USPAS Accelerator Physics 2024
Hampton VA / Northern Illinois University

Chapter 12+
Quantum Excitation, Low Emittance Lattices and Synchrotron Light Sources
(with particular thanks to Andy Wolski and Pantaleo Raimondi)

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http://www.toddsatogata.net/2024-USPAS

Happy birthday to Doug Engelbart, Phil Collins, and Kid Cudi!

Happy National Croissant Day, National Maxwell Day, and Plan for a Vacation Day!

12.0: Electron Emittances

- Steve introduced synchrotron radiation this morning
  - Bending high energy electrons radiate power
    - Mostly forward but a little bit transverse (1/γ forward cone)
  - Average total particle energy is restored by RF acceleration
    - (Almost) entirely forward
- Question:
  - Why don't electron transverse emittances shrink to zero?
- Answer:
  - Transverse noise "thermal equilibrium"
  - Limiting factor is NOT RF cavity alignment or other general accelerator system noise
  - Synchrotron radiated power dominates. It is **quantized**, so power radiated is a **stochastic lumpy noisy** process rather than smooth and continuous
    - **Difference maps**, though noisy rather than periodic

Quantum Mechanics (No grease monkey, I, but a quantum mechanic)

- Characteristic energy of emitted photons

freq $$\omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}$$ energy $$u_c = \hbar\omega_c = \frac{3}{2}\frac{\hbar c\gamma^3}{\rho}$$

quantum! $$\hbar = 6.582 \times 10^{-16} \ [\mathrm{eV}\cdot\mathrm{s}]$$

- We can simplify using our old friends to find $\frac{p}{c} = B\rho \quad p = \beta\gamma mc \approx E/c$

(12.3) $$\boxed{u_c \ [\mathrm{keV}] \approx 0.665 \, E^2 \ [\mathrm{GeV}^2] \, B \ [\mathrm{T}]}$$

1 keV photon: hc/E ~ 1.25 nm (X-rays)

Visible light: 360-830 nm (3.44-1.5 eV)

Donald Kerst
UIUC 2.5 MeV
First Betatron, 1940
NOT sync radiation!

Wikipedia: 1947 70 MeV electron GE **synchrotron** “first light”
B~1 T for ~3 eV photons   checks out ☺

Photon number/spectrum Quantum excitation

(12.3) $$\boxed{u_c\ [\mathrm{keV}] \approx 0.665\, E^2\ [\mathrm{GeV}^2]\, B\ [\mathrm{T}]}$$

1 keV photon: hc/E = 1.25 nm

- How many photons? We integrate over a quantum spectrum of photon energy *u* to find N (photons/sec) and apply random walk

$$N = \int n(u) du \qquad \mathcal{P}(\omega) = u\, n(u)\, \frac{du}{d\omega} = \hbar u\, n(u)$$

- Quantum excitation more naturally scales as

(12.6)

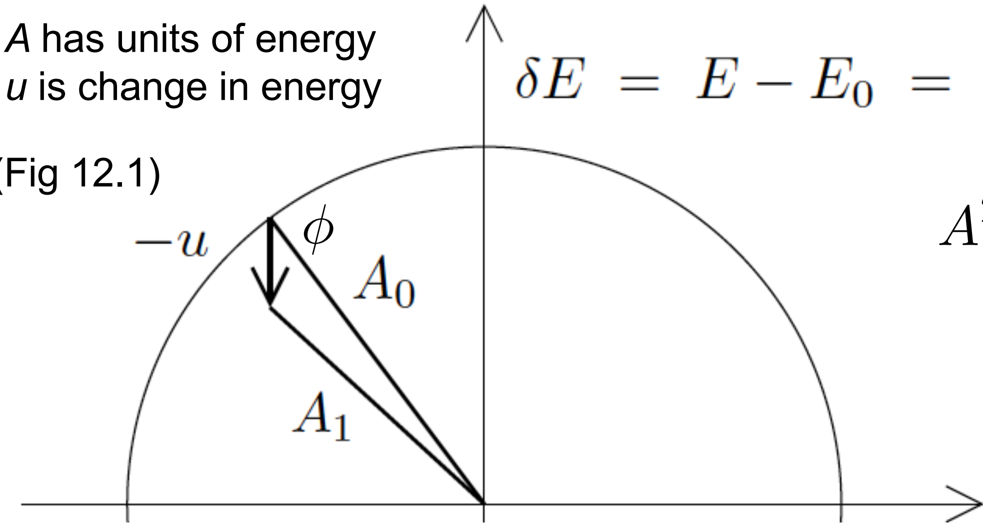
$$\boxed{N\langle u^2\rangle = \int_0^\infty n(u)\, u^2\, du = \frac{55}{24\sqrt{3}} r_0 \hbar\, m c^4\, \frac{\gamma^7}{\rho^3} \approx 8.5 \times 10^{-8}\, \frac{\gamma^7}{\rho^3 [\mathrm{m}^3]} \left[\frac{\mathrm{eV}^2}{\mathrm{s}}\right]}$$

12.1: Energy Spread and Equilibrium

- Consider single particle change in energy -*u* per turn producing change in longitudinal phase space amplitude *A*

A has units of energy
u is change in energy

(Fig 12.1)



$$\delta E = E - E_0 = A\cos(\phi)$$

Law of cosines

$$A_1^2 = A_0^2 + u^2 - 2A_0 u\cos\phi$$

$$\langle A_1^2 - A_0^2\rangle = \langle u^2\rangle - 2\langle A_0 \cancelto{0}{u\cos}\phi\rangle$$

Multiply by N photons/turn

- Average over phase ϕ and photon distribution to find the average square amplitude growth over time
  - At equilibrium this is equal to the damping rate

$$\frac{d\langle A^2\rangle}{dt} = N\langle u^2\rangle = -\frac{\langle A^2\rangle}{\tau_s/2} \qquad (11.32)$$

