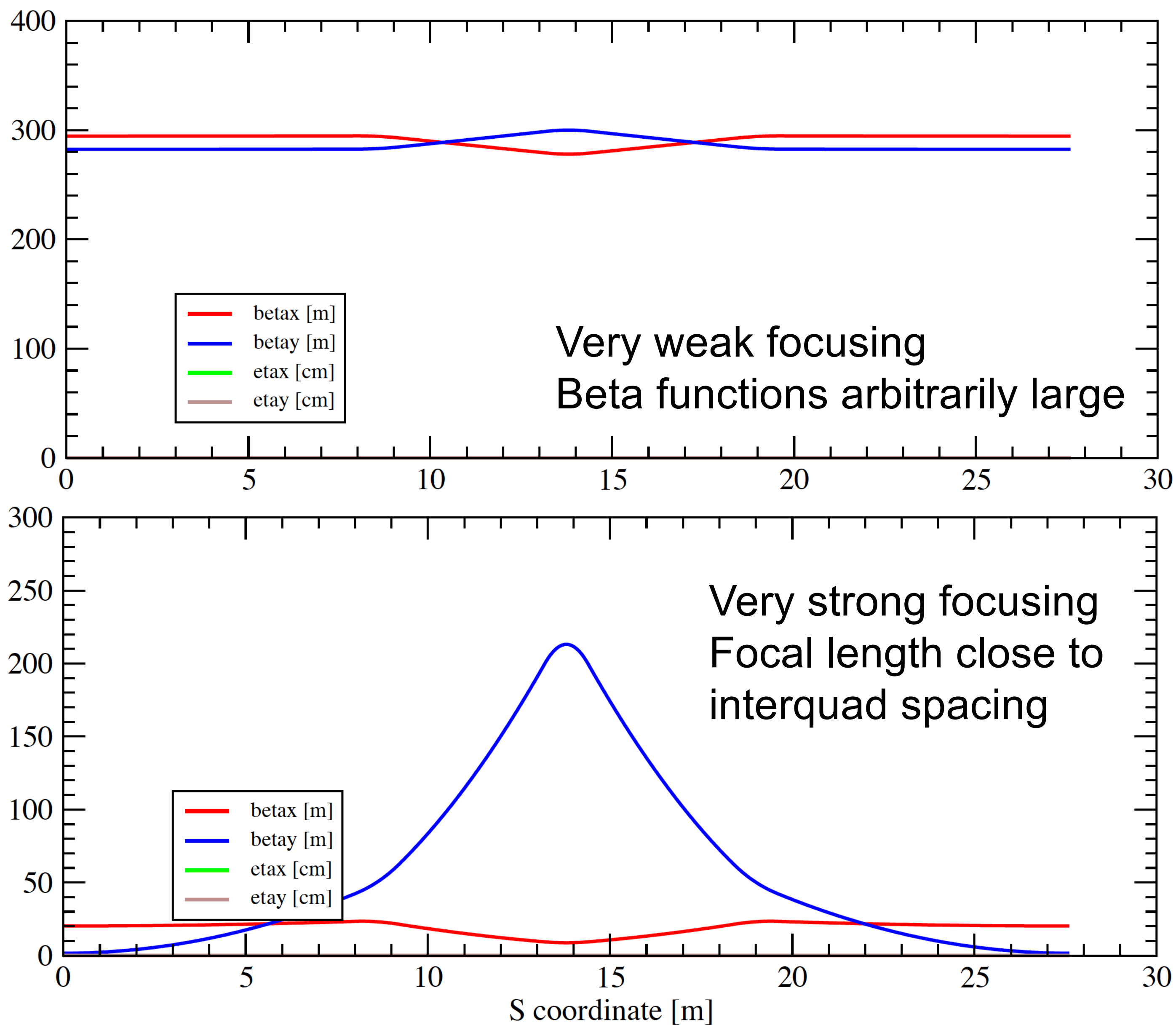
Triplet Focusing: "Round" beams

madx file link

```
// Element definitions
D: DRIFT, L=4.0;
QF: QUADRUPOLE, L=1,K1=0.05;
QD: QUADRUPOLE, L=1,K1=-0.0935;

// Lattice definitions
TRIPLET: LINE=(2*D,QF,D,QD,D,QF,2*D);
```

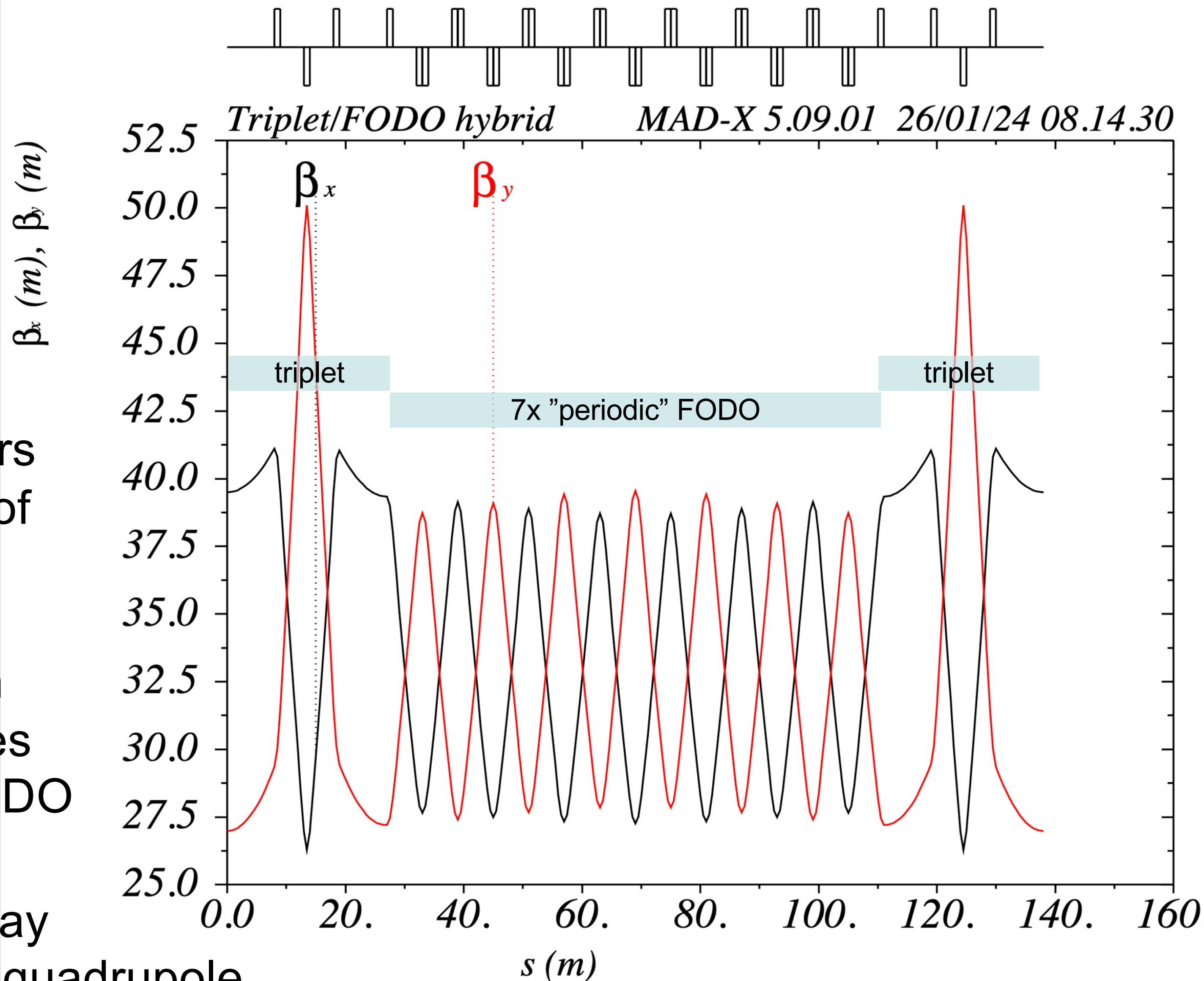


Triplet Focusing: Absurd Extremes



First Matching: Triplet/FODO lattice

madx file link



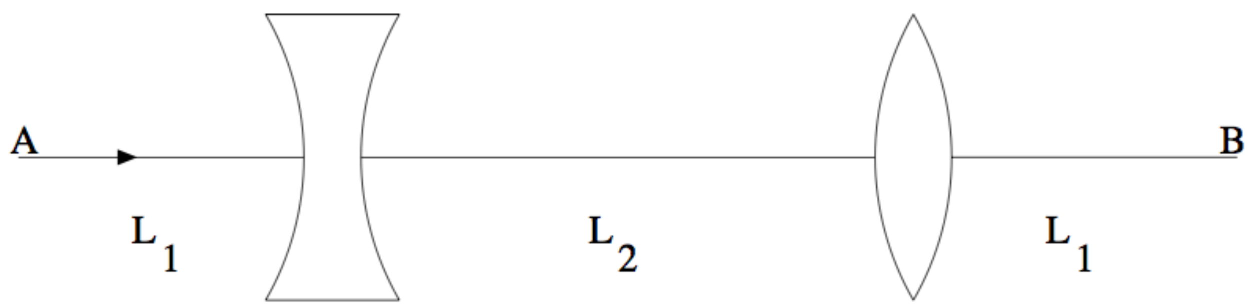
Optics matching:

- All twiss parameters are equal at ends of two sections to be matched
- Here need to match $\alpha_{x,y}$=0 and $\beta_{x,y}$ values at ends of triplet/FODO

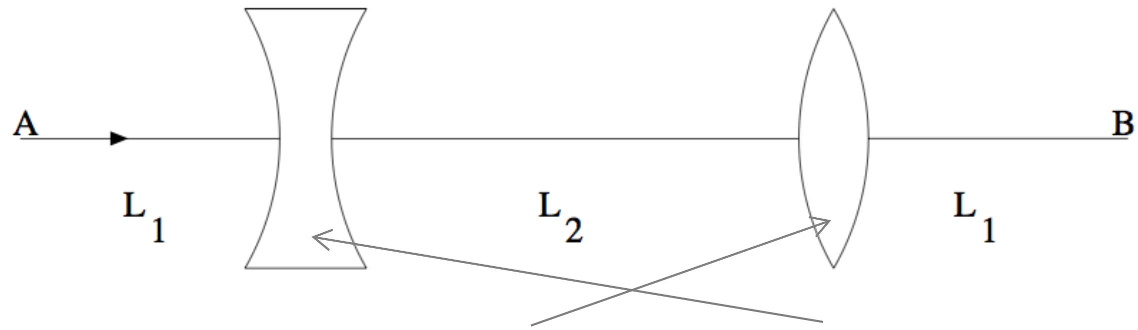
Use $\alpha_{x,y}$=0 halfway through FODO quadrupole

π/2 Insertion

- Insertions and matching: **modular** accelerator design
- FODO sections have very regular spacings of quads
  - Periodicity of quadrupoles => periodicity of focusing
- But we may need some long quadrupole-free sections
  - RF, injections, extraction, experiments, long instruments
- Can we design a periodic "module" that fits in a FODO lattice with a long straight section, and matches to FODO optics?
  - Yes: the minimal periodic option is the π/2 insertion
  - Matching lattice functions $(\beta,\alpha)_{x,y}$ at locations A,B



π/2 Insertion



$$M = \begin{pmatrix} 1 & L_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & L_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & L_1 \\ 0 & 1 \end{pmatrix}$$

$$M = \begin{pmatrix} 1 + \frac{L_2}{f} - \frac{L_1 L_2}{f^2} & 2L_1 + L_2 - \frac{L_1^2 L_2}{f^2} \\ -\frac{L_2}{f^2} & 1 - \frac{L_1 L_2}{f^2} - \frac{L_2}{f} \end{pmatrix} = \begin{pmatrix} \cos\mu + \alpha \sin\mu & \beta \sin\mu \\ -\gamma \sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

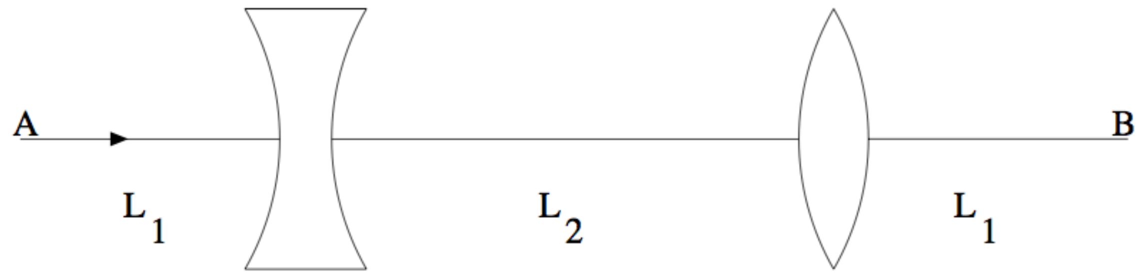
periodic boundary conditions

$$\cos\mu = 1 - \frac{L_1 L_2}{f} \qquad \beta \sin\mu = \left(2 - \frac{L_1 L_2}{f^2}\right) L_1 + L2 \qquad \gamma \sin\mu = \frac{L_2}{f^2}$$

$$m_{21} \text{ term}: \quad L_2 = f^2 \gamma \sin\mu \qquad (\text{recall } \gamma \equiv (1+\alpha^2)/\beta > 0)$$

