Lecture 11: Synchrotron Radiation – Classical Damping

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"At GE [in 1947], Pollack … assemble[d] … a 70-MeV electron synchrotron to test the idea [of the phase-stability principle]. Fortunately … the doughnut-shaped electron tube was transparent, which allowed a technician to look around the shielding with a large mirror to check for sparking in the tube. Instead, he saw a bright arc of light, which the GE group quickly realised was actually coming from the electron beam."

A.L.Robinson, ".. History of Synchrotron Radiation".

A) Spectrum & distribution pattern

B) Energy loss, longitudinal damping

C) Continuous acceleration

D) Transverse damping, partition nos.

- LARMOR (19th century) knew how charged particles radiate
- POLLOCK 1947 first observation
- ELECTRONS

  0) Radiate copiously : protons "not at all"

  1) H, V, S motion is damped + stabilised CLASSICAL

  2) H, S motion is driven QUANTUM ($\hbar \neq 0$)

  3) Dynamical equilibrium delivers natural emittances

BUT for THIS lecture $\boxed{\hbar = 0}$

Table 11.1 Partial chronological list (2016)

typically

$E = 3$ GeV

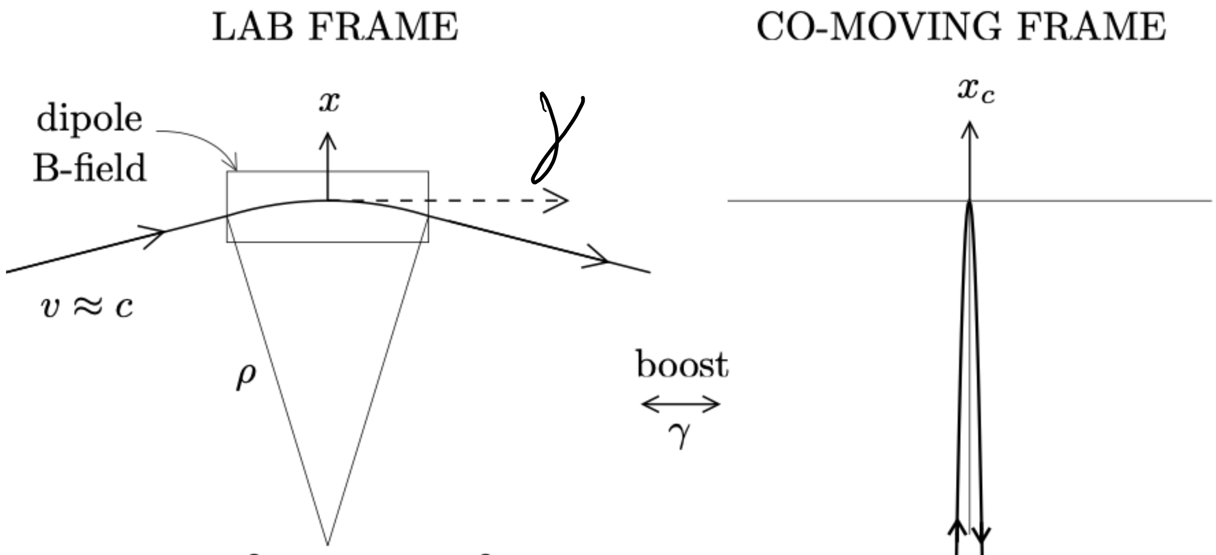
$m_e = 0.511$ MeV

$\gamma = 6{,}000$

Operating years	Name	Location	Energy [GeV]
1961-	SURF	Gaithersburg	0.18
1968-87	Tantalus	Madison	0.24
1972-75	Solidi Roma	Frascati	1.0
1973-88	ACO	Orsay	0.54
1973-	SSRL	Stanford	3.0
1974-93	DORIS	Hamburg	5.0
1974-	INS-SOR	Tokyo	0.3
1979-	CHESS	Ithaca	5.5
1981-2006	DCI	Orsay	1.0
1997-	HSRC	Hiroshima	0.7
1982-2014	NSLS-I	Upton	2.8
1982-	Photon Factory	Tsukuba	2.5
1986-	MAX-I	Lund	0.55
1987-2006	Super-ACO	Orsay	0.8
1991-	BSRF	Beijing	2.5
1991-	NSRL	Hefei	0.8
1992-	ESRF	Grenoble	6.0
1993-	ALS	Berkeley	1.9
1993-2012	DORIS III	Hamburg	5.0
1993-	ELETTRA	Trieste	2.4
1995-	APS	Lemont	1.9
1997-	HSRC	Hiroshima	0.7
1997-	LNLS	Campinas	1.4
1997-	Spring-8	Sayo	8.0
1998-	BESSY II	Berlin	1.7
1999-	Indus 2	Indore	2.5
1999-	SIBIR	Moscow	2.5
2000-	ANKA	Karlsruhe	2.5
2001-	SLS	Villigen	2.8
2004-	CLS	Saskatoon	2.9
2004-	SLRI	Suranari	1.2
2006-	Australian Synch.	Clayton	3.0
2006-	Diamond	Abingdon	3.0
2006-	SOLEIL	Orsay	3.0
2007-	SSRF	Shanghai	3.5
2009-	PETRA III	Hamburg	6.5
2010-	ALBA	Barcelona	3.0
2015-	NSLS-II	Upton	3.0
2015-	Taiwan PS	Hsinchu	3.0
2016-	MAX-IV	Lund	3.0