12.1: Longitudinal Equilibrium Energy Spread

- We can average around the ring as well as the distribution and longitudinal phases to connect *A* and the "natural" energy spread:

$$\sigma_{\mathrm{E}}^2 = \frac{1}{2}\langle A^2 \rangle = \frac{\tau_s}{4}\langle N \langle u^2 \rangle \rangle_s$$

- Then we can finally find the equilibrium energy spread

(12.13)
$$\boxed{\left(\frac{\sigma_E}{E_0}\right)^2 = \frac{C_q}{J_s}\gamma^2 \frac{\langle 1/\rho^3(s)\rangle_s}{\langle 1/\rho^2(s)\rangle_s}}$$

J_s : partition function

Averages: "radiation integrals"

$$\begin{aligned} C_q &= \frac{55}{32\sqrt{3}}\frac{\hbar}{mc} \\ &= 3.84\times 10^{-13}\ [\mathrm{m}] \qquad \text{electrons} \\ &= 2.09\times 10^{-16}\ [\mathrm{m}] \qquad \text{protons} \end{aligned}$$

Only geometry: live with it!
o(few e-4) for most light sources

12.2: Horizontal Energy Loss Effects

- Energy losses at dispersive locations

$$x_{TOT} = \eta_x \left(\frac{\delta E}{E_0}\right) + x_\beta$$

Conserved quantity without radiation

$$a_x^2 = \gamma_x x_\beta^2 + 2\alpha_x x_\beta x'_\beta + \beta_x x_\beta^2$$

Emit a photon with energy u

$$\Delta a_x^2 = \mathcal{H} \left(\frac{u}{E_0}\right)^2$$

$$\mathcal{H} \equiv \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta'_x + \beta_x \eta'_x$$

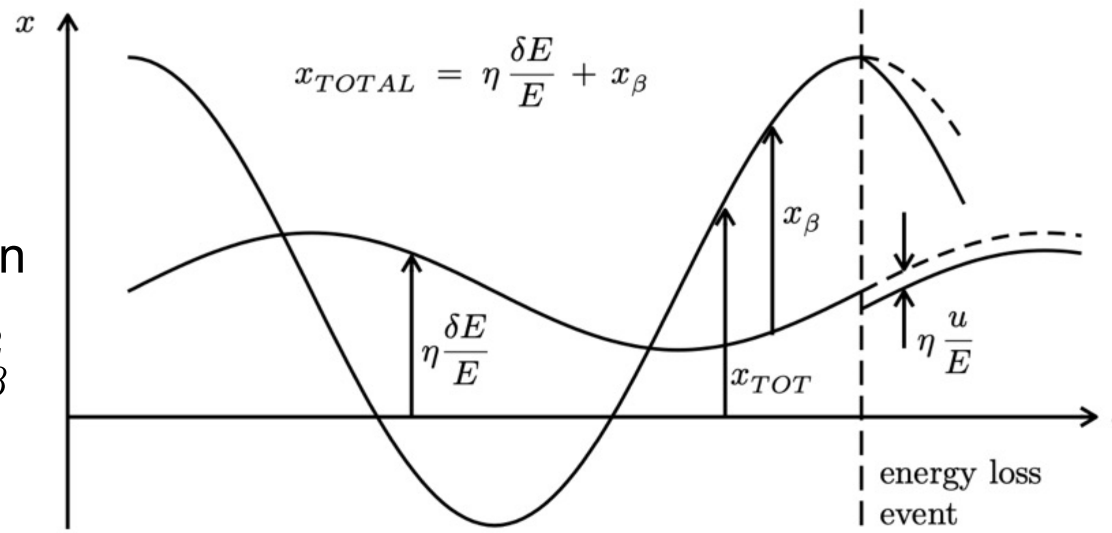


Figure 12.2 The excitation of the horizontal betatron displacement and angle in an energy loss event such as photon emission, or a noisy RF system.

The rate of betatron excitation (averaged around the ring)

$$\frac{d\langle a_x^2\rangle}{dt} = \frac{1}{E_0^2}\langle N\langle u^2\rangle \mathcal{H}\rangle_s \quad (12.22)$$

competes with the horizontal damping

$$\frac{d\langle a_x^2\rangle}{dt} = -2\frac{\langle a_x^2\rangle}{\tau_x} \quad (12.23)$$

so that at equilibrium the natural horizontal unnormalised emittance is

$$\epsilon_x \equiv \frac{1}{2}\langle a_x^2\rangle = \frac{\tau_x}{4E_0^2}\langle N\langle u^2\rangle \mathcal{H}\rangle_s \quad (12.24)$$

12.2: Horizontal Equilibrium Emittance

- The equilibrium emittance, balanced between synchrotron radiation damping and quantum excitation effects, is

$$\boxed{\epsilon_x = \frac{C_q}{J_x}\gamma^2 \frac{\langle \mathcal{H}(s)/\rho^3(s)\rangle_s}{\langle 1/\rho^2\rangle_s}}$$

Jx: Partition function, not action!