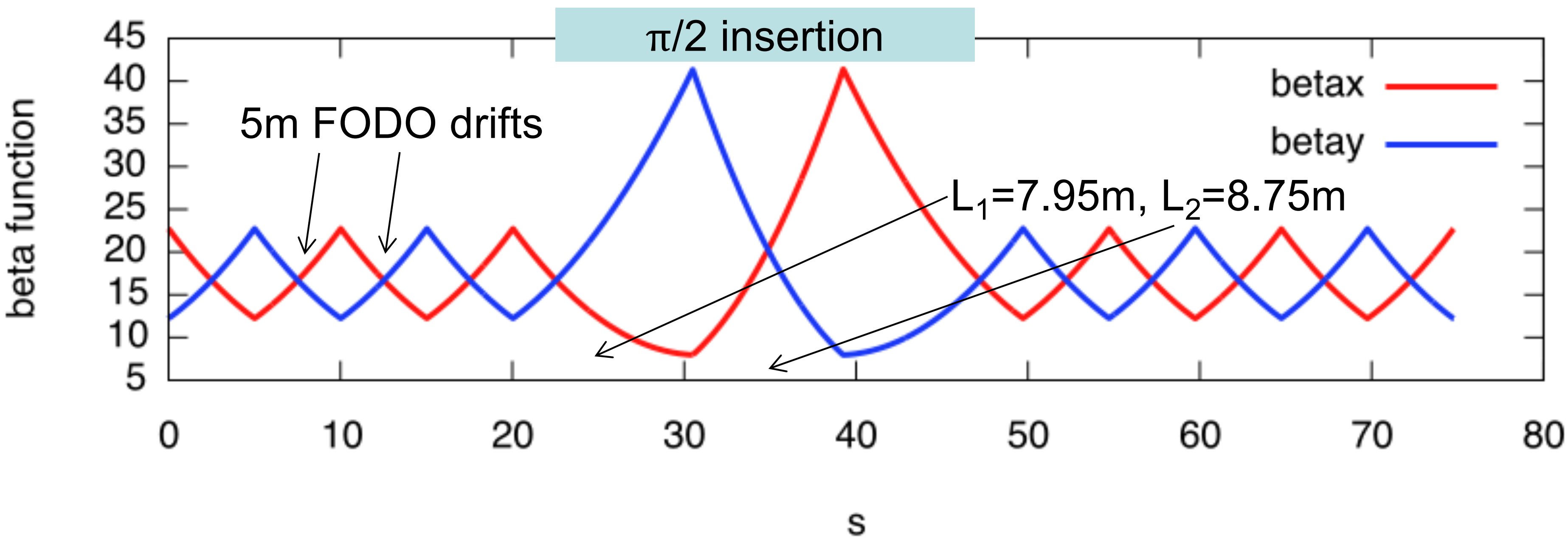
$$\text{Maximum } L_2 \text{ when} \qquad \sin\mu = 1 \qquad \mu = \frac{\pi}{2} \qquad \cos\mu = 0$$

π/2 Insertion



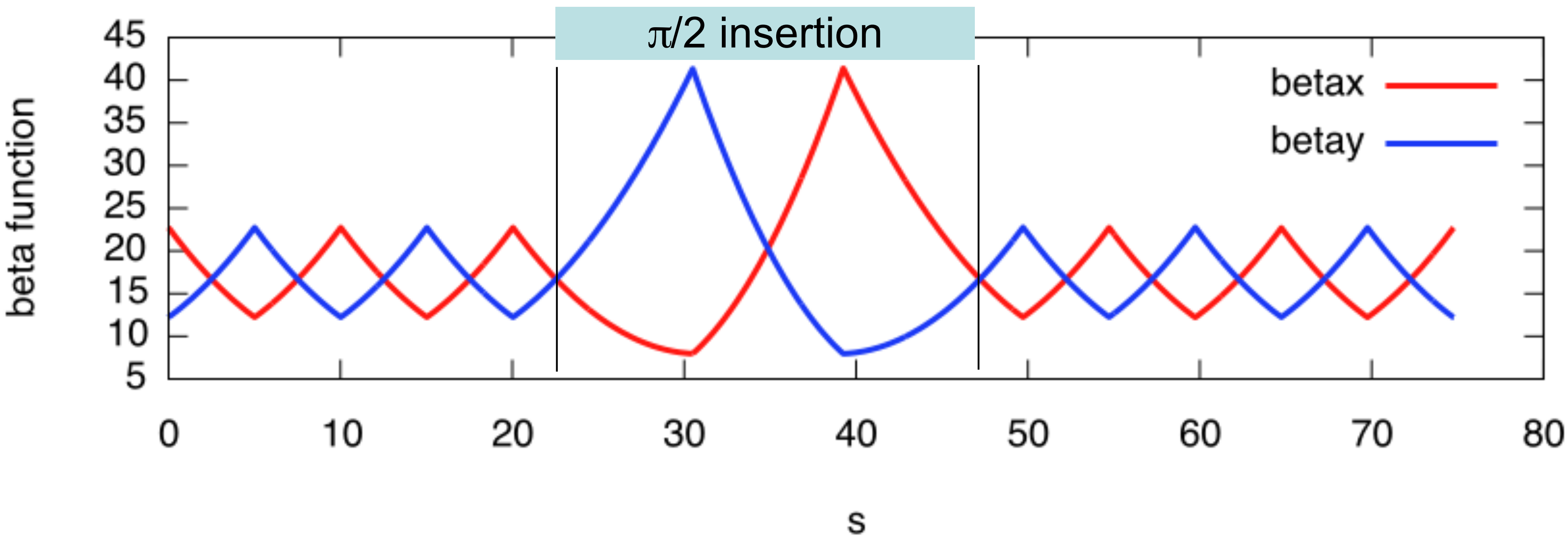
$$\text{Design constraints} \; : \; f = \frac{\alpha}{\gamma} \quad L_2 = \frac{\alpha^2}{\gamma} \quad L_1 = \beta - L_2$$

$$\mathbf{M}_{\pi/2} = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} = J \qquad (\text{recall } J^2 = -I)$$

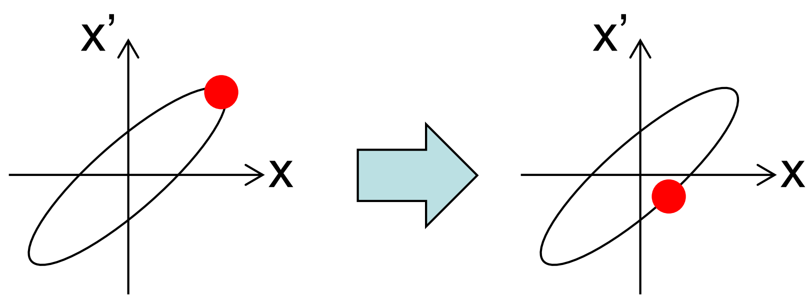


π/2 Insertion

$$\mathbf{M}_{\pi/2} = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} = J \qquad (\text{recall } J^2 = -I)$$



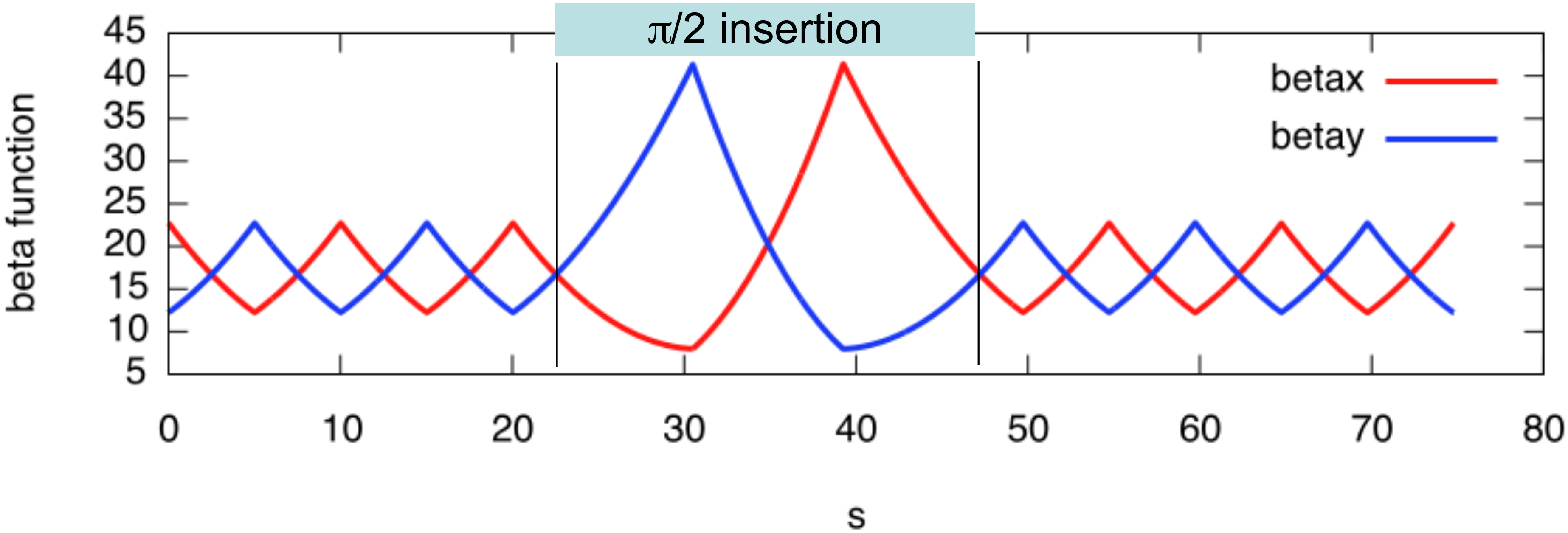
Particles advance 90 degrees (π/2) in phase in this insertion but the Twiss parameters are completely periodic



Play with equation 3.34 and components of $M_{\pi/2}$

π/2 Insertion

$$\mathbf{M}_{\pi/2} = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} = J \qquad (\text{recall } J^2 = -I)$$

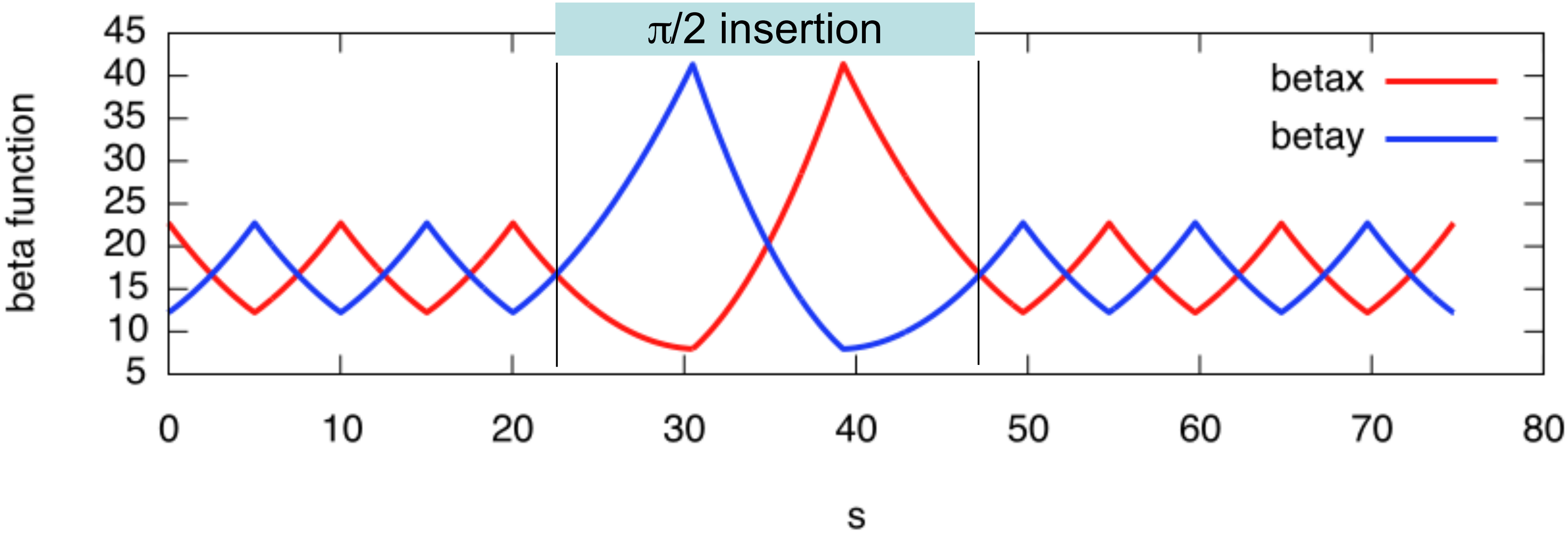


Q1: Why does this work for both planes even though we just designed for one plane?