A) Spectrum & distribution pattern

11.1 Larmor radiation in lab & co-moving frames



$$a \;=\; \frac{d^2x}{dt^2} \;=\; -\frac{(\beta c)^2}{\rho} \qquad\qquad a_c \;=\; \gamma^2\, a$$

Larmor total $P = \frac{1}{6\pi\epsilon_0} \cdot \frac{q^2 a_c^2}{c^3}$

boosted, becomes $\boxed{P = \frac{1}{6\pi\epsilon_0} \cdot \frac{q^2 c}{\rho^2} \cdot \gamma^4 \beta^4}$

more conveniently

$$\boxed{P = \frac{1}{6\pi\epsilon_0} \cdot \frac{e^4}{m^4 c^5} \cdot B^2 E^2}$$

Energy

dipole field

Protons don't radiate: $\left(\frac{m_p}{m_e}\right)^4 \simeq 10^{13}$

Q: WHY are muon colliders attractive?

A1: electrons & muons are point-like

A2: $m_\mu = 105.7$ MeV $(m_\mu / m_e) \simeq 200$

BUT muon decay time is $2.2\ \mu s$ $\times \gamma_\mu$!!

UNIVERSAL SPECTRUM SHAPE ($\omega = 2\pi f$)

$$\boxed{P(\omega) = \frac{P}{\omega_c} S\left(\frac{\omega}{\omega_c}\right)}$$

where $S(\xi) = \frac{9\sqrt{3}}{8\pi} \cdot \xi \int_{\xi}^{\infty} K_{5/3}(\bar{\xi})\, d\bar{\xi}$

Modified Bessel fn.