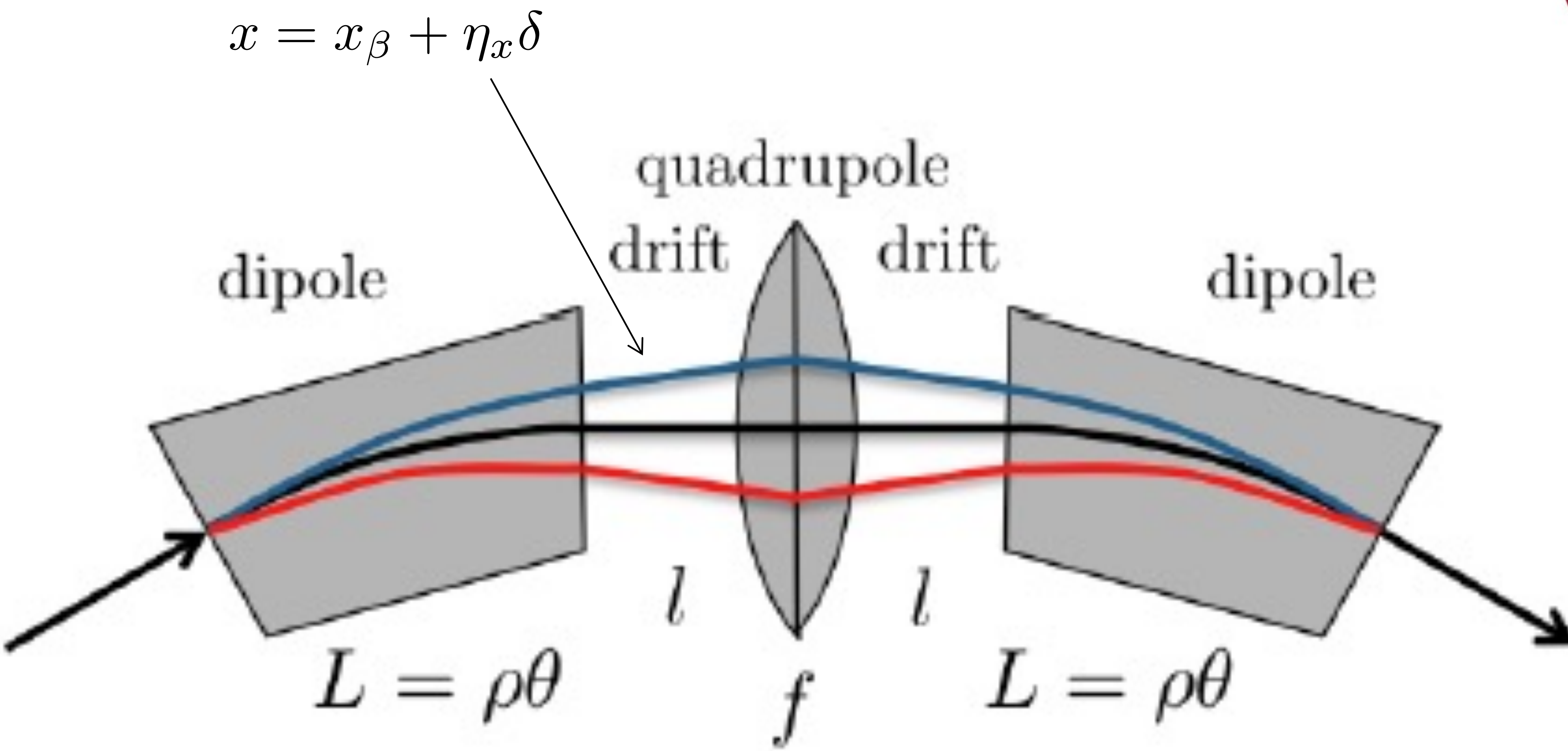
- This depends on details of the lattice, in particular dispersion located in the dipoles (at locations of synchrotron radiation emission that couple dimensions)

$$\boxed{\mathcal{H}(s) = \beta_x \eta_x'^2 + 2\alpha_x(s)\eta_x\eta_x' + \gamma_x\eta_x^2}$$

$$(\text{“Curly} - \text{H function''})$$

- Vertical is like this but dispersion and curly-H are zero!