Hint: $\text{Design constraints} \; : \; f = \frac{\alpha}{\gamma} \quad L_2 = \frac{\alpha^2}{\gamma} \quad L_1 = \beta - L_2$

π/2 Insertion

$$\mathbf{M}_{\pi/2} = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} = J \qquad (\text{recall } J^2 = -I)$$

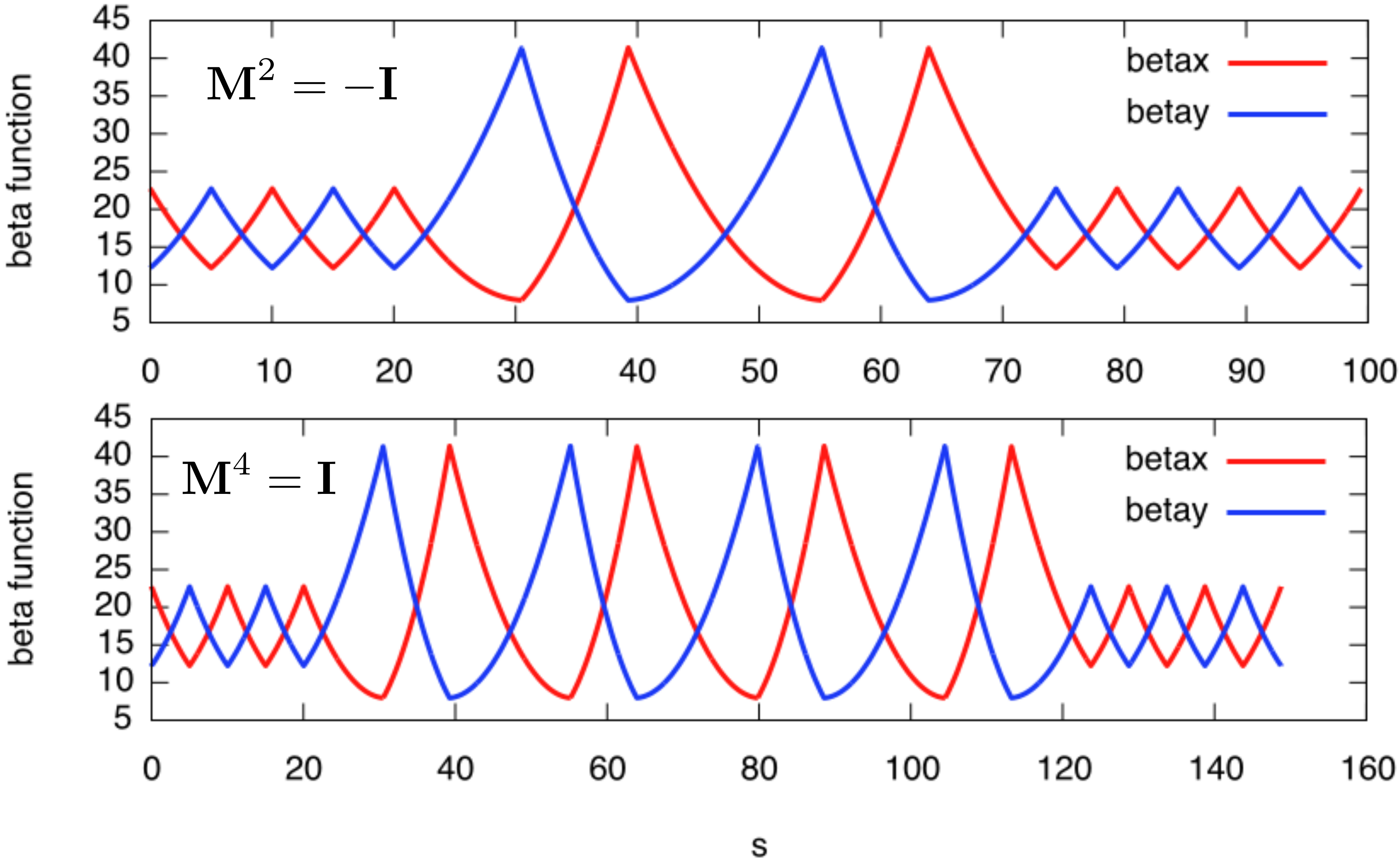


Q2: Can we set α=0 so this becomes an (x,x') exchanger?

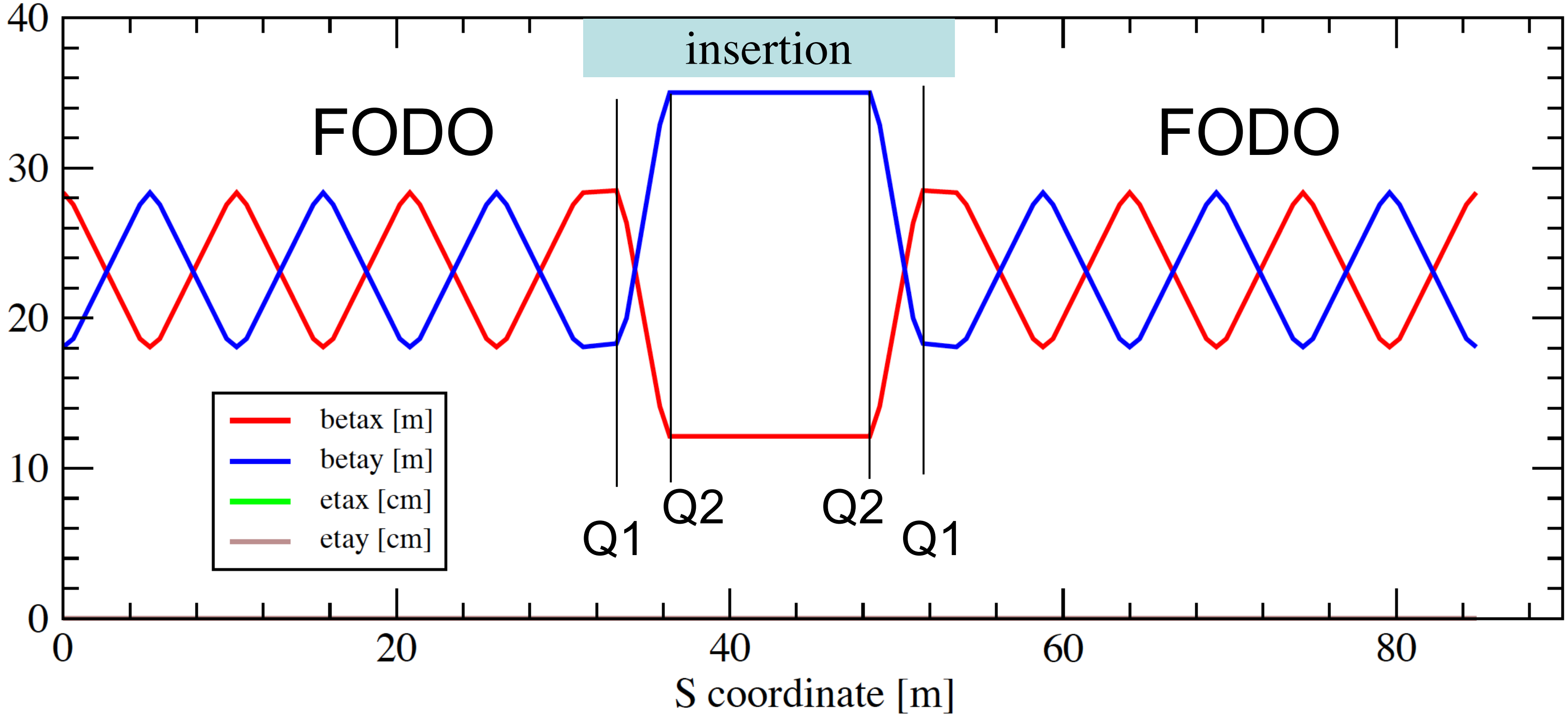
$$M_{\mathrm{xx'\ exchange}} = \begin{pmatrix} 0 & \beta \\ -1/\beta & 0 \end{pmatrix}$$

Hint: $\text{Design constraints} \ : \ f = \dfrac{\alpha}{\gamma} \quad L_2 = \dfrac{\alpha^2}{\gamma} \quad L_1 = \beta - L_2$

Multiple π/2 Insertions



Symmetric Two-Doublet Insertion



Q: What does the symmetry imply for optics behavior?
Q: What about an antisymmetric two doublet insertion?

Class Photo (FODO) Time: 10:40(ish)

USPAS accelerator physics in the days (2011) of 33 students and one TA!

From (x,x') Exchange to (x,y) Exchange

- The π/2 solution prompted a question about (x,x') exchange
- Steve briefly discussed coupling from a theoretical and practical standpoint yesterday...
- Q: is it possible to construct a lattice insertion that exchanges horizontal and vertical phase spaces?
- A: Yes. This was developed in the 90s at Cornell and is called a Mobius insertion.
  - Could "trivially" be implemented with a very long solenoid

Mobius Insertion

- Fully coupled equal-emittance optics for e^+e^- CESR collisions (round beam e^+e^- collisions)
  - Symmetrically exchange horizontal/vertical motion in insertion
  - Horizontal/vertical motion are coupled
    - Only one transverse tune degree of freedom!

$$Q_{x,y}: \ \text{unrotated tunes} \qquad Q_{1,2} = \frac{Q_x + Q_y}{2} \pm \frac{1}{4} \qquad Q_1 - Q_2 = \frac{1}{2}$$

  - Match insertion to points where $\beta_x = \beta_y$ and $\alpha_x = \alpha_y$ with phase advances that differ by π between planes
    - Normal insertion: $\mathbf{M}_{\text{erect}} = \begin{pmatrix} \mathbf{T} & 0 \\ 0 & -\mathbf{T} \end{pmatrix}$
    - Rotated by 45 degrees around s axis: $\mathbf{M}_{\text{mobius}} = \begin{pmatrix} 0 & \mathbf{T} \\ \mathbf{T} & 0 \end{pmatrix}$
  - A purely transverse example of an **emittance exchanger**

S. Henderson, R. Talman, et al., "Investigation of the Möbius Accelerator at CESR", Proc. of the 1999 Particle Accelerator Conference, New York, NY; R. Talman, "A Proposed Möbius Accelerator", Phys. Rev. Lett **74**, 1590-3 (1995).

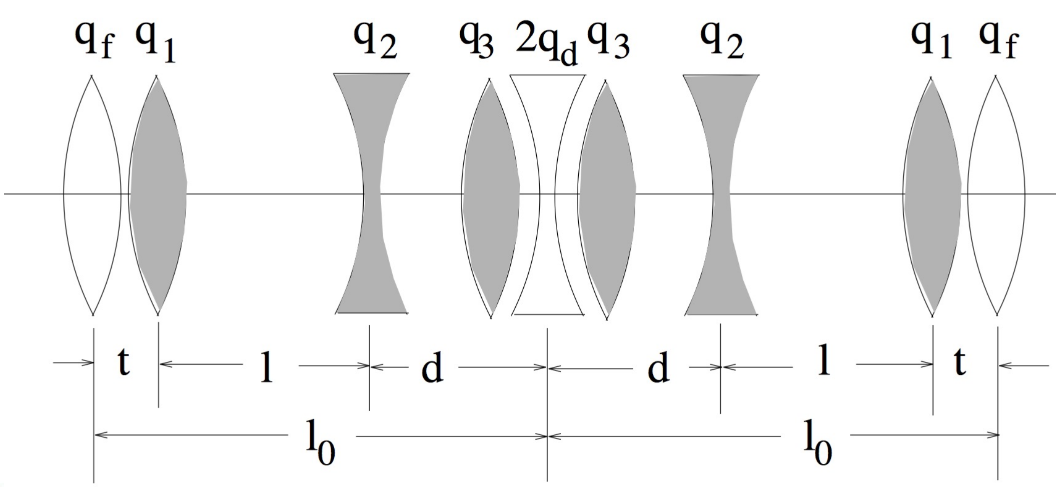
Talman 1993 Mobius Paper

- https://www.classe.cornell.edu/public/CBN/1993/Mobius.ps

The MÖBIUS ACCELERATOR

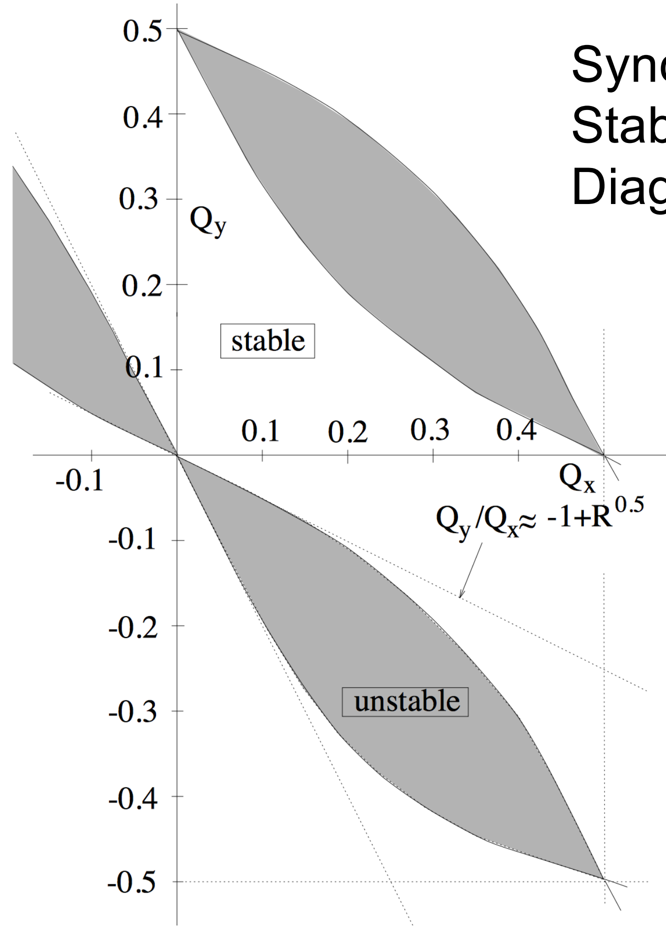
Richard Talman

Laboratory of Nuclear Studies
Cornell University
Ithaca, NY 14853



Shaded = Skew quadrupole

Figure 2: Lattice section needed to switch between ordinary and Möbius operation. For ordinary operation the unshaded elements are run as normal equal-tune FODO elements. For Möbius operation the central element q_d is turned off and the shaded, skew quadrupole elements are powered.