CHARACTERISTIC FREQUENCY

$$\boxed{\omega_c = \frac{3}{2} \frac{c\gamma^3}{\rho}}$$

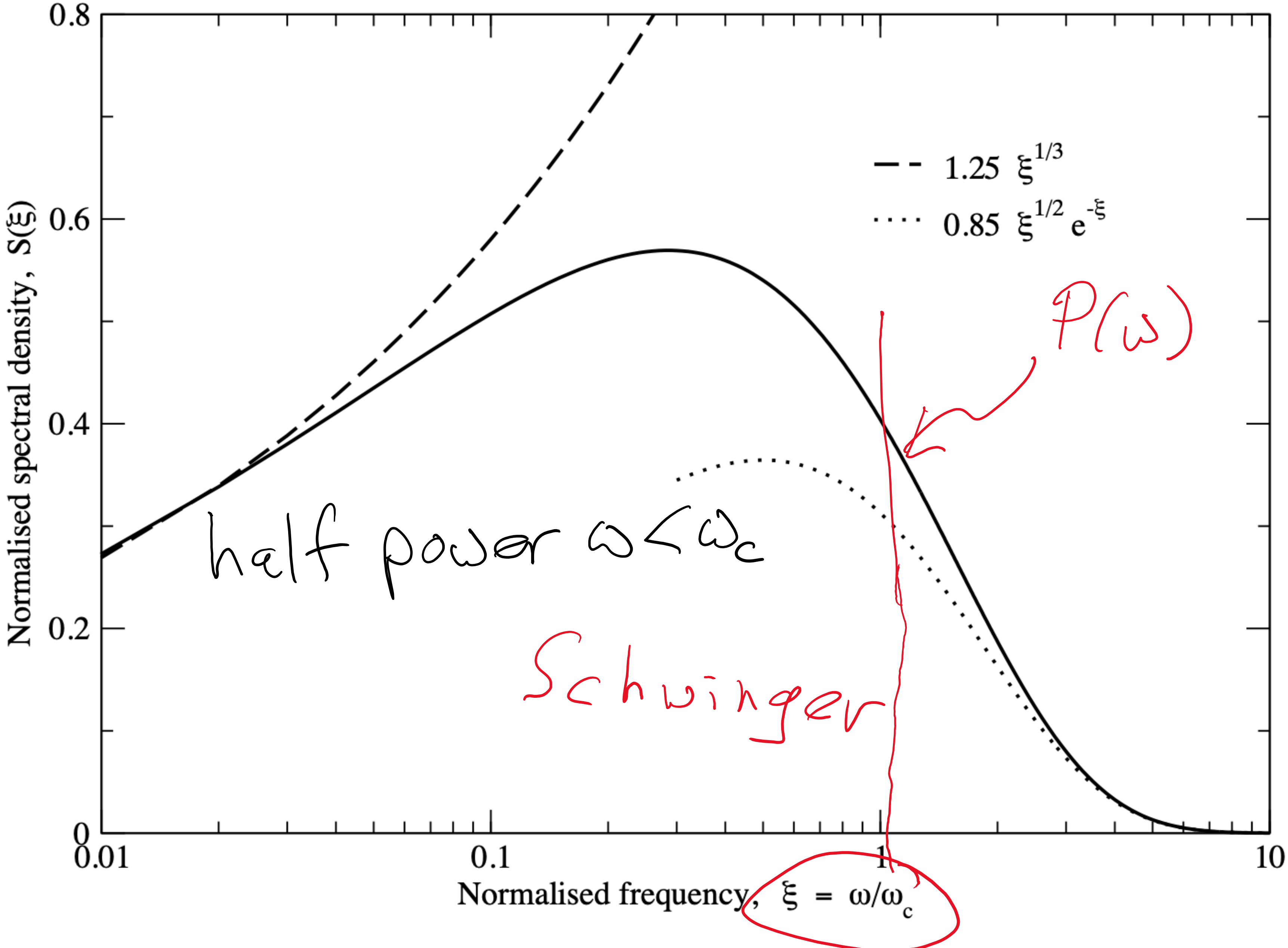
Typically $E = 3$ GeV, $B = 0.5$ T

WAVELENGTH

$$\boxed{\lambda_c[m] = \frac{1.86 \times 10^{-9}}{B[T]\, E^2[GeV^2]}}$$

$\lambda_c \simeq 4 \times 10^{-10}$ m

11.2 Universal shape of the SR spectrum



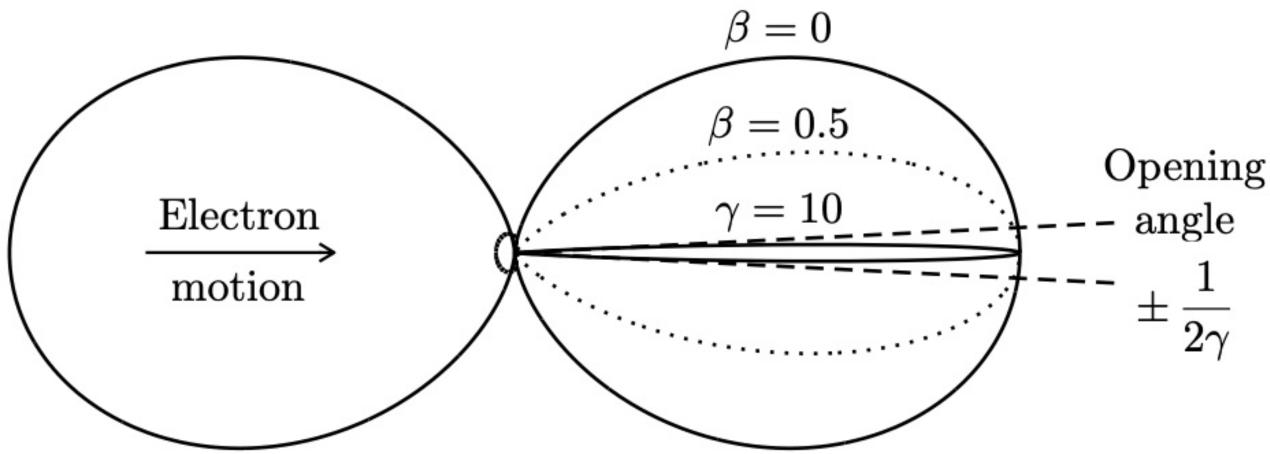
ANGULAR DISTRIBUTION in synchrotron plane

$$\frac{dP}{d\Omega} = \frac{P_0}{(1-\beta\cos(\theta))^3}\cdot\left[1 - \frac{\sin^2(\theta)}{\gamma^2(1-\beta\cos(\theta))^2}\right]$$

$$\gamma \gg 1 \qquad \boxed{\frac{dP}{d\Omega} \simeq \frac{P_1}{(1+\gamma^2\theta^2)^3}\cdot\left[1 - \frac{4\gamma^2\theta^2}{(1+\gamma^2\theta^2)^2}\right]}$$

HEADLIGHT EFFECT

11.3 Radiation pattern for 3 values of β & γ



$\boxed{\beta \ll 1}$ Classical 2 lobe dipole pattern

$\boxed{\gamma \gg 1}$ EG 6,000, opening angle

$\pm \frac{1}{2\gamma} \simeq \pm 0.1$ mrad

B) Energy loss, longitudinal damping

ENERGY LOSS PER TURN

1 particle, 1 turn $\quad U = \oint P(s) \cdot \frac{ds}{c}$

gives IDEALLY $\boxed{U_0 = C_g E_0^4 \cdot \frac{c}{2\pi} \langle G^2 \rangle}$

where $\langle\,\rangle$ angle brackets denote a ring average

and $G = \frac{1}{\rho}$ is local bending strength

and $\boxed{C_g = \frac{4\pi}{3} \frac{r_0}{(mc^2)^3}}$ ← classical radius

$= 8.85 \times 10^{-5}\ [\mathrm{m.GeV^{-3}}]$ ELECTRONS

$= 7.78 \times 10^{-18}\ [\mathrm{m.GeV^{-3}}]$ PROTONS

MUONS??