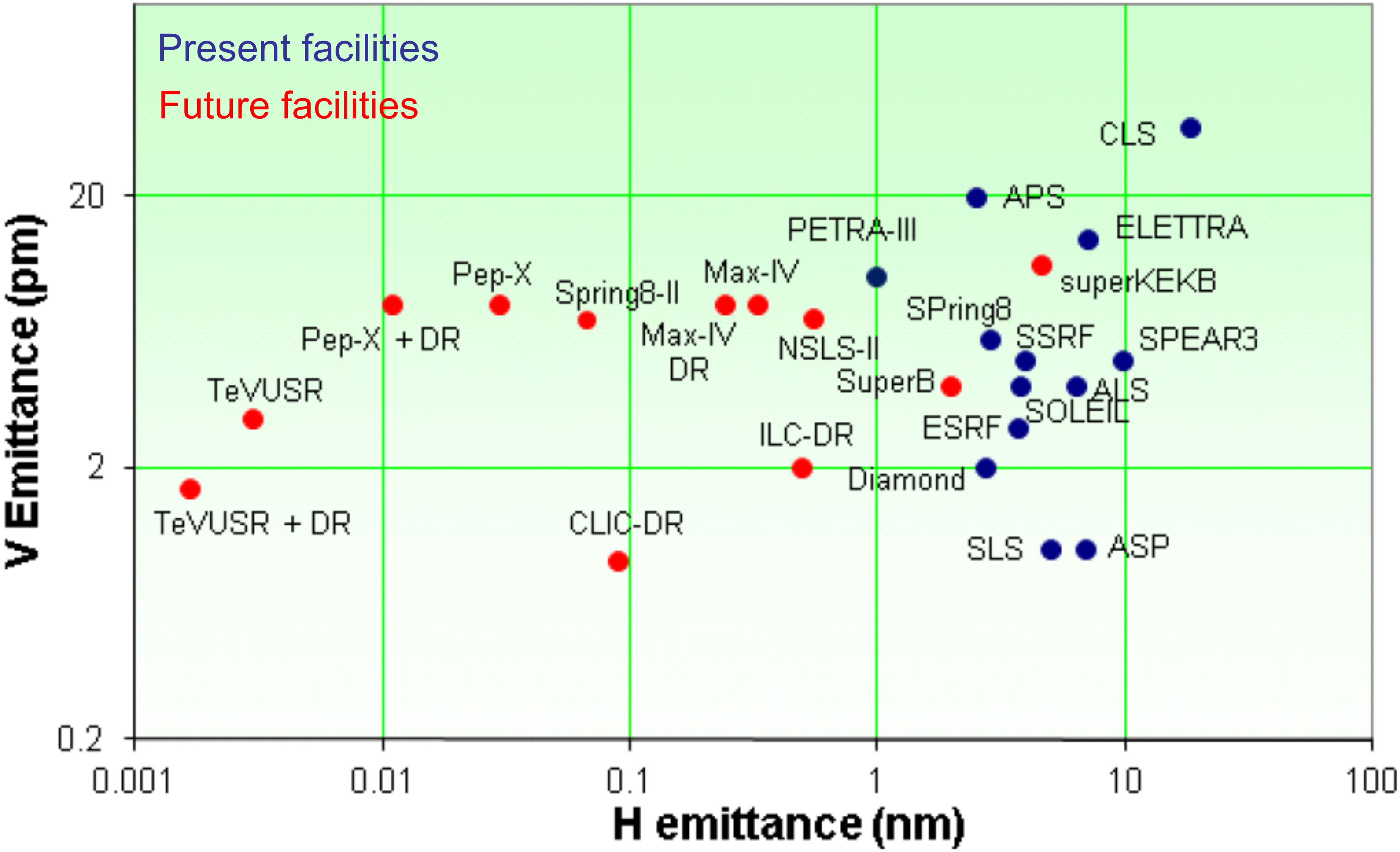
Review and Bonus: Double Bend Achromat



- DBA is also known as a Chasman-Green lattice
  - Used in early third-generation light sources (e.g. NSLS at BNL)
  - But now we know about synchrotron radiation, $\mathcal{H}$ functions

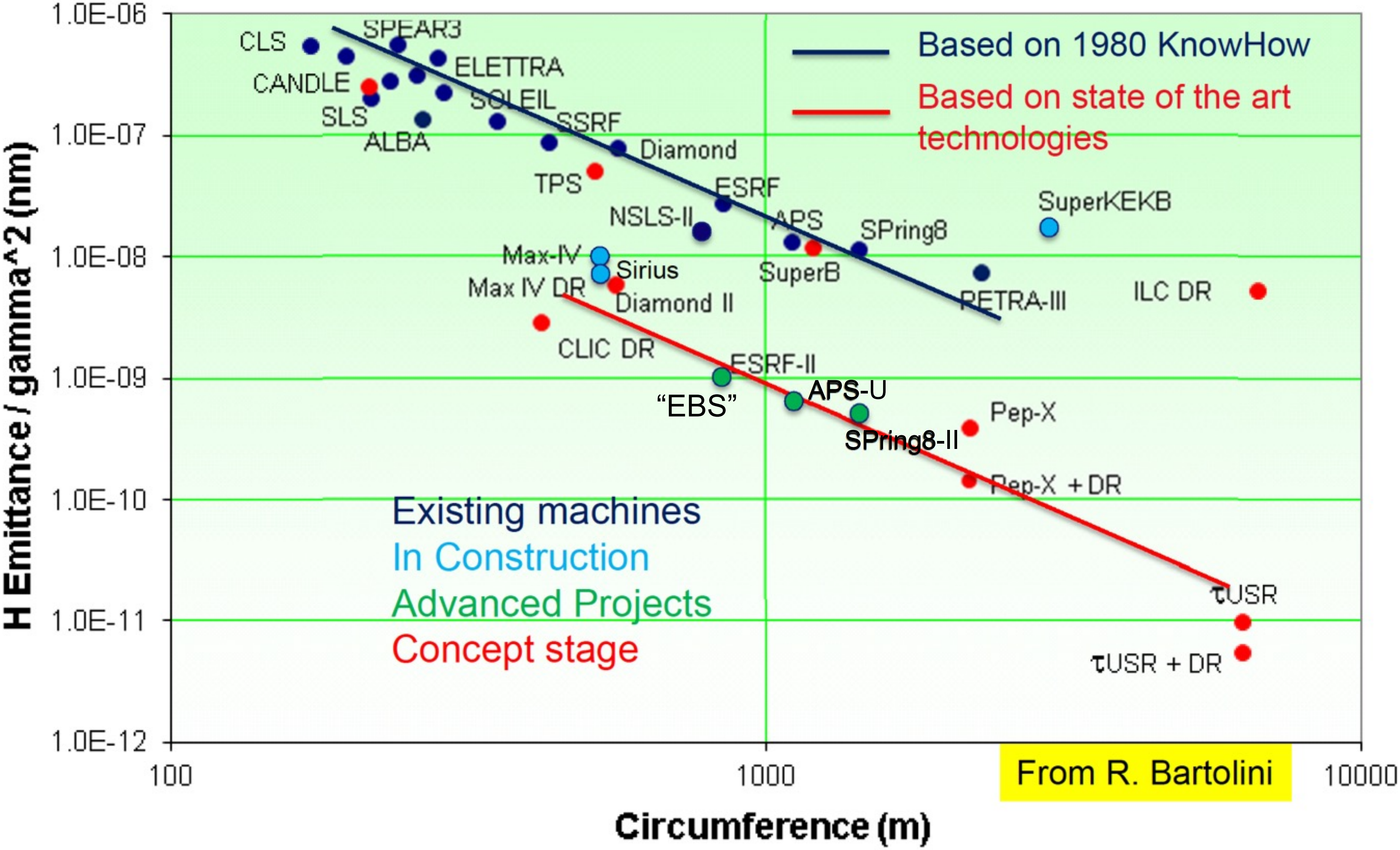
One View of Light Source Emittances

Note the logarithmic emittance scales and relatively constant vertical emittance!