Synchrotron Stability Diagram

Example 4: A Möbius insertion in a light so option to lower the bunch density in order to emittance growth and particle losses

Mobius is still in literature! (2022)

Equilibrium Parameters in Coupled Storage Ring Lattices and Practical Applications
V. Ziemmann and A. Streun
PHYSICAL REVIEW ACCELERATORS AND BEAMS 25, 050703 (2022)

Example 4: A Möbius insertion in a light source is an option to lower the bunch density in order to minimize emittance growth and particle losses

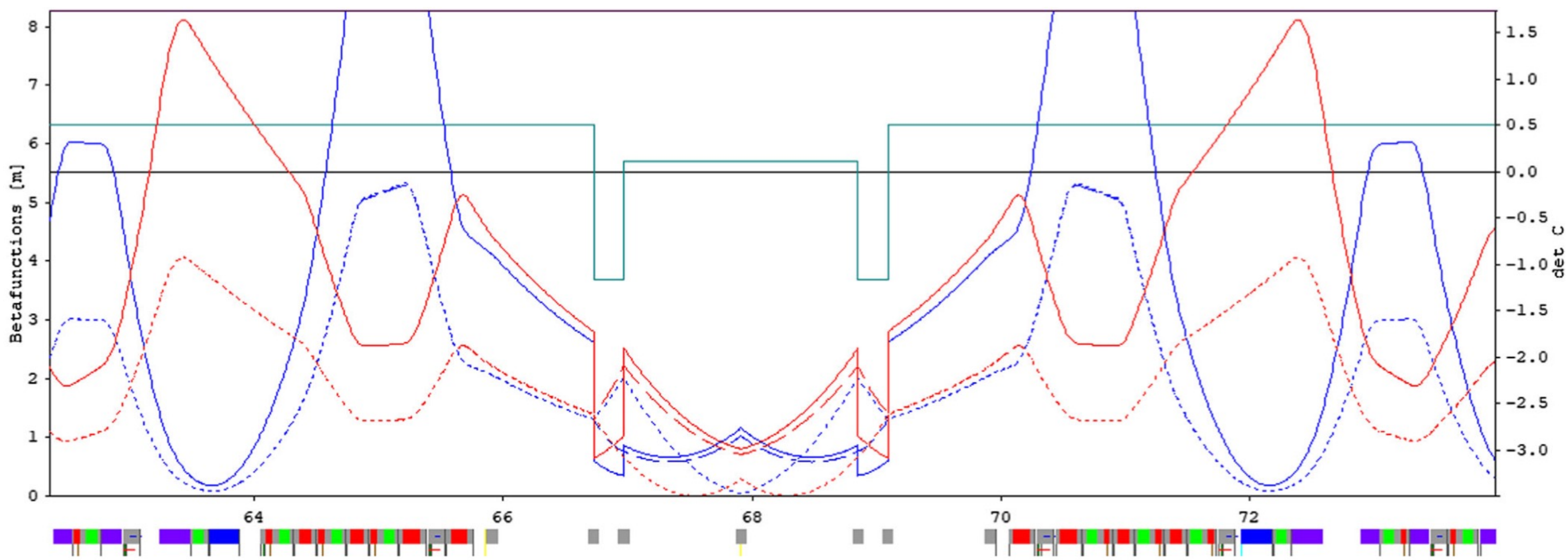
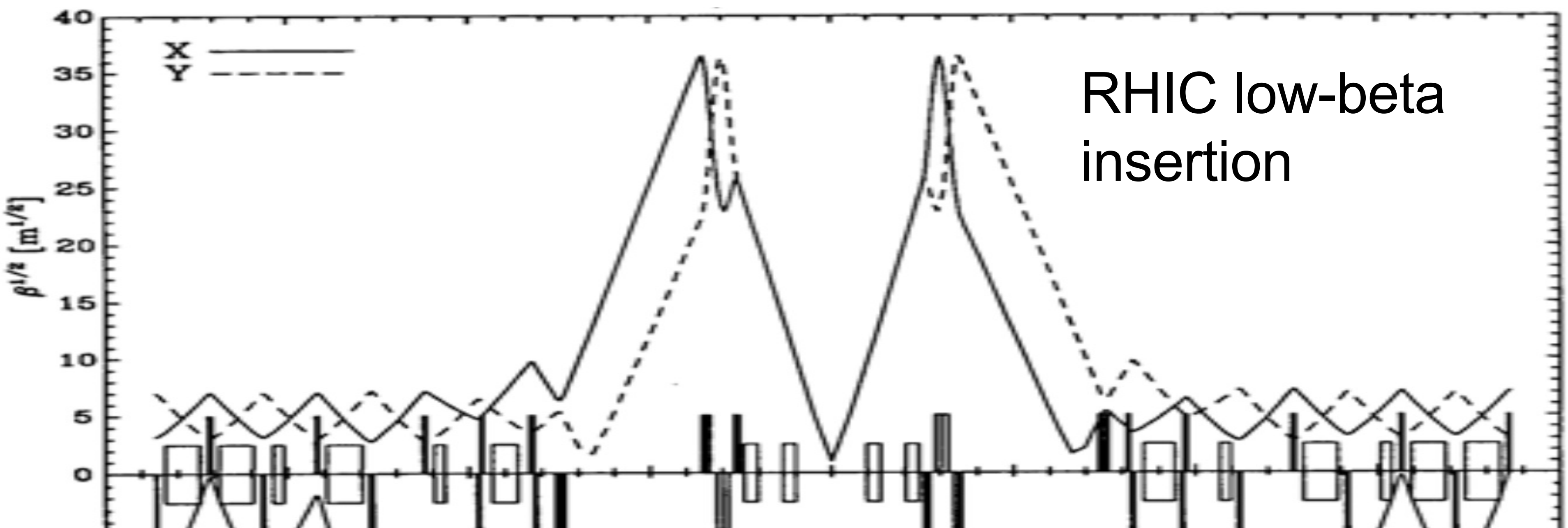


FIG. 7. Möbius insertion in straight 4 of the SLS 2.0 storage ring. Solid lines show the normal mode beta functions (blue a-mode, red b-mode), dashed and dotted lines the projected beta functions β_{xa}, β_{yb} and β_{xb}, β_{ya} (blue β_x, red β_y). The square line shows the determinant of the coupling matrix $\det C = 1 - g^2$ (axis at right).

Low-beta Insertions: intro

- We have one final “dipole-free” insertion to discuss
  - (In practice it may well not be dipole free)
- Low beta insertions are fundamentally quads that focus into special long drift spaces
  - To this point we have avoided overfocusing
  - Low beta insertions are **intentionally** overfocused to create a **minimum beam size** (or waist) in a drift

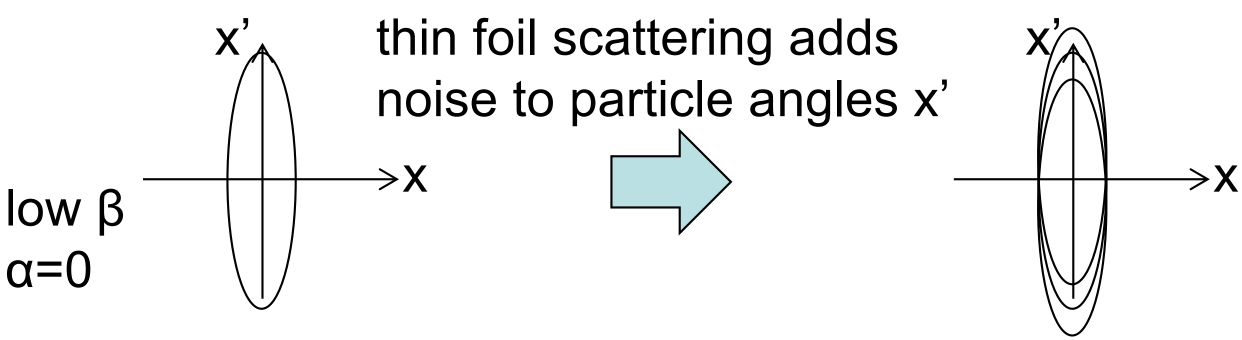


Low-beta Insertions: Uses

- Low-beta is most famously used to maximize collider luminosity by minimizing beam size at interaction point

$$L = f_{rev}\, M\, \frac{N^2}{4\pi \sigma_H^* \sigma_V^*} \qquad (1.11)$$

- Also used to maximize beam divergence
  - *Minimizes* emittance growth from interactions with materials, e.g. ion stripping foils or diagnostic screens



Steve mentioned this application Thursday

Low-beta Insertions

- Recall homework 2.2 about β evolution and phase advance in a drift

$$\beta(s) = \beta^\star + \frac{s^2}{\beta^\star}$$

$$\alpha^\star = 0$$
$$\gamma^\star = 1/\beta^\star$$

where β* is the minimum value of β and s is the s-coordinate distance from this minimum

Smaller $\beta^\star$ gives steeper parabolic increase!

β must be quite large at the quadrupoles surrounding the low-beta insertion to create a small β*

Phase advance across straight section : $2 \arctan\left(\frac{L_{\text{insertion}}}{\beta^\star}\right)$

For $\mathrm{L}_{\text{insertion}} \gg \beta^\star$, , phase advance is π

Low-beta Insertion guidelines

1. Calculate the periodic solution in the arc
2. Start from the IP, introduce the drift space needed for the insertion device (detector ...)
3. Install a quadrupole triplet (or doublet?) fix the aperture requirements and the achievable field gradient
4. Set the desired beta*, drive the triplet at high field, so that the beam is focused back
5. Introduce additional quadrupoles to match the beam parameters to the values at the beginning of the arc

Parameters to be optimized & matched to the periodic solution:

$$\begin{array}{cccc} \beta_x & \alpha_x & D_x & \mu_x \\ \beta_y & \alpha_y & D_y & \mu_y \end{array}$$

Use a code (e.g. madx) to optimize and match!

(D' is normally accepted at the IP)

8 (at least) individually powered quad magnets are needed to match the insertion

Slide from D. Pellegrini

Combining Beam Separation and Low Beta Quads

Both dipoles and quadrupoles need to be close to the IP, not always integrable into the detector.

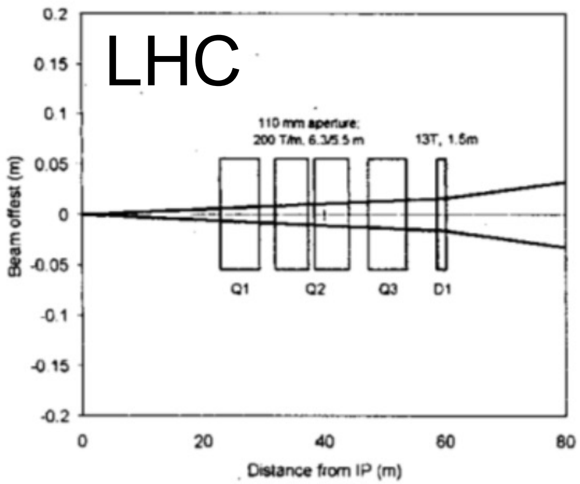


Figure 1: Quadrupole-first IR.

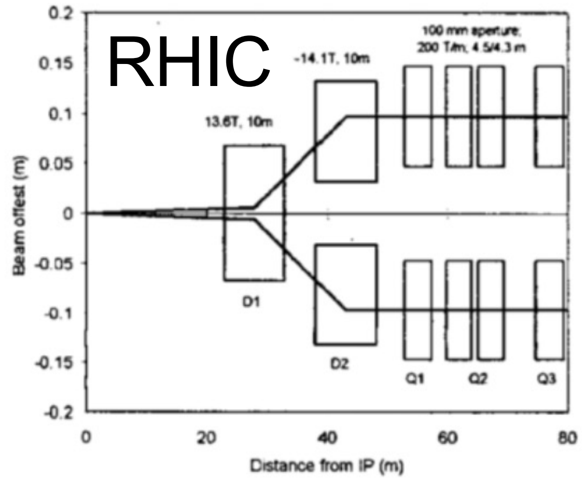


Figure 2: Dipoles-first IR.

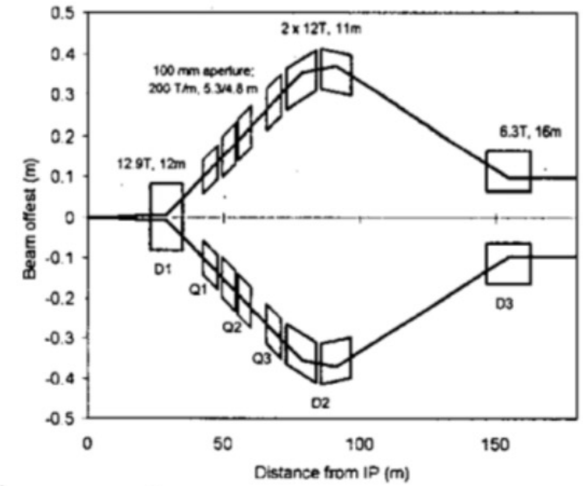


Figure 3: IR with quads between the separation dipoles.

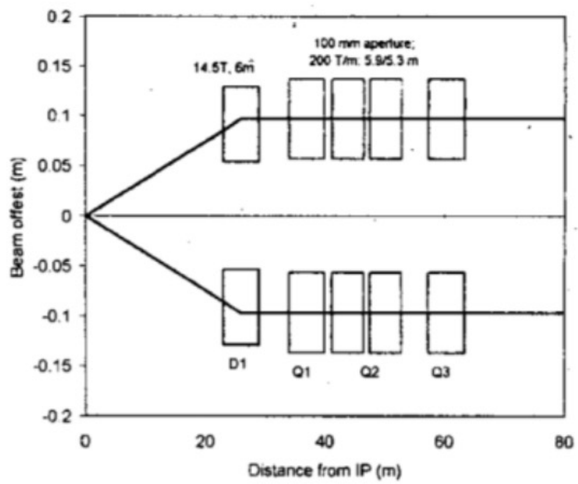


Figure 4: Dipole-first IR with large crossing angle.