EG: ISOMAGNETIC RING

- All dipoles have same bending radius ρ

$$\langle G^n \rangle = \frac{\oint G^n \, ds}{\oint ds}$$

$$\boxed{\langle G^n \rangle_{\text{ISO}} = \frac{1}{R \rho^{n-1}}} \quad \text{where } 2\pi R = C$$

and $$U_0 = \frac{C_\gamma E_0^4}{\rho}$$

ELECTRONS $$\boxed{U_0 [\text{keV}] = 88.5 \frac{E_0^4 [\text{GeV}^4]}{\rho [\text{m}]}}$$ ISO!!

EG1: $B = 0.4\,\text{T}$, $E = 3\,\text{GeV} \Rightarrow U_0 = 0.29\,\text{MeV}$ NSLS-II

EG2: 50 TeV protons, $\rho = 14\,\text{km} \Rightarrow U_0 = 3.5\,\text{MeV}$ FCC/VLHC

LONGITUDINAL DAMPING

Previously SLIP FACTOR $\eta_s = \frac{1}{\gamma_T^2} - \frac{1}{\gamma^2}$

NOW use "COMPACTION FACTOR" (Electron speak)

$$\boxed{\alpha = \langle \eta G \rangle} = \frac{1}{\gamma_T^2}$$

So longitudinal displacement z evolves like

$$\boxed{z_{n+1} = z_n - \alpha C \cdot \delta_n} \qquad \text{(A)}$$

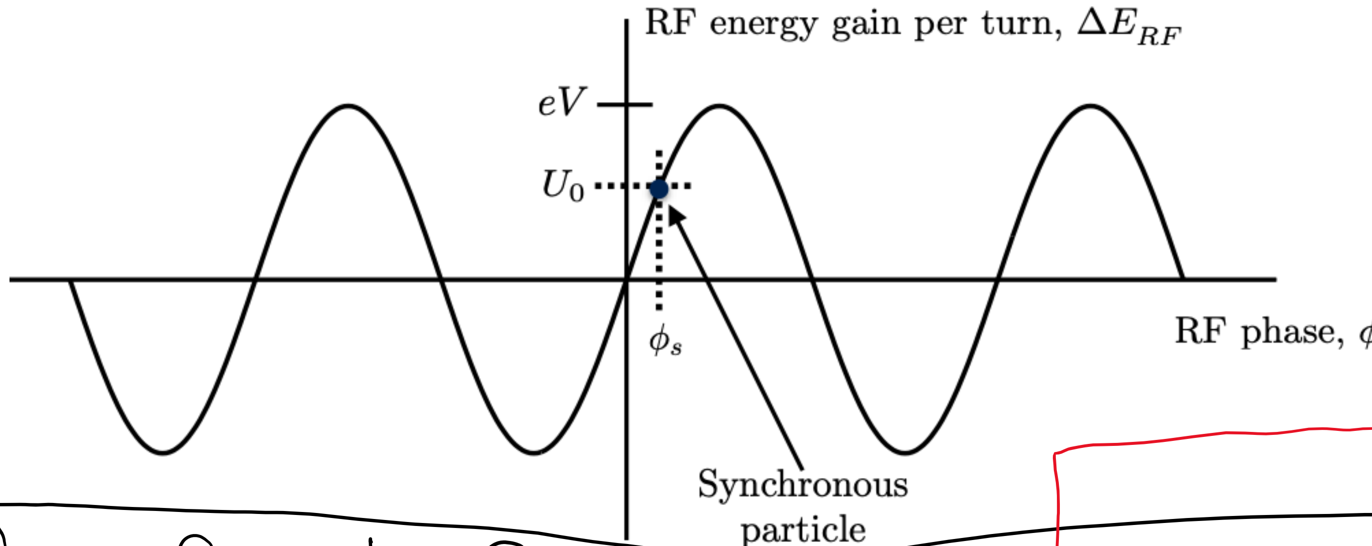
On every turn must replenish (on average) energy loss

TOTAL RF VOLTAGE

$$\boxed{U_0 = eV \cdot \sin(\phi_s)}$$

SYNCHRONOUS PHASE

11.4 Energy gain from RF for an electron of phase φ



(B) $$\boxed{\delta_{n+1} = \delta_n + \frac{1}{E_0}\left[eV\left(\sin(\phi_n) - \sin(\phi_s)\right) - \left(U(\delta_n) - U_0\right)\right]}$$

$$\left(\gamma \gg 1 \Rightarrow \delta = \frac{\Delta p}{p} \approx \frac{\Delta E}{E_0}\right)$$

VARIATION with δ_n is VITAL!!