R. Bartolini, ICFA Beam Dynamics Newsletter 57 (2012)

Another View of Light Source Emittances



P. Raimondi, IPAC'17, THPPA3

Radiation Integrals

- There are several radiation integrals that come into play in evaluation of effects of radiation on dynamics of ultra-relativistic particles in a storage ring or beamline, including one (I_5) that depends on curly-H.

$$I_1 \equiv \oint \frac{\eta_x(s)}{\rho(s)} ds \qquad \text{momentum compaction } \alpha_p = \frac{I_1}{L}$$

$$I_2 \equiv \oint \frac{ds}{\rho^2(s)} \qquad I_4 \equiv \oint \frac{\eta_x(s)}{\rho(s)} \left(\frac{1}{\rho^2(s)} + 2k_1(s) \right) ds$$

$$I_3 \equiv \oint \frac{ds}{|\rho(s)|^3} \qquad I_5 \equiv \oint \frac{\mathcal{H}_x}{|\rho^3(s)|} ds$$

These integrals only depend on the lattice design

Relation of Integrals to Partition Numbers

- For a horizontal planar ring we can write relationships between the radiation integrals and partition numbers mentioned by Steve/Chapter 11:

$$J_x = 1 - \frac{I_4}{I_2} \qquad J_y = 1 \qquad J_s = 2 + \frac{I_4}{I_2}$$

- See Handbook of Accelerator Physics, Chao and Tigner, p. 210
- You usually get **radiation integrals** and not partition numbers from lattice design codes (madx, elegant...)

SYNCH_1 First synchrotron radiation integral
SYNCH_2 Second synchrotron radiation integral
SYNCH_3 Third synchrotron radiation integral
SYNCH_4 Fourth synchrotron radiation integral
SYNCH_5 Fifth synchrotron radiation integral
SYNCH_6 Sixth synchrotron radiation integral
SYNCH_8 Eighth synchrotron radiation integral

- `radiation_integrals` — A flag indicating, if set, that radiation integrals should be computed and included in output. *N.B.: Radiation integral computation is not correct for systems with vertical bending, nor does it take into account coupling. See the* `moments_output` *command if you need such computations.*

Equilibrium Horizontal Emittance

- The evolution of horizontal emittance, including both damping and quantum excitation, is

$$\frac{d\epsilon_x}{dt} = -\frac{2}{\tau_x}\epsilon_x + \frac{2}{J_x \tau_x} C_q \gamma^2 \frac{I_5}{I_2} \qquad J_x = 1 - \frac{I_4}{I_2}$$

damping

Quantum excitation

$$\text{“quantum constant”} \ C_q = \frac{55}{32\sqrt{3}} \frac{\hbar}{mc} \approx 3.83 \times 10^{-13} \ \text{m}$$

- This is at an equilibrium for the “natural” emittance

$$\boxed{\epsilon_0 = C_q \frac{\gamma^2}{J_x} \frac{I_5}{I_2}}$$

  - This only depends on beam energy and radiation integrals!

Equilibrium Energy Spread

- We can average the quantum excitation effects on beam momentum offset to find the evolution of energy spread:

$$\frac{d\sigma_\delta^2}{dt} = C_q \gamma^2 \frac{2}{J_u \tau_u} \frac{I_3}{I_2} - \frac{2}{\tau_u} \sigma_\delta^2 \qquad J_u = 2 + \frac{I_4}{I_2}$$

Quantum excitation | damping

- We can also find the equilibrium energy spread and bunch length

$$\boxed{\sigma_{\delta 0}^2 = C_q \gamma^2 \frac{I_3}{J_u I_2}} \qquad \text{bunch length} : \boxed{\sigma_{z0} = \frac{\alpha_p c}{Q_s} \sigma_{\delta 0}}$$

  - Note the lack of RF parameters! This equilibrium distribution is again determined only by the lattice (and collective effects). We can shorten bunch length by raising RF voltage

Evaluating Radiation Integrals

- If bends have no quadrupole component (a modern separated function synchrotron), $J_x \approx 1$ and $J_u \approx 2$
- To find the equilibrium emittance, we then need to evaluate two synchrotron radiation integrals