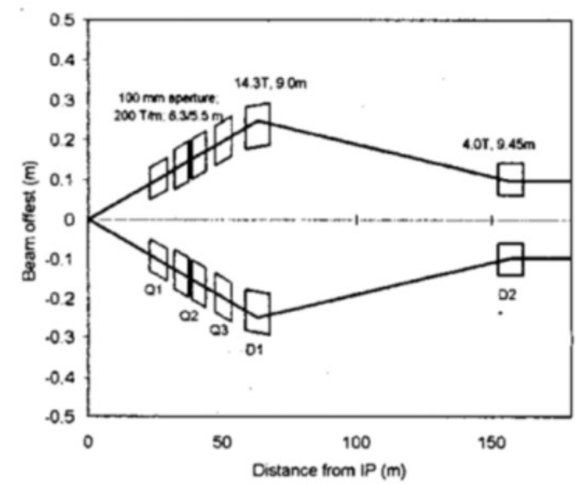


Figure 5: Quadrupole-first IR with large crossing angle.

LHC went for case 1

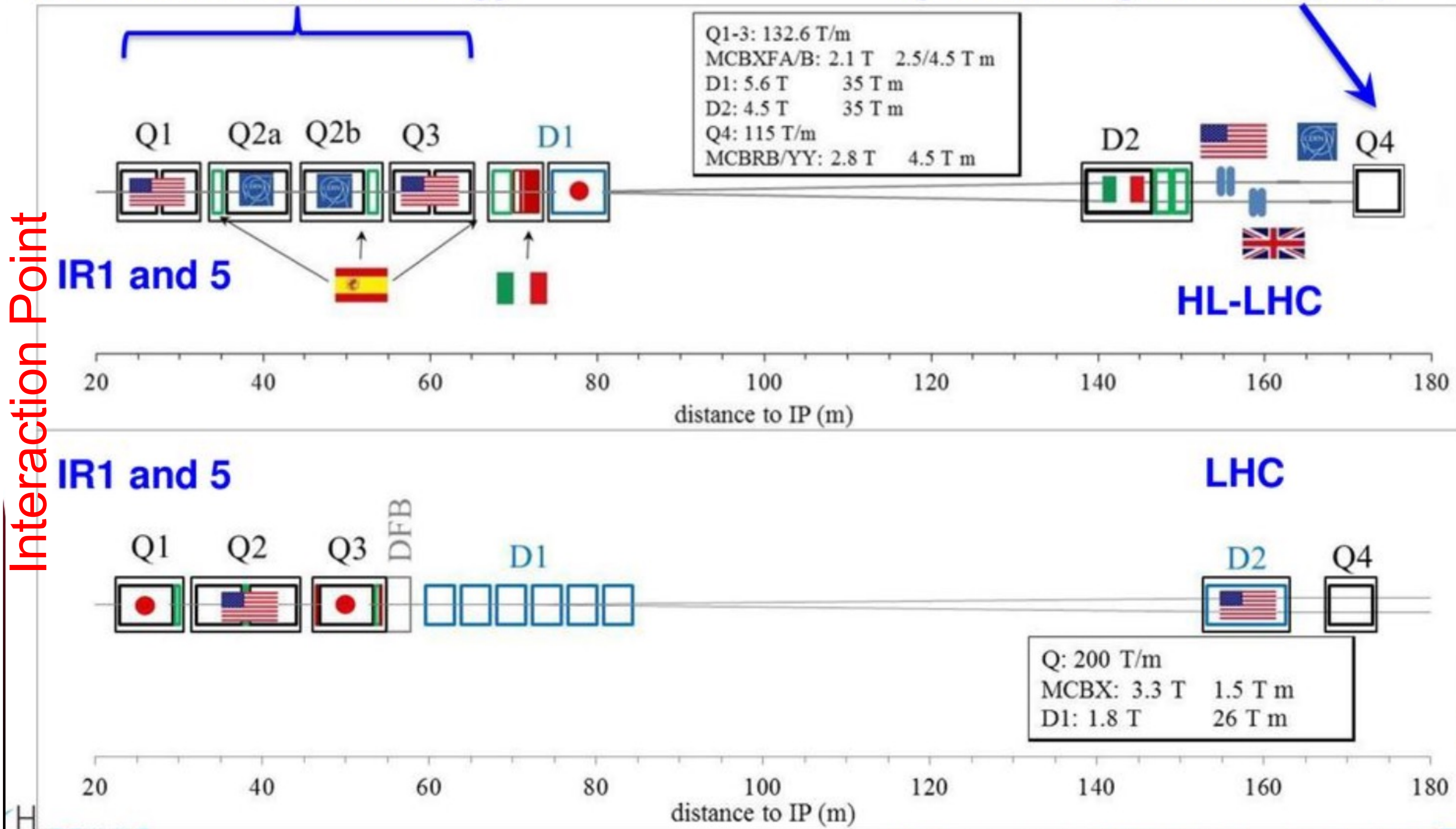
J. Strait et al., proceedings of PAC 2003

Slide from D. Pellegrini

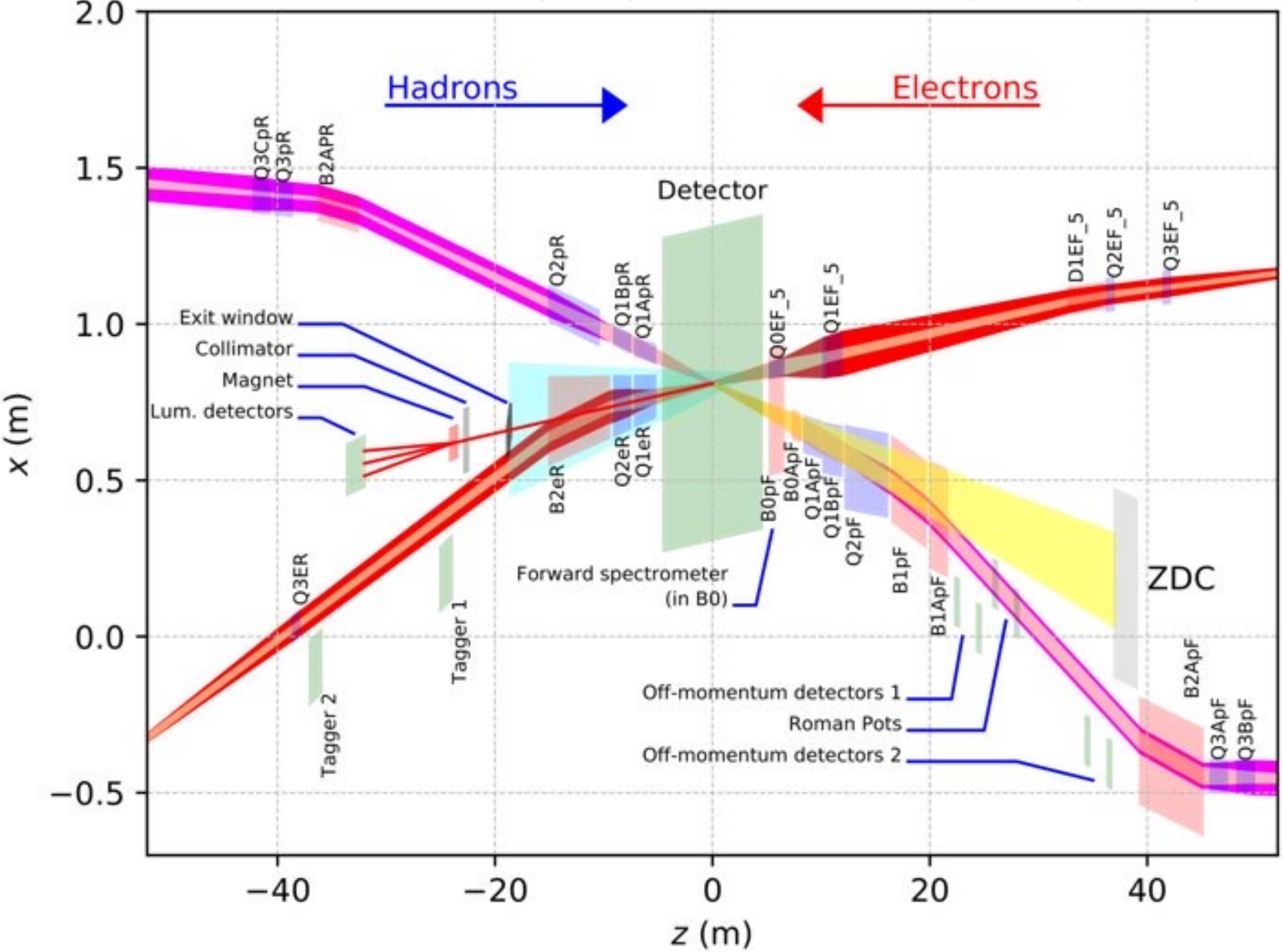
The HL-LHC Interaction Region

Reduction of triplet gradient (140 T/m to 133 T/m). Additional margin taking for the brand new technology

Staged installation of larger aperture quadrupole for other optics configurations (flat)



Electron-Ion Collider (EIC) Interaction Region (2022)



Dispersion Review

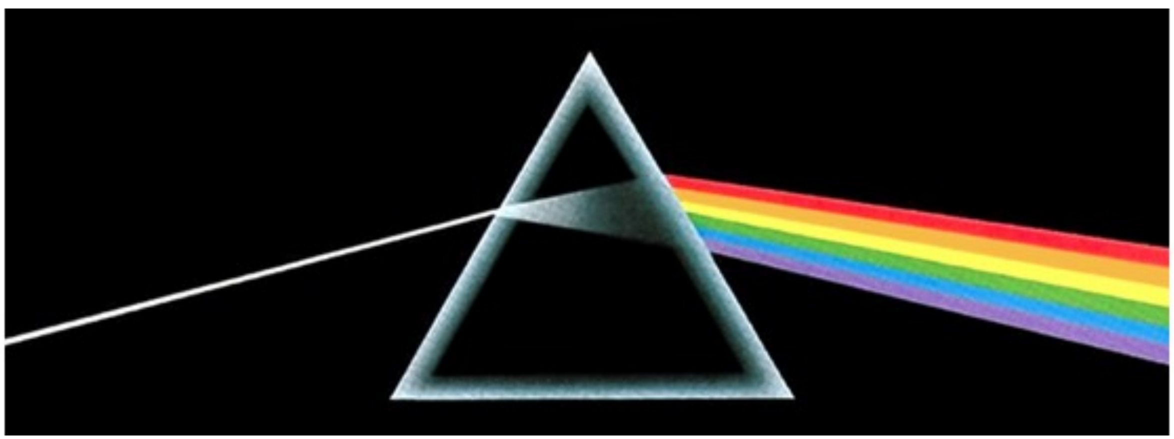
- Review and reformulation of Tuesday PM material (a long time ago)
- **Dispersion** $\eta(s)$ is defined as the change in particle position with fractional momentum offset $\delta \equiv \Delta p/p_0$

$$x(s) = \text{betatron} + \eta_x(s)\delta \qquad \eta_x(s) \equiv \frac{dx}{d\delta}$$

Dispersion **originates** from momentum dependence of dipole bends

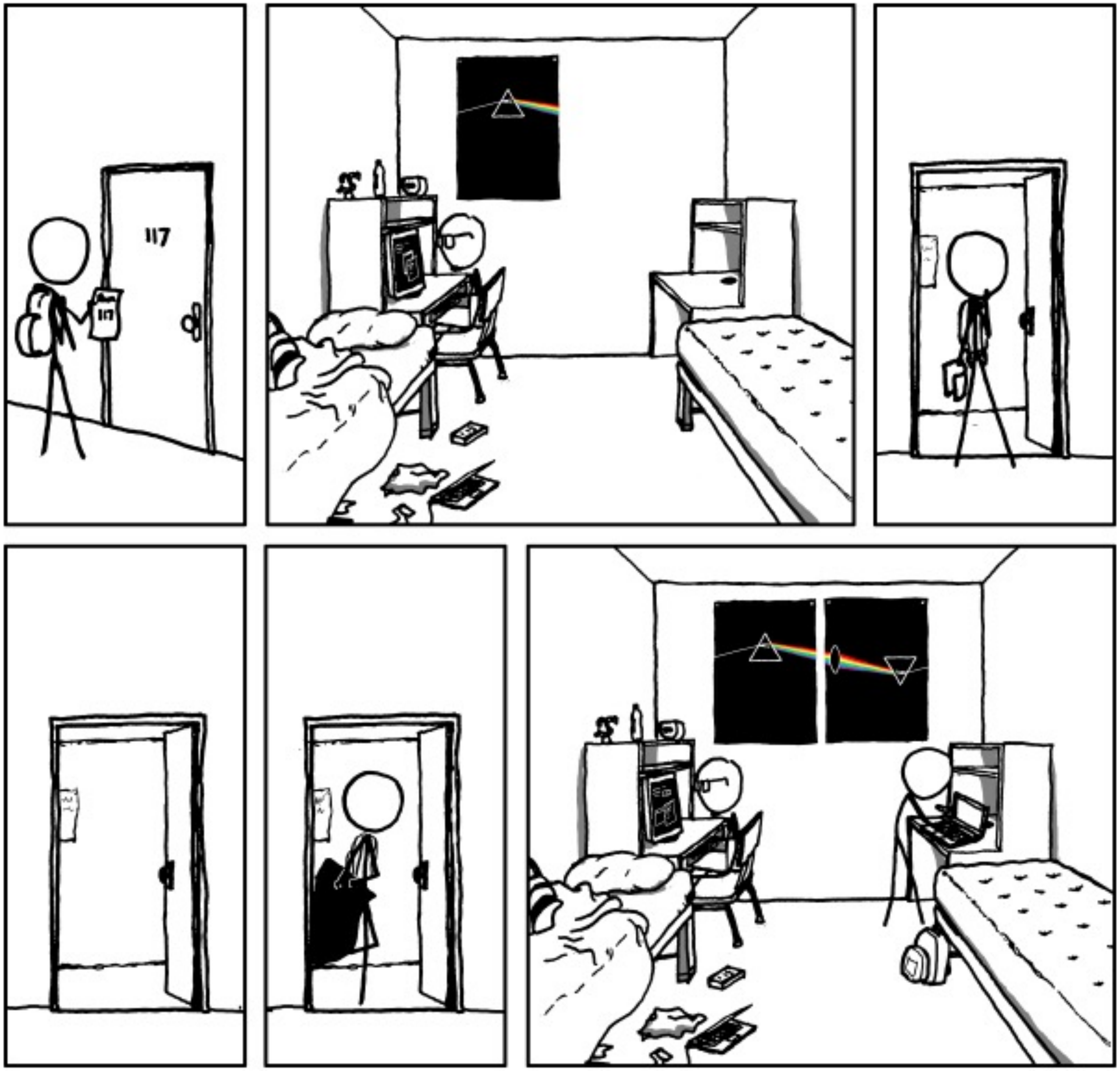
Equivalent to separation of optical wavelengths in prism