<u>IF</u> longitudinal oscillations are slow (synchrotron tune Q_s is small)

<u>THEN</u> combine (A) & (B) to give

$$\boxed{\frac{d^2\delta}{dt^2} + \frac{2}{\tau_s}\frac{d\delta}{dt} + \left(\frac{2\pi Q_s}{T}\right)^2 \delta = 0}$$

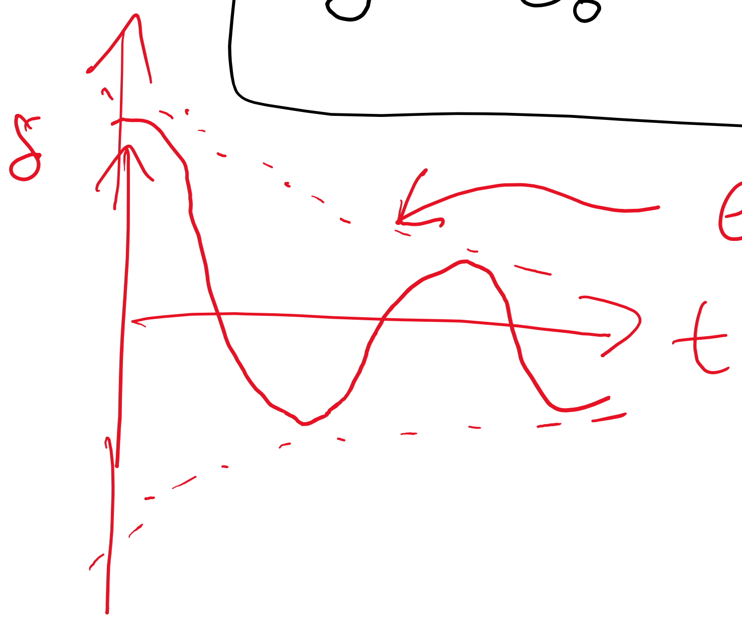
where t is time (seconds)

$T = \frac{C}{c}$ is circulation time

Q: Solution? Damped harmonic oscillator!!

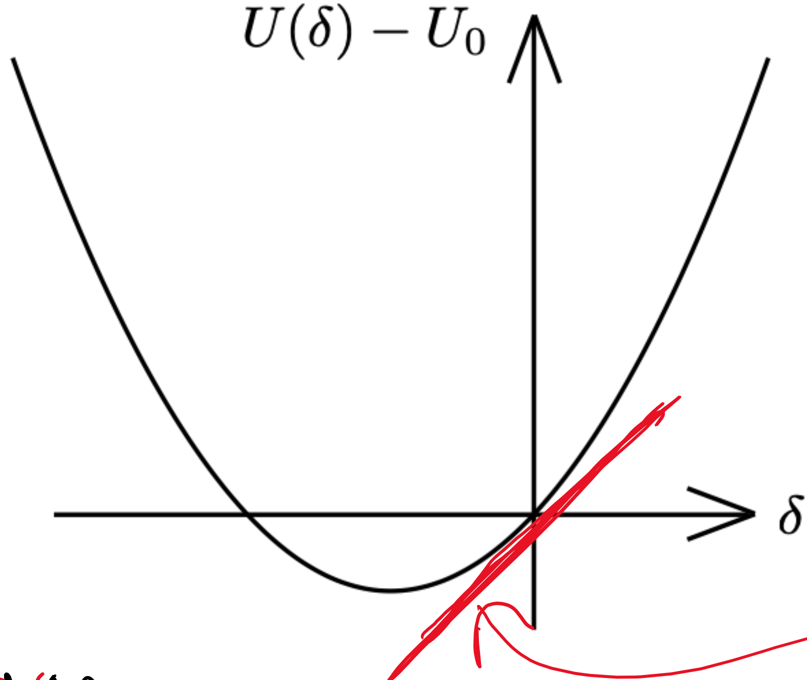
DAMPED MOTION SOLUTION

$$\boxed{\delta = \delta_0 \, e^{-t/\tau_s} \cos\left(2\pi Q_s \cdot \frac{t}{T}\right)}$$



WITH $$\boxed{\tau_s = \frac{2T}{\left.\frac{dU}{dE}\right|_0}}$$

11.5 Energy loss per turn
U vs momentum offset δ



$$\boxed{\tau_s = \frac{2T}{\left.\frac{dU}{dE}\right|_0}}$$

VARIATION?

$$U(\delta) \sim \oint P(\delta)\, ds \sim \oint B^2 E^2\, ds$$

where

$$B(\delta) \sim G + K(\eta\delta + x_{co})$$

$$E(\delta) \sim E_0(1+\delta)$$

from errors

So $\left.\frac{dU}{dE}\right|_0$ is USUALLY positive $\Rightarrow$ motion is damped. BUT...