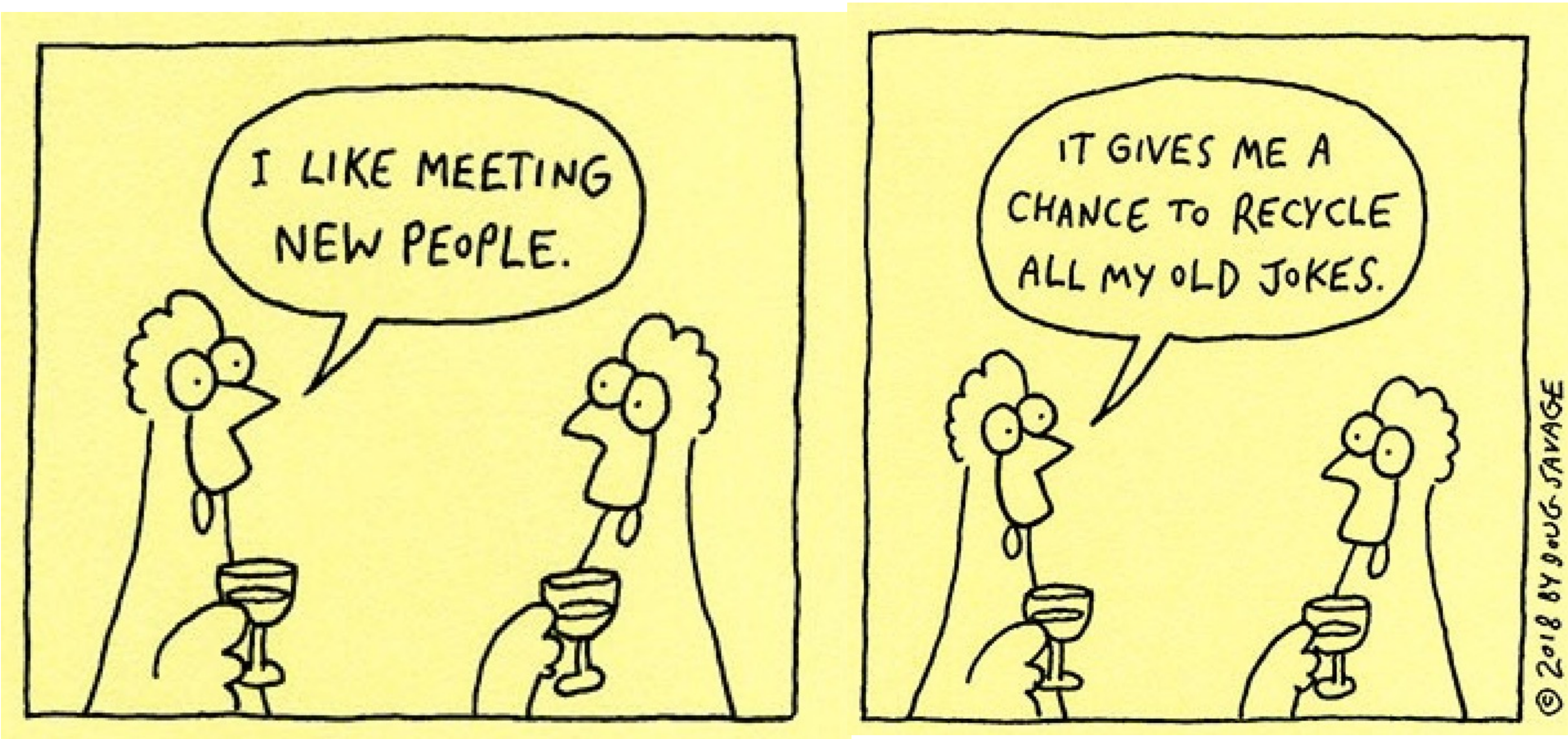
- I_2 depends on only detailed knowledge of dipole magnets
  - e.g. for all dipole magnets being the same, total bend 2π

$$I_2 = \oint \frac{ds}{\rho^2(s)} = \frac{2\pi B}{(B\rho)} \approx \frac{2\pi c B}{U/e}$$

- Evaluating I_5 depends on detailed knowledge of optics

$$I_5 \equiv \oint \frac{\mathcal{H}_x}{|\rho^3(s)|} ds \qquad \mathcal{H}(s) = \beta_x \eta_x'^2 + 2\alpha_x \eta_x \eta_x' + \gamma_x \eta_x^2$$

(Take a breath)



FODO Lattice I_5

- Just like our excursions into the FODO lattice before, we had calculated our optical functions in terms of
  - Thin quadrupole focal length f
  - Dipole bending radius ρ (for dispersion contributions)
  - Dipole lengths $L = \rho\theta$ (full space between quadrupoles)

- These calculations are usually done with computer programs that find the optical functions and integrate $\mathcal{H}$ for us.
  - But Wolski (see below) writes out some of the logic to progress through a FODO lattice and evaluate some reasonably realistic approximations

$$\theta \ll 1 \qquad \Rightarrow \qquad \rho \gg 2f \qquad \Rightarrow \qquad 4f \gg L$$

A. Wolski, 2011 CERN Accelerator School Lectures, Greece

FODO Lattice I_5

- Similar to the dogleg, the analysis is most easily done in an expansion of small dipole bend angle θ

$$\frac{I_5}{I_2} = \left(4 + \frac{\rho^2}{f^2}\right)^{-\frac{3}{2}} \left[8 - \frac{\rho^2}{2f^2}\theta^2 + O(\theta^4)\right]$$

$$\approx \left(1 - \frac{\rho^2}{16f^2}\theta^2\right)\left(1 + \frac{\rho^2}{4f^2}\right)^{-\frac{3}{2}}$$

$$\sin\frac{\mu}{2} = \frac{\rho\theta}{2f}$$

$$\rho >> 2f \quad \Rightarrow \quad \frac{I_5}{I_2} \approx \left(1 - \frac{L^2}{16f^2}\right)\frac{8f^3}{\rho^3}$$

$$4f >> L \quad \Rightarrow \quad \frac{I_5}{I_2} \approx \frac{8f^3}{\rho^3}$$

Approximate Natural Emittance of FODO Lattice

- We can then write the approximate natural horizontal emittance of the FODO lattice, again with $J_x \approx 1$

$$\epsilon_0 = C_q \frac{\gamma^2}{J_x} \frac{I_5}{I_2} \approx C_q \gamma^2 \left(\frac{2f}{L} \right)^3 \theta^3$$

  - Proportional to square of beam energy γ
  - Proportional to cube of bending angle per dipole
    - Increase number of cells to reduce bending angle per dipole and thus reduce FODO emittance.
  - Proportional to cube of quadrupole focal length
    - Stronger quads gives stronger focusing, lower natural emittance
  - Inversely proportional to cube of the cell (or dipole) length
    - Longer cells also reduce overall natural emittance