White light with many frequencies (momenta) enters, all with same initial trajectories (x,x')



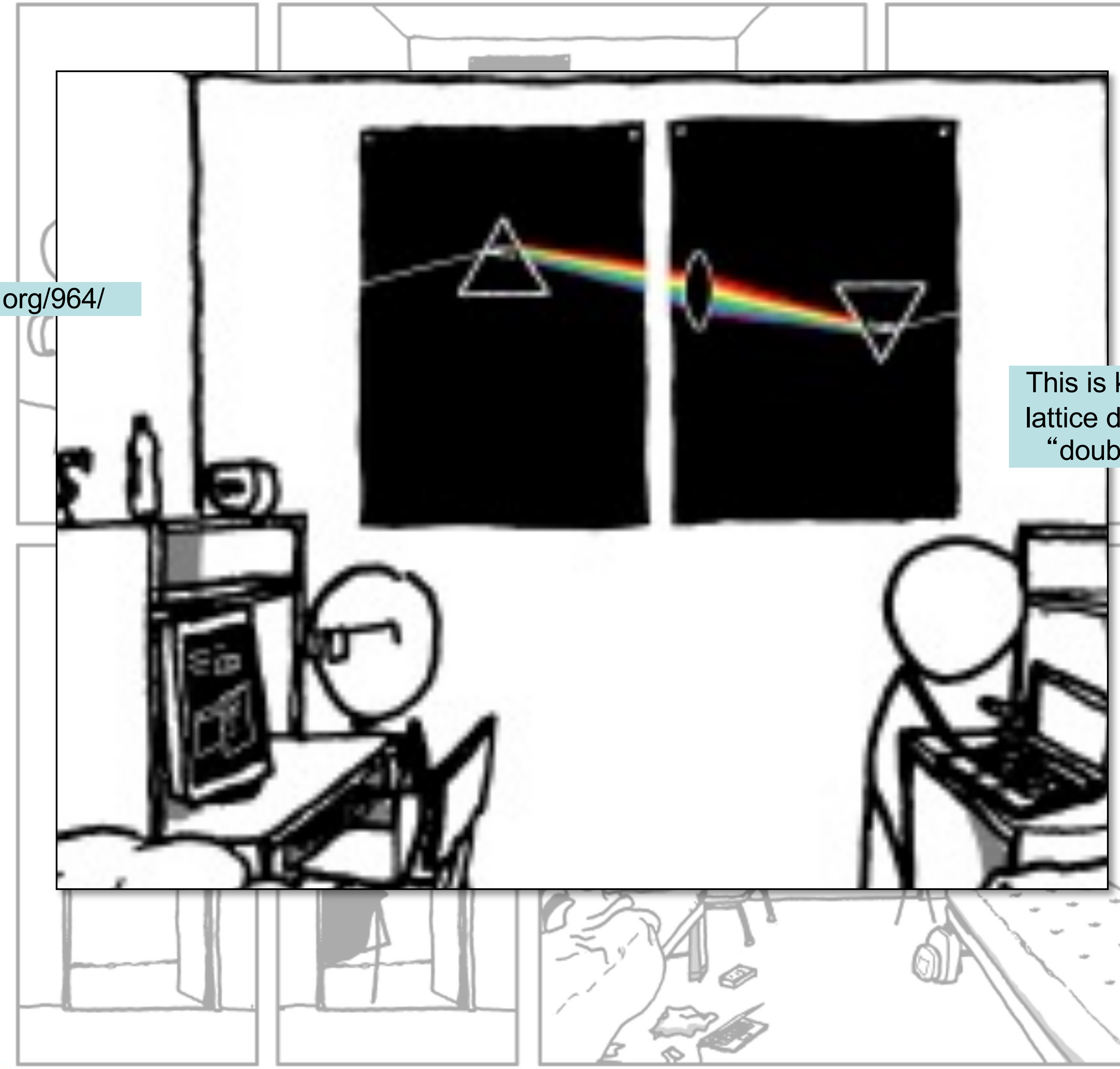
Different positions due to different bend angles of different wavelengths (frequencies, momenta) of incoming light

(xkcd interlude)



(xkcd interlude)

http://www.xkcd.org/964/



This is known in accelerator lattice design language as a "double bend achromat"

"Review": Sector Dipole (and "Weak" Sector Dipole)

Wednesday Magnet lecture

$$M_{\text{sector dipole}} = \begin{pmatrix} \cos\theta & \rho\sin\theta & 0 & 0 & 0 & \rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & 0 & 0 & 0 & \sin\theta \\ 0 & 0 & 1 & \rho\theta & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\sin\theta & -\rho(1-\cos\theta) & 0 & 0 & 1 & -\rho(\theta-\sin\theta) \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- "Weak" dipole: $\theta \ll 1$ with $L=\rho\theta$ constant

$$M_{\text{thin sector dipole}} = \begin{pmatrix} 1 & L & 0 & 0 & 0 & \theta L/2 \\ 0 & 1 & 0 & 0 & 0 & \theta \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\theta & -\theta L/2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

We will be simplifying by only looking at (x,x',δ):

$$M_{\text{thin sector dipole}} = \begin{pmatrix} 1 & L & \theta L/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}$$

Dispersion

- Add explicit momentum dependence to equation of motion

$$x'' + K(s)x = \frac{\delta}{\rho(s)}$$

Perturb our zero-dispersion solution (2nd order ODE) to find

$$x(s) = C(s)x_0 + S(s)x_0' + D(s)\delta_0$$
$$x'(s) = C'(s)x_0 + S'(s)x_0' + D'(s)\delta_0$$

Here D(s) is a **matrix element**
η(s) is the **dispersion function**
Reality is nonlinear: ... δ_0^2, δ_0^3 ...

$$\begin{pmatrix} x(s) \\ x'(s) \\ \delta(s) \end{pmatrix} = \begin{pmatrix} C(s) & S(s) & D(s) \\ C'(s) & S'(s) & D'(s) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_0' \\ \delta_0 \end{pmatrix}$$

The trajectory has two parts:

$$\boxed{x(s) = \text{betatron} + \eta_x(s)\delta \qquad \eta_x(s) \equiv \frac{dx}{d\delta}}$$

Periodic Dispersion

- Substituting and noting dispersion is periodic, $\eta_x(s+C)=\eta_x(s)$

$$\begin{pmatrix}\eta_x(s)\\ \eta_x'(s)\\ \delta(s)\end{pmatrix}=\begin{pmatrix}C(s) & S(s) & D(s)\\ C'(s) & S'(s) & D'(s)\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\eta_x(s)\\ \eta_x'(s)\\ \delta_0\end{pmatrix} \qquad \text{achromat}: D=D'=0$$

- If we take $\delta_0=1$ we can solve this in a clever way

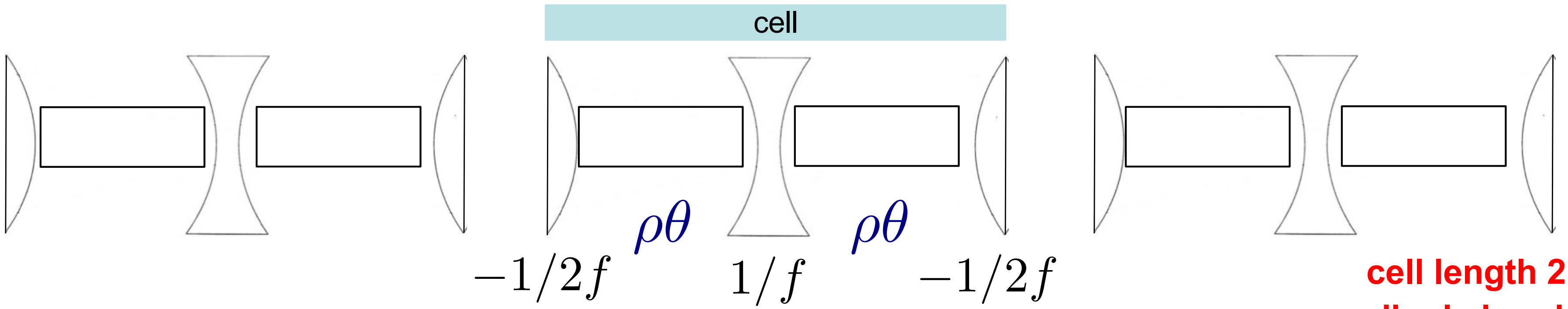
$$\begin{pmatrix}\eta_x(s)\\ \eta_x'(s)\end{pmatrix}=\begin{pmatrix}C(s) & S(s)\\ C'(s) & S'(S)\end{pmatrix}\begin{pmatrix}\eta_x(s)\\ \eta_x'(s)\end{pmatrix}+\begin{pmatrix}D(s)\\ D'(s)\end{pmatrix}=M\begin{pmatrix}\eta_x(s)\\ \eta_x'(s)\end{pmatrix}+\begin{pmatrix}D(s)\\ D'(s)\end{pmatrix}$$

$$(I-M)\begin{pmatrix}\eta_x(s)\\ \eta_x'(s)\end{pmatrix}=\begin{pmatrix}D(s)\\ D'(s)\end{pmatrix} \quad \Rightarrow \quad \boxed{\begin{pmatrix}\eta_x(s)\\ \eta_x'(s)\end{pmatrix}=(I-M)^{-1}\begin{pmatrix}D(s)\\ D'(s)\end{pmatrix}}$$

- Solving gives

$$\boxed{\begin{aligned}\eta(s)&=\frac{[1-S'(s)]D(s)+S(s)D'(s)}{2(1-\cos\mu)}\\ \eta'(s)&=\frac{[1-C(s)]D'(s)+C'(s)D(s)}{2(1-\cos\mu)}\end{aligned}}$$

FODO with dipoles



cell length 2L
dipole bend angle θ

- A periodic lattice **without** dipoles has no **intrinsic** dispersion
- Consider FODO with long dipoles and thin quadrupoles
  - Each dipole has total length $\rho\theta$ so each dipole is of length $L = \rho\theta$
  - Assume a large accelerator with many FODO cells so $\theta \ll 1$

$$M_{-2f} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad M_{\text{thin sector dipole}} = \begin{pmatrix} 1 & L & \theta L/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \quad M_f = \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M_{\text{FODO}} = M_{-2f} M_{\text{dipole}} M_f M_{\text{dipole}} M_{-2f}$$

$$M_{\text{FODO}} = \begin{pmatrix} 1 - \frac{L^2}{2f^2} & L\left(2 + \frac{L}{f}\right) & L\left(2 + \frac{L}{2f}\right)\theta \\ -\frac{L}{2f^2}\left(1 - \frac{L}{2f}\right) & 1 - \frac{L^2}{f^2} & \left(2 - \frac{L}{2f} - \frac{L^2}{4f^2}\right)\theta \\ 0 & 0 & 1 \end{pmatrix}$$