C) Continuous acceleration

TIME OUT — Briefly return to CONSERVATIVE motion

$V(\delta=0)$ NO radiation, = (PROTONS?)

$\phi_s \neq 0$ ACCELERATION !!

Hamiltonian

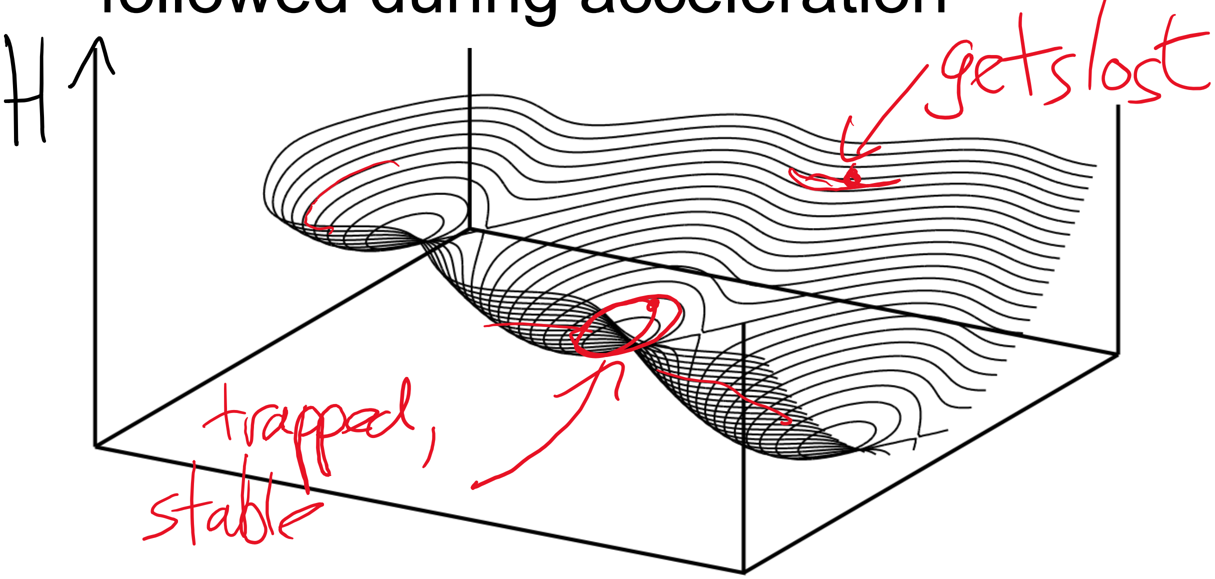
$$\boxed{H(\phi,\delta) = \frac{1}{2}\alpha\,\omega_{RF}\,\delta^2 - \frac{eV}{TE_0}\left(\cos(\phi) + \phi\sin(\phi_s)\right)}$$

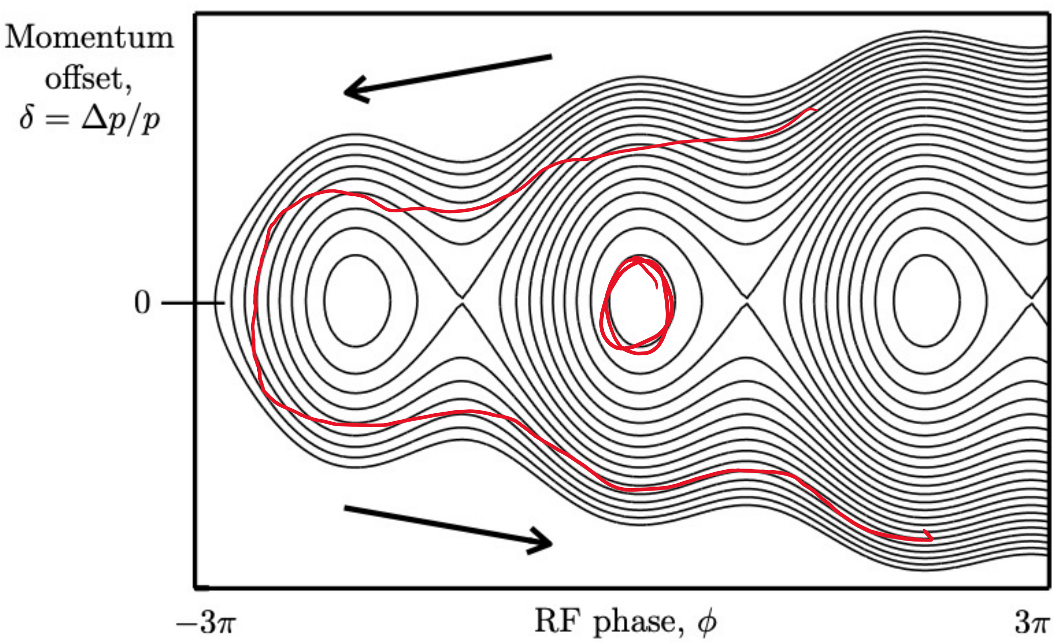
reproduces (A) & (B) (with $U=0$) through canonical equations

$$\frac{d\delta}{dt} = \frac{\partial H}{\partial \phi}, \qquad \frac{d\phi}{dt} = -\frac{\partial H}{\partial \delta}$$

NOW we can visualize the motion ...