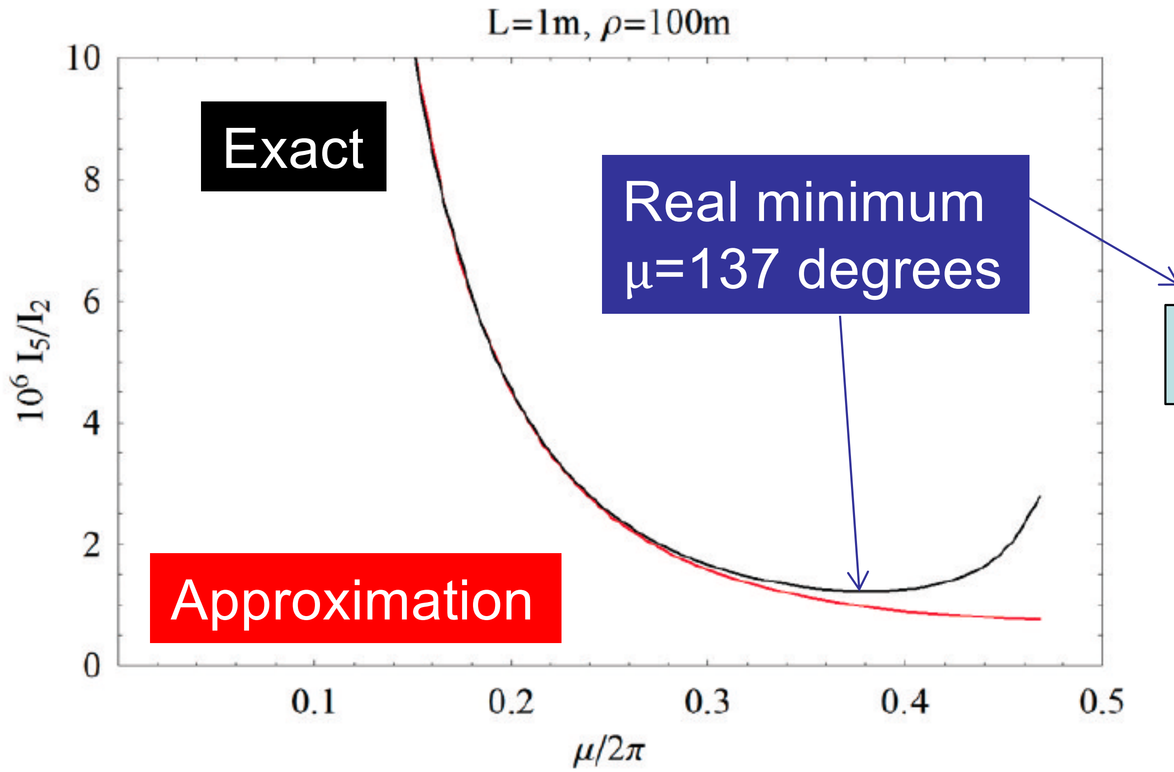
A. Wolski, 2011 CERN Accelerator School Lectures, Greece

Minimum Emittance of FODO Lattice?

- The stability criterion for FODO lattices with these parameters is $f \geq L/2$ with a minimum of $f/L = 1/2$
  - Estimated FODO lattice minimum emittance

$$\epsilon_0 \approx C_q \gamma^2 \theta^3$$

  - But approximations start to break down for strong focusing



$$\epsilon_0 \approx 1.2\, C_q \gamma^2 \theta^3$$

We Can Do Better!

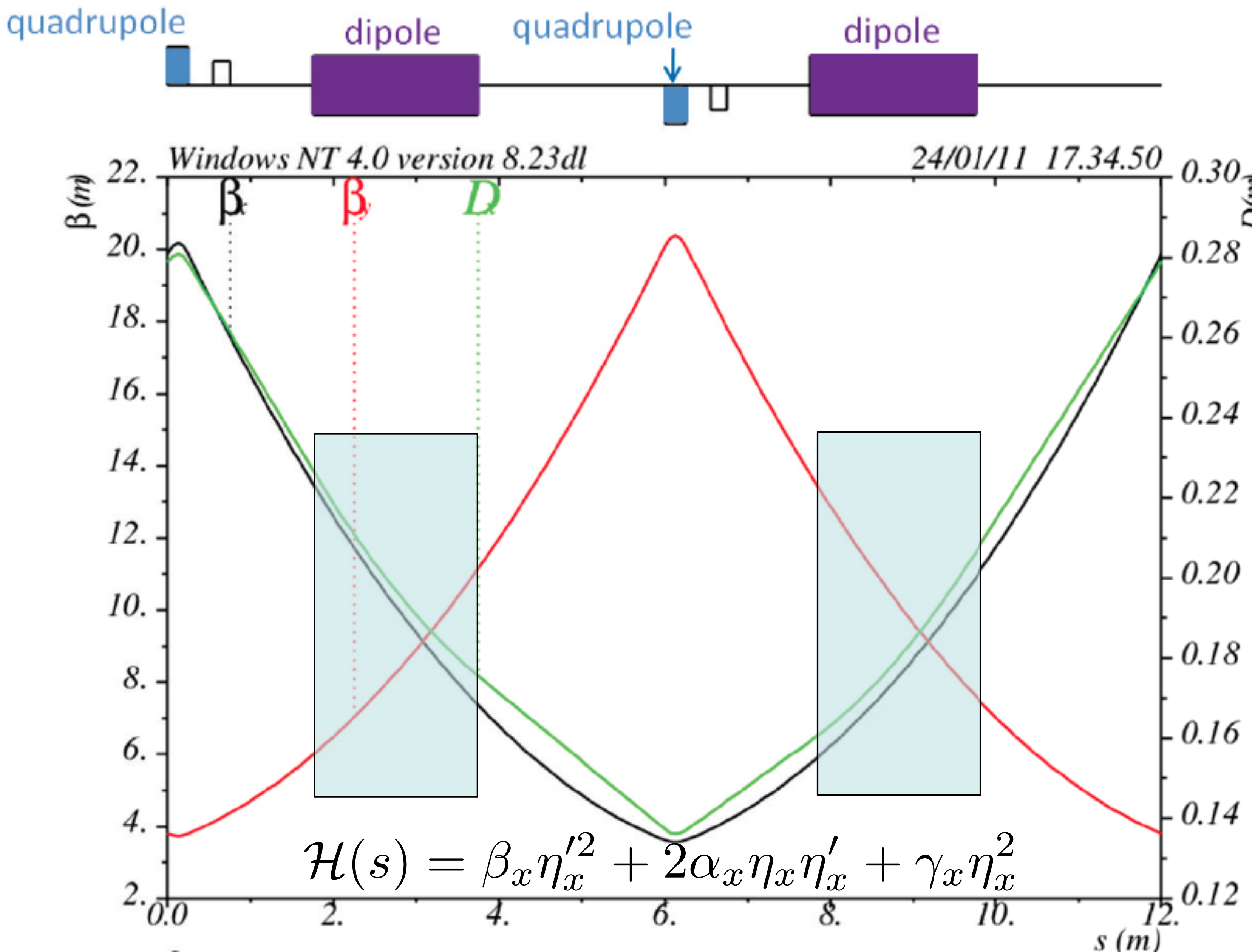
- It turns out that this emittance isn't usually good enough for modern third-generation light source requirements
  - 1-2 orders of magnitude too big
- How do we fix this?
  - Beam energy determines some properties of the sync light
  - So the remaining handle we have is the optics