FODO with dipoles

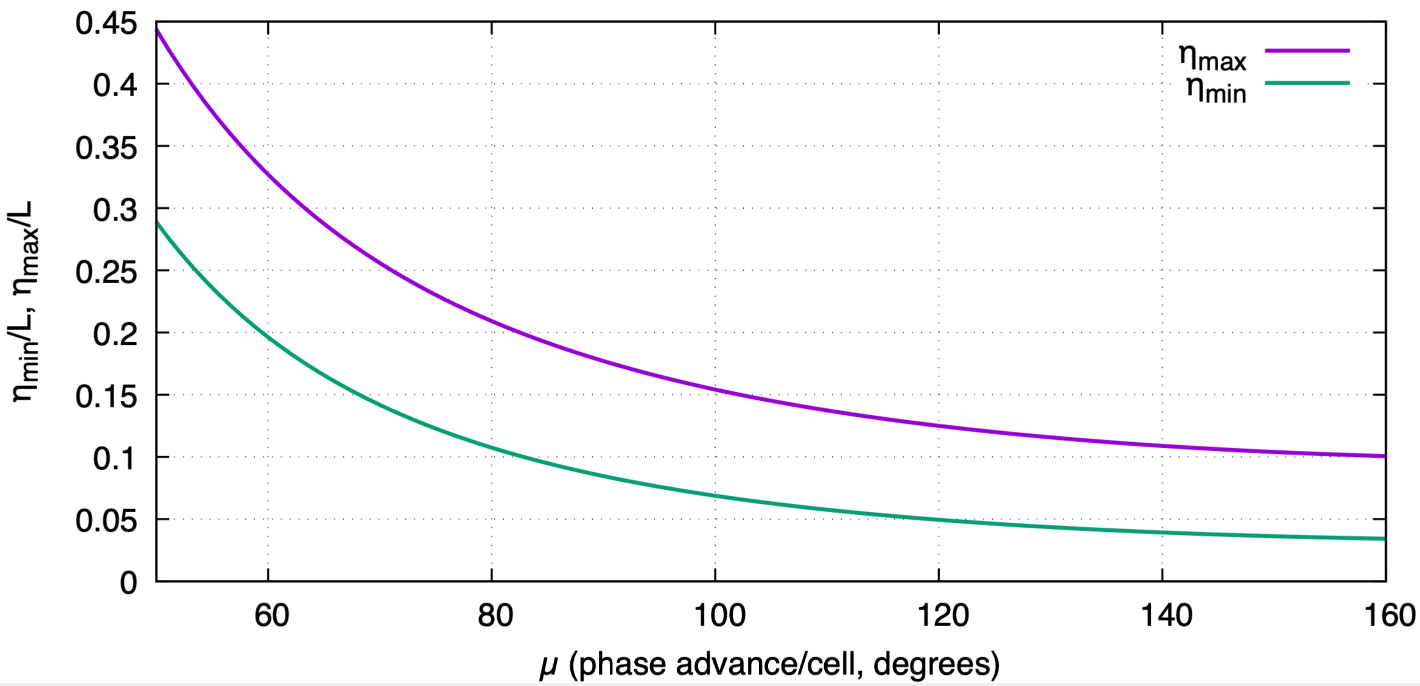
- Like $\hat{\beta}$ before, this choice of periodicity gives us $\hat{\eta}_x$

$$\hat{\eta_x} = 2L\theta \left[\frac{1 + \frac{1}{2}\sin\frac{\mu}{2}}{\sin^2\frac{\mu}{2}} \right] \qquad \eta_x' = 0 \text{ at max}$$

cell length 2L

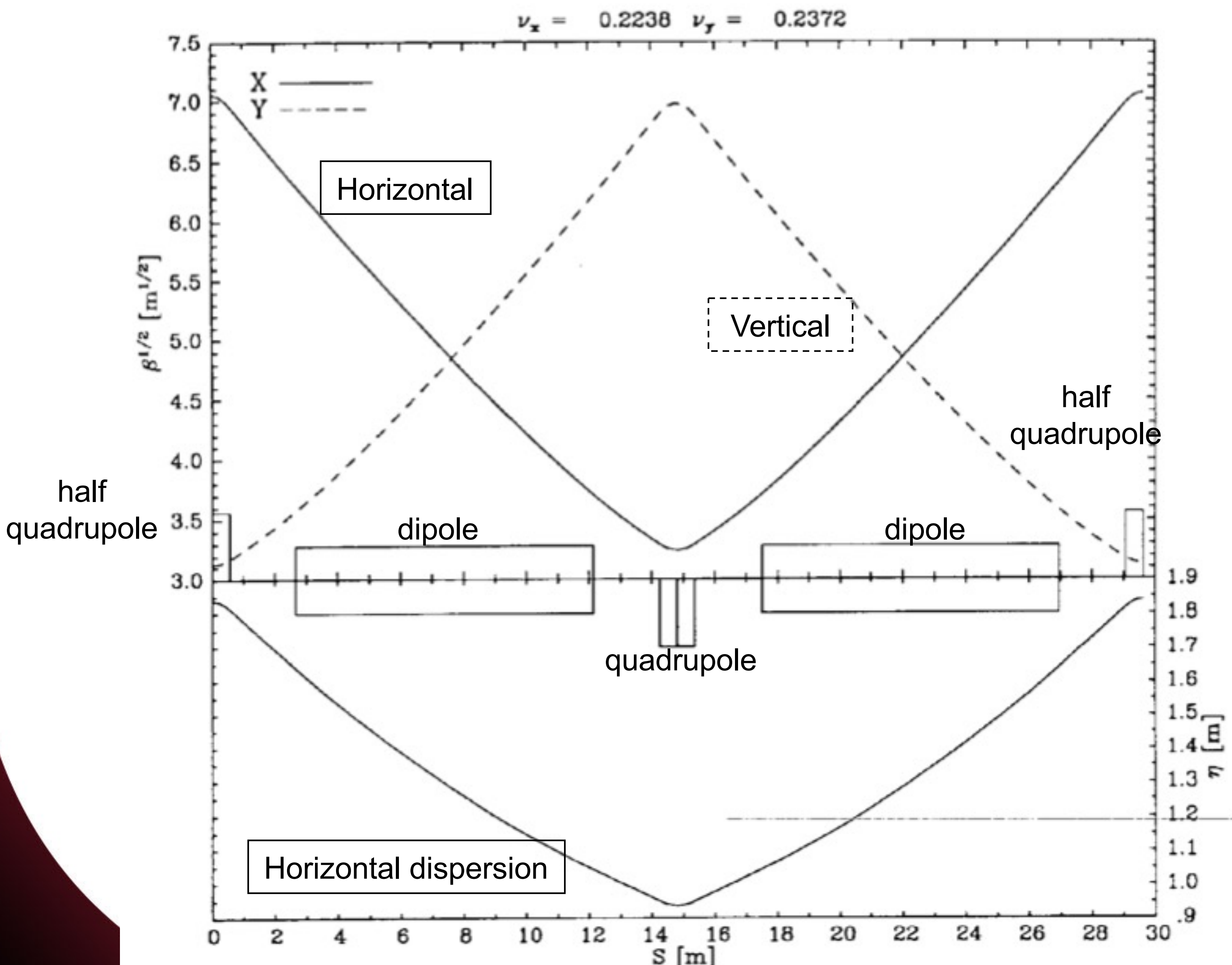
- Changing periodicity to defocusing quad centers gives $\check{\eta}_x$

$$\check{\eta_x} = 2L\theta \left[\frac{1 - \frac{1}{2}\sin\frac{\mu}{2}}{\sin^2\frac{\mu}{2}} \right] \qquad \eta_x' = 0 \text{ at min}$$



For θ=33 mrad

RHIC FODO Cell



192 dipoles/ring
θ=32.7 mrad
L=9.8m

87 deg/cell
$\sin(\mu/2)\sim 0.688$

Calculated
η_{max}=1.8m
η_{min}=0.9m

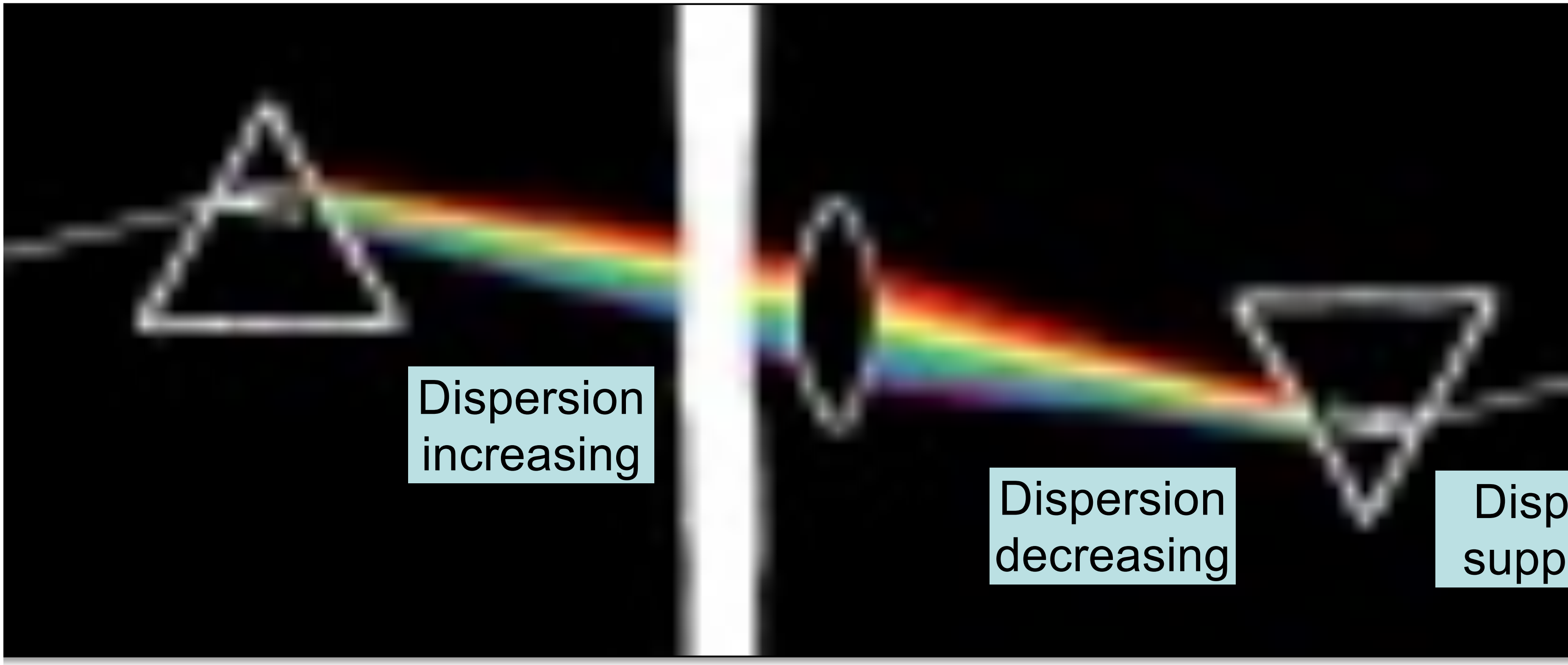
Dispersion suppressors

- The FODO dispersion solution is non-zero everywhere
  - But in straight sections we often want $\eta_x = \eta'_x = 0$
    - e.g. to keep beam small in wigglers/undulators in a light source
    - e.g. to keep longitudinal/transverse separate in straight sections
  - We can "match" between these two conditions with with a **dispersion suppressor**, a **non-periodic** set of magnets that transforms FODO (η_x, η'_x) to zero.

- Remember that dispersion is momentum-dependent transverse position

$$x(s) = \text{betatron} + \eta_x(s)\delta \qquad \eta_x(s) \equiv \frac{dx}{d\delta}$$

(xkcd interlude revisited)

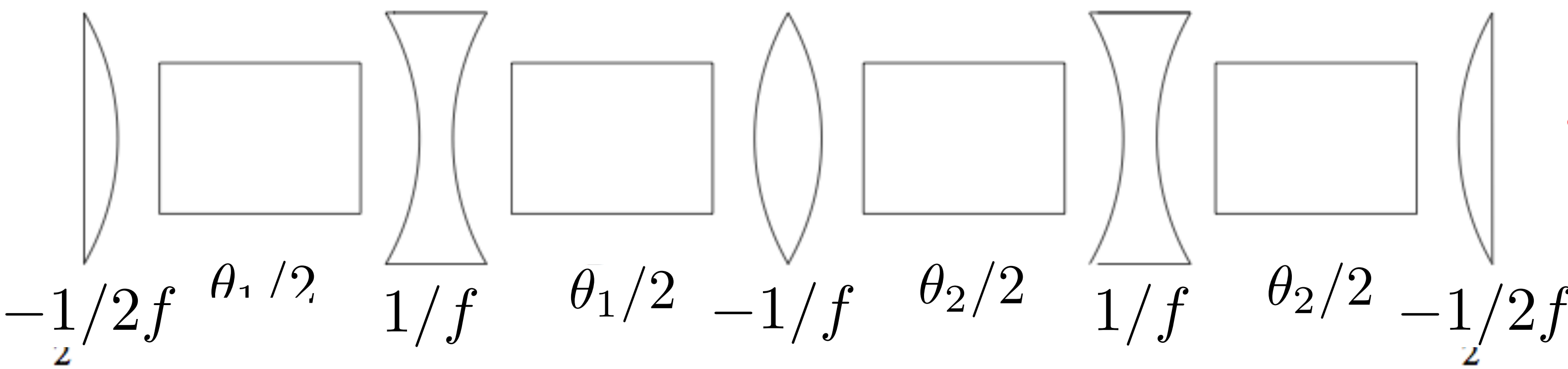


A "double bend" achromat is the simplest example of a dispersion suppressor: 1 quad and 1 dipole!