11.6 Hamiltonian CONTOURS followed during acceleration

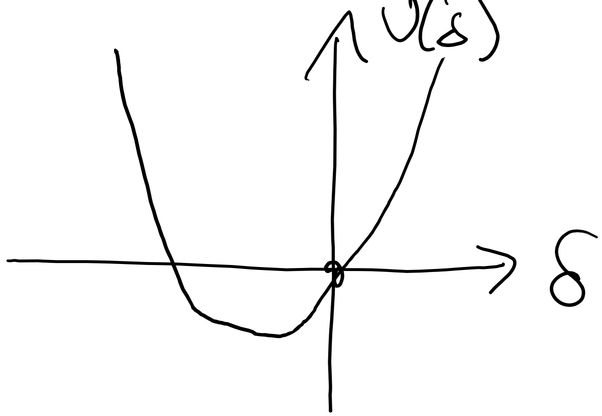




... by following the contours

- Some particles are trapped/stored in a lake/"RF bucket"
- Others get lost
- Speed is proportional to steepness

TIME IN - turn radiation back on



3 CONTROL VARIABLES

ϕ_s SYNCHRONOUS PHASE

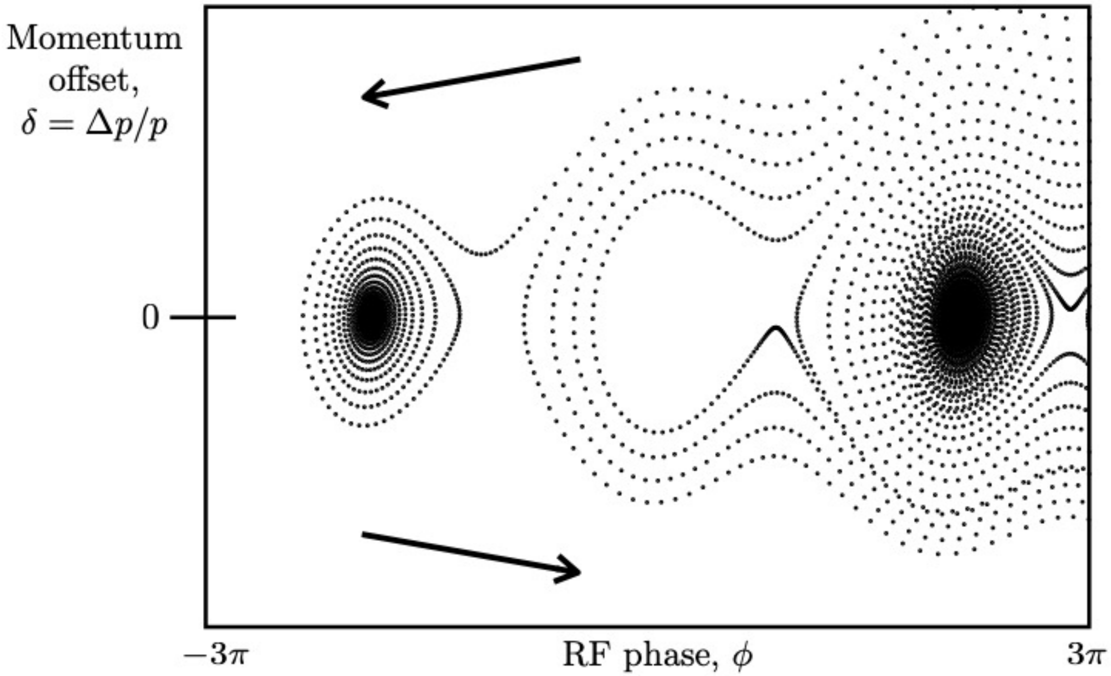
Q_s SYNCH. TUNE

τ_s DAMPING TIME

describe the motion

- Particles no longer follow contours...

11.7 Damped MOTION under acceleration & radiation



$\phi_s = 0.4$

$Q_s = 0.02$

$T_s = 300$ turns

- some particles are lost to an aperture [$\delta < 0$]
- others are trapped in a bucket
- NONE escape, once trapped

$\Rightarrow$ THINK STRANGE ATTRACTORS!!