You are almost *never* this lucky, e.g. FODO dispersion

Dispersion suppressors

- The FODO dispersion solution is non-zero everywhere
  - But in straight sections we often want $\eta_x = \eta'_x = 0$
    - e.g. to keep beam small in wigglers/undulators in a light source
  - We can "match" between these two conditions with with a **dispersion suppressor**, a **non-periodic** set of magnets that transforms FODO (η_x, η'_x) to zero.



be careful about definitions

- Consider two FODO cells with different total bend angles θ_1, θ_2
  - Same quadrupole focusing to not disturb $\beta_x,\ \mu_x$ much
  - We want this to match $(\eta_x, \eta'_x) = (\hat{\eta}_x, 0)$ to $(\eta_x, \eta'_x) = (0, 0)$
  - $\alpha_x = 0$ at ends to simplify periodic matrix

Dispersion suppressors

Zero dispersion area

slope η' =0

$$\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos 2\mu_x & \beta_x \sin 2\mu_x & D(s) \\ -\frac{\sin 2\mu_x}{\beta_x} & \cos 2\mu_x & D'(s) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{\eta}_x \\ 0 \\ 1 \end{pmatrix}$$

FODO peak dispersion,

slope η' =0

multiply matrices $\Rightarrow$

$$D(s) = \frac{L}{2}\left(1 + \frac{L}{8f}\right)\left[\left(3 - \frac{L^2}{4f^2}\right)\theta_1 + \theta_2\right]$$

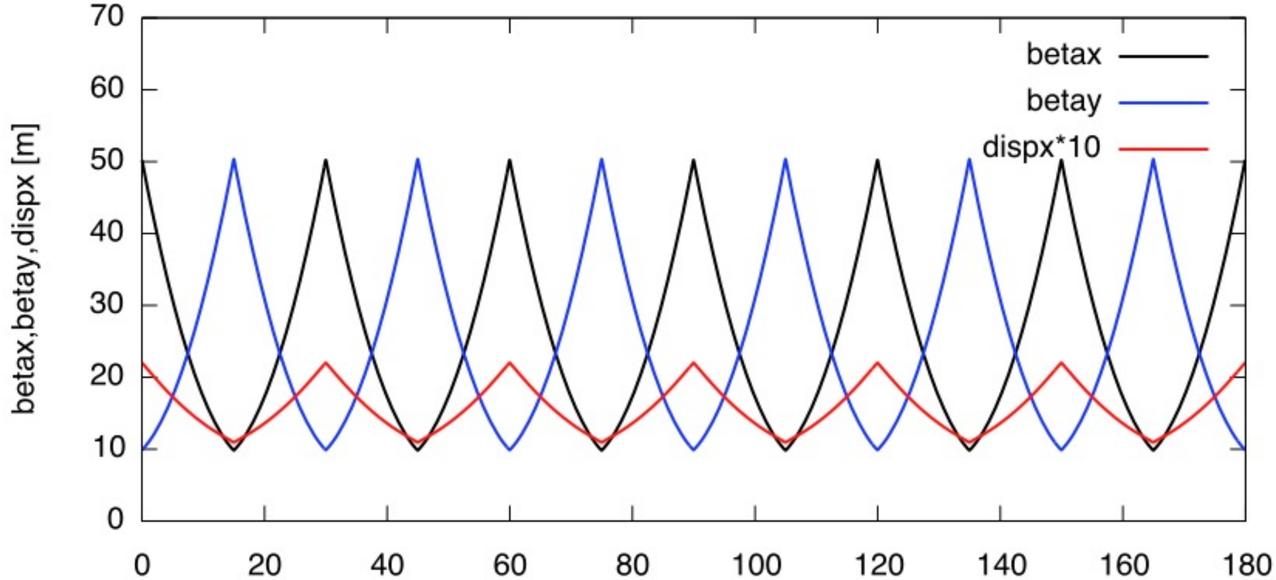
$$D'(s) = \left(1 - \frac{L}{8f} - \frac{L^2}{32f^2}\right)\left[\left(1 - \frac{L^2}{4f^2}\right)\theta_1 + \theta_2\right]$$

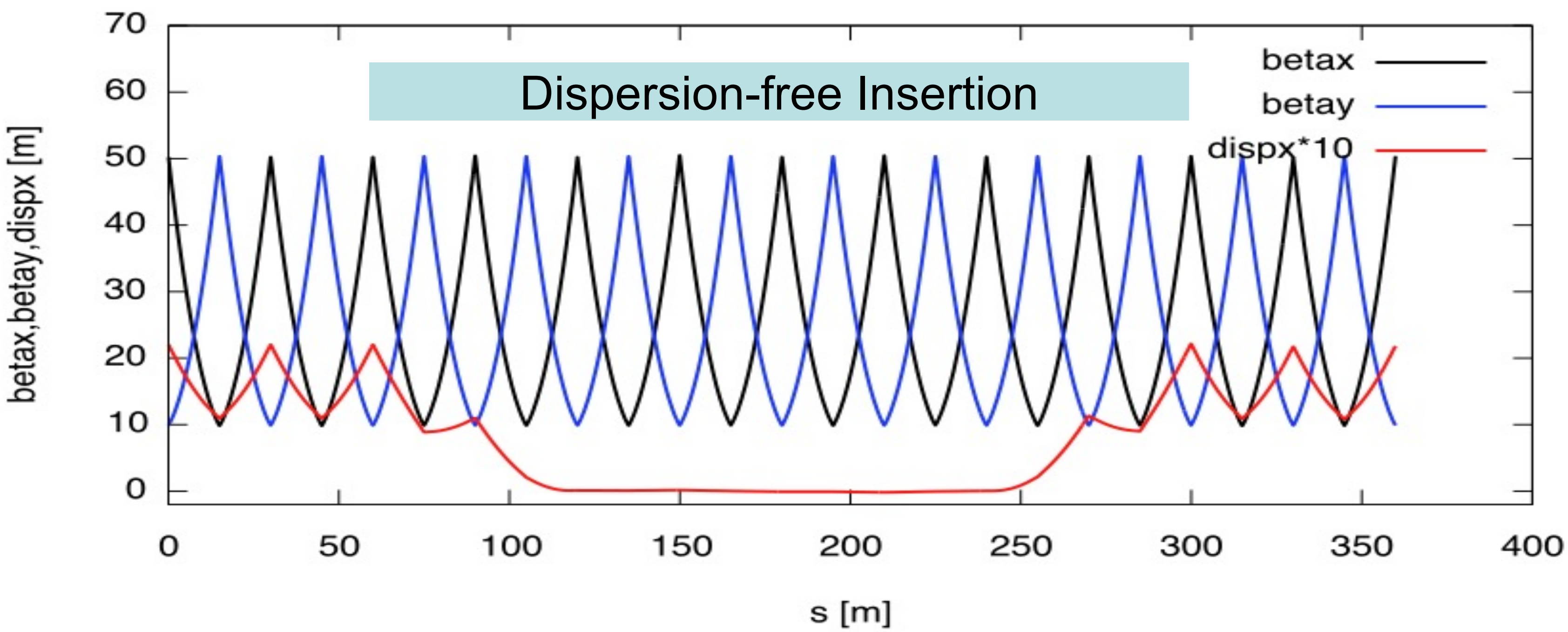
$$\hat{\eta}_x = \frac{4f^2}{L}\left(1 + \frac{L}{8f}\right)(\theta_1 + \theta_2)$$

$$\boxed{\theta_1 = \left(1 - \frac{1}{4\sin^2\frac{\mu}{2}}\right)\theta \qquad \theta_2 = \left(\frac{1}{4\sin^2\frac{\mu}{2}}\right)\theta}$$

$\boxed{\theta = \theta_1 + \theta_2}$ two cells, one FODO bend angle $\rightarrow$ reduced bending

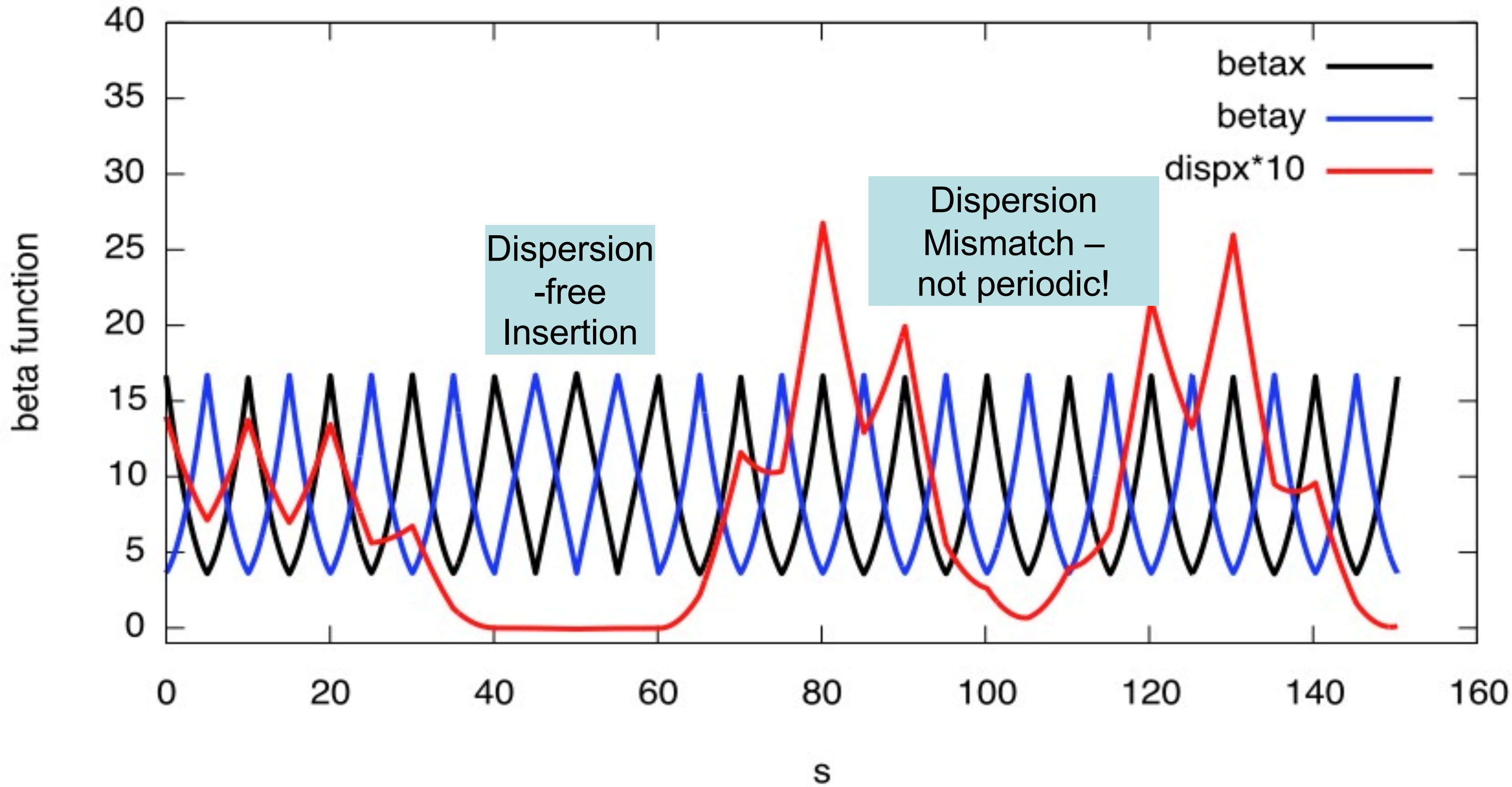
FODO Cell Dispersion and Suppressor



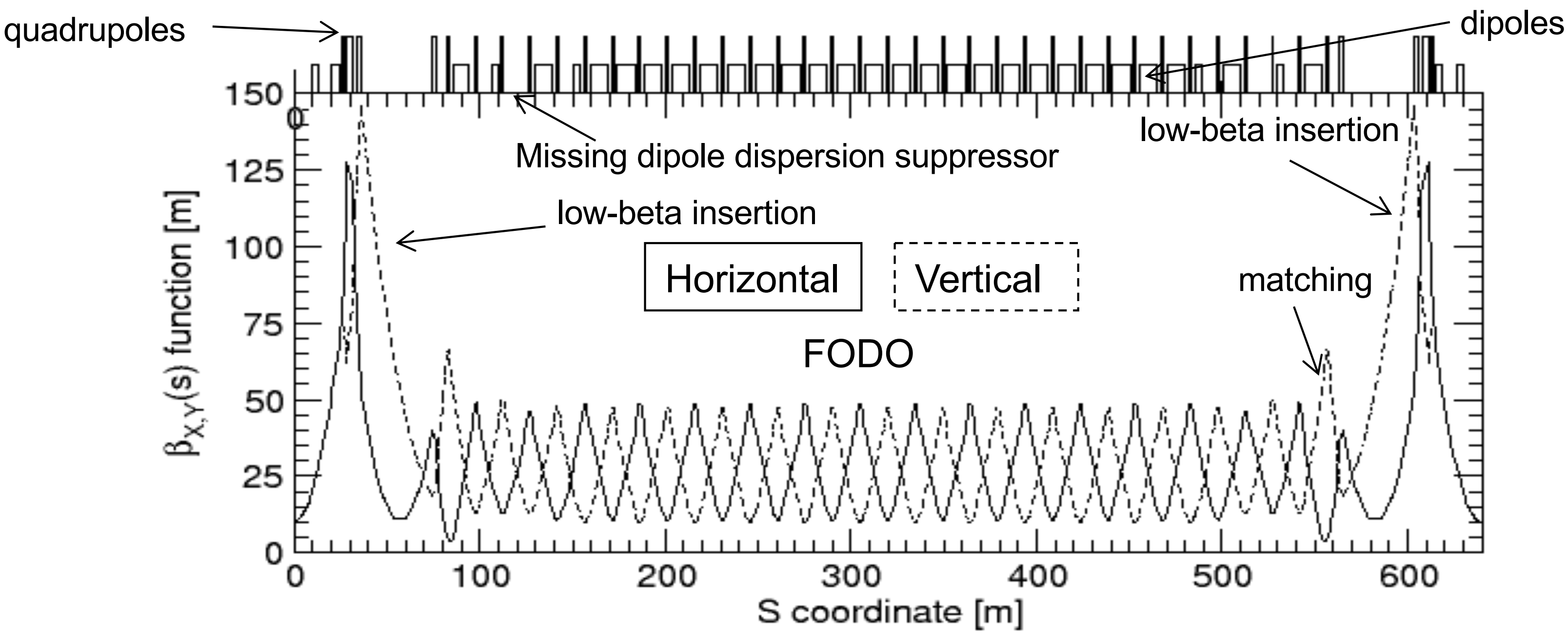


Mismatched Dispersion

- What does mismatched dispersion look like?
  - For example, this is what happens when the second dispersion suppressor is eliminated and the dipole-free FODO cells run right up against the FODO cells with dipoles



RHIC Lattice Revisited



- Note modular design, including low-beta insertions
  - Used for experimental collisions
  - Minimum beam size σ (with zero dispersion)
    - **maximize luminosity**
  - Large σ, beam size in "low beta quadrupoles"
  - Other facilities also have **longitudinal bunch compressors** (this afternoon)