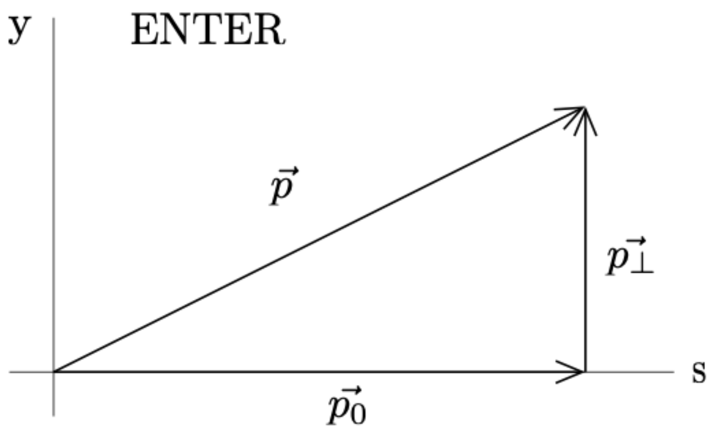
D) Transverse damping, partition numbers

VERTICAL DAMPING

V is easier to understand than H — η_V is zero!!

- Consider an electron losing ΔU energy in a single dipole ...
- ... and (in effect) recovering ΔU from RF system "in that dipole"

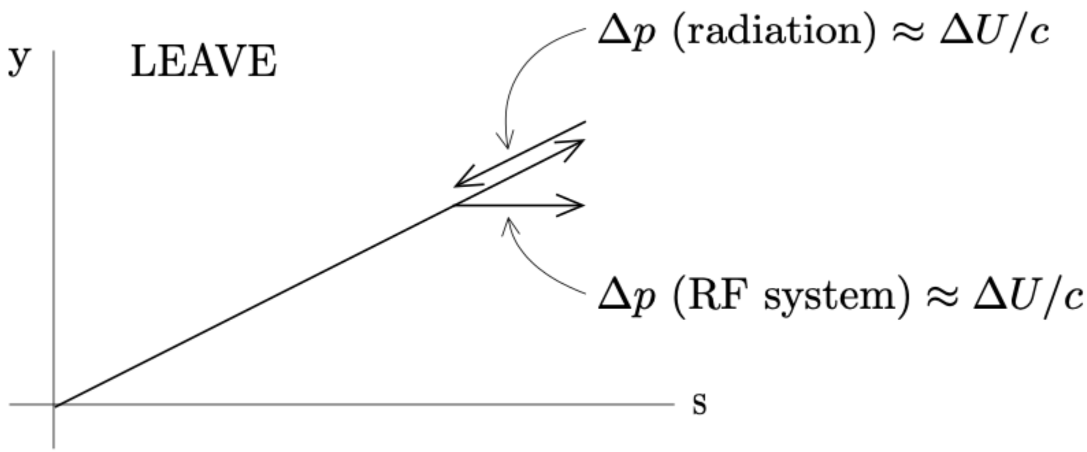
11.8 Vertical damping through a single dipole



$$y' = \frac{p_\perp}{p_0}$$

$$y'_{NEW} = \frac{\left(1 - \frac{\Delta U}{E_0}\right) p_\perp}{p_0}$$

$$= \left(1 - \frac{\Delta U}{E_0}\right) y'_{OLD}$$



VERTICAL BETATRON OSCILLATIONS DAMP

$$\boxed{y = a_y \cdot e^{-\frac{t}{\tau_y}} \cos\left(2\pi Q_y \cdot \frac{t}{T}\right)}$$

$$\boxed{\tau_y = 2T \frac{E_0}{U_0}}$$

Characteristic time!

Twice as long as it would take to lose E_0 at U_0 per turn

PARTITION NUMBERS

The H, V, & S damping times are connected (e.g. through horizontal dispersion) by the partition numbers $J_{x,y,s}$

$$\boxed{\tau_{x,y,s} = \frac{\tau_0}{J_{x,y,s}}}$$

characteristic time

where "it can be shown" that

$$\boxed{J_x + J_y + J_s = 4} \qquad J_y = 1$$

EG NSLS-II, $E_0 = 3$ GeV, $U_0 = 0.29$ MeV

$\Rightarrow \tau_0 = 2{,}000$ turns //

3-D STABILITY

Both J_x & J_s must be positive

$$J_x = 1 - \mathcal{D}$$

$$J_s = 2 + \mathcal{D}$$

$$\boxed{\mathcal{D} = \frac{\langle \eta G^3 \rangle + 2\langle \eta G K \rangle}{\langle G^2 \rangle}}$$

E.G. SEPARATED FUNCTION MAGNETS

$GK = 0$ $\quad \mathcal{D} \sim \frac{\eta}{\rho} \sim \frac{2}{50} \ll 1$

and stability is "guaranteed"

(but large machines beware)

EG COMBINED FUNCTION OPTICS

$\mathcal{D}$ can easily be of order 1 if $GK \neq 0$

This doesn't matter

- in hadron rings: some are CF, some are SF
- if electron storage time is short
  : e.g. Cornell electron inj. synchrotron

Q: What prevents electron beam sizes vanishing?

$\hbar \neq 0$!!