$$I_5 \equiv \oint \frac{\mathcal{H}_x}{|\rho^3(s)|} ds \qquad \mathcal{H}(s) = \beta_x \eta_x'^2 + 2\alpha_x \eta_x \eta_x' + \gamma_x \eta_x^2$$

  - Minimizing η and η' in the dipoles will minimize the overall integral of $\mathcal{H}$ and thus I_5

- How do the dispersion functions look though FODO dipoles?

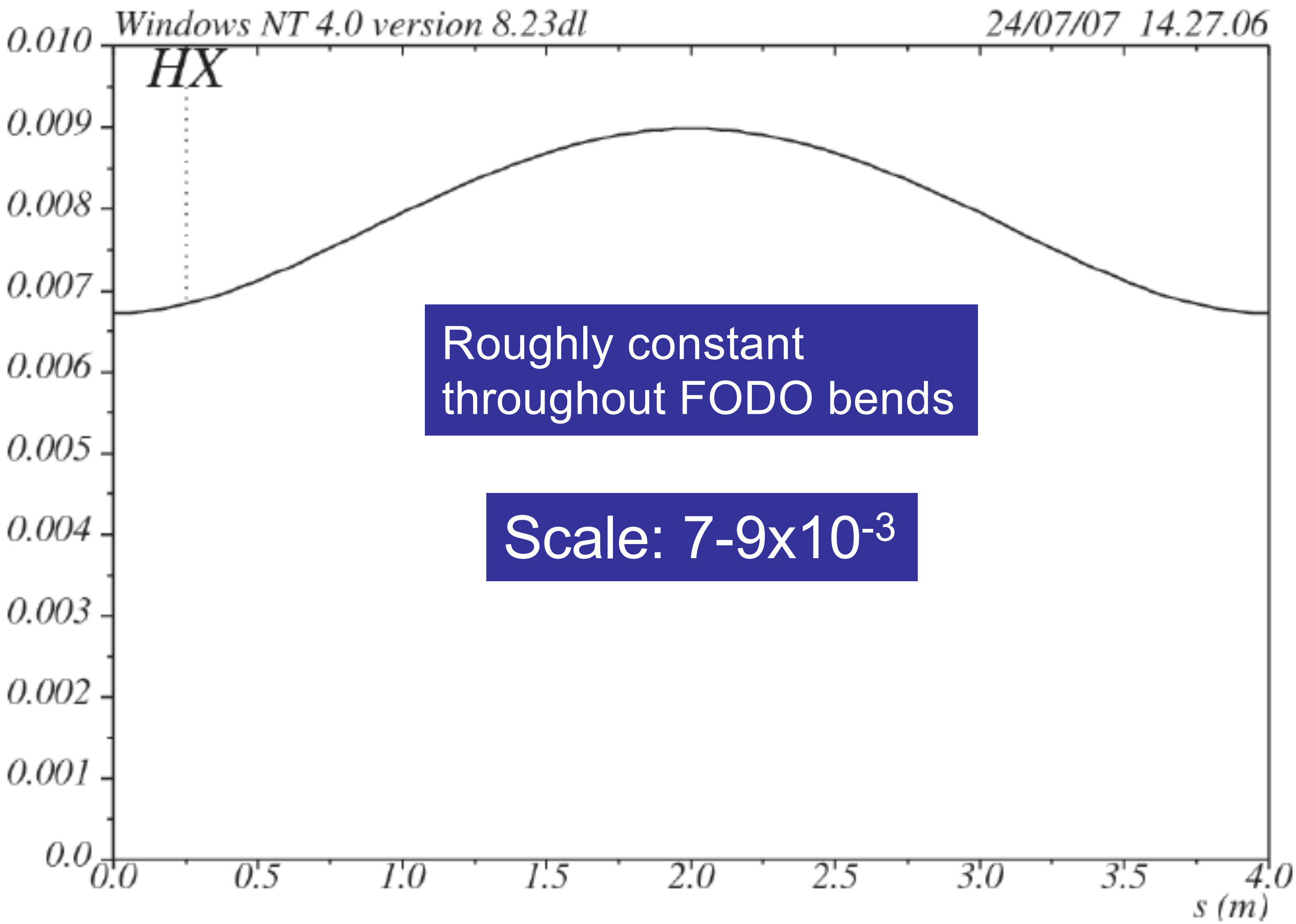
FODO Cell Optics



$$\mathcal{H}(s) = \beta_x \eta_x'^2 + 2\alpha_x \eta_x \eta_x' + \gamma_x \eta_x^2$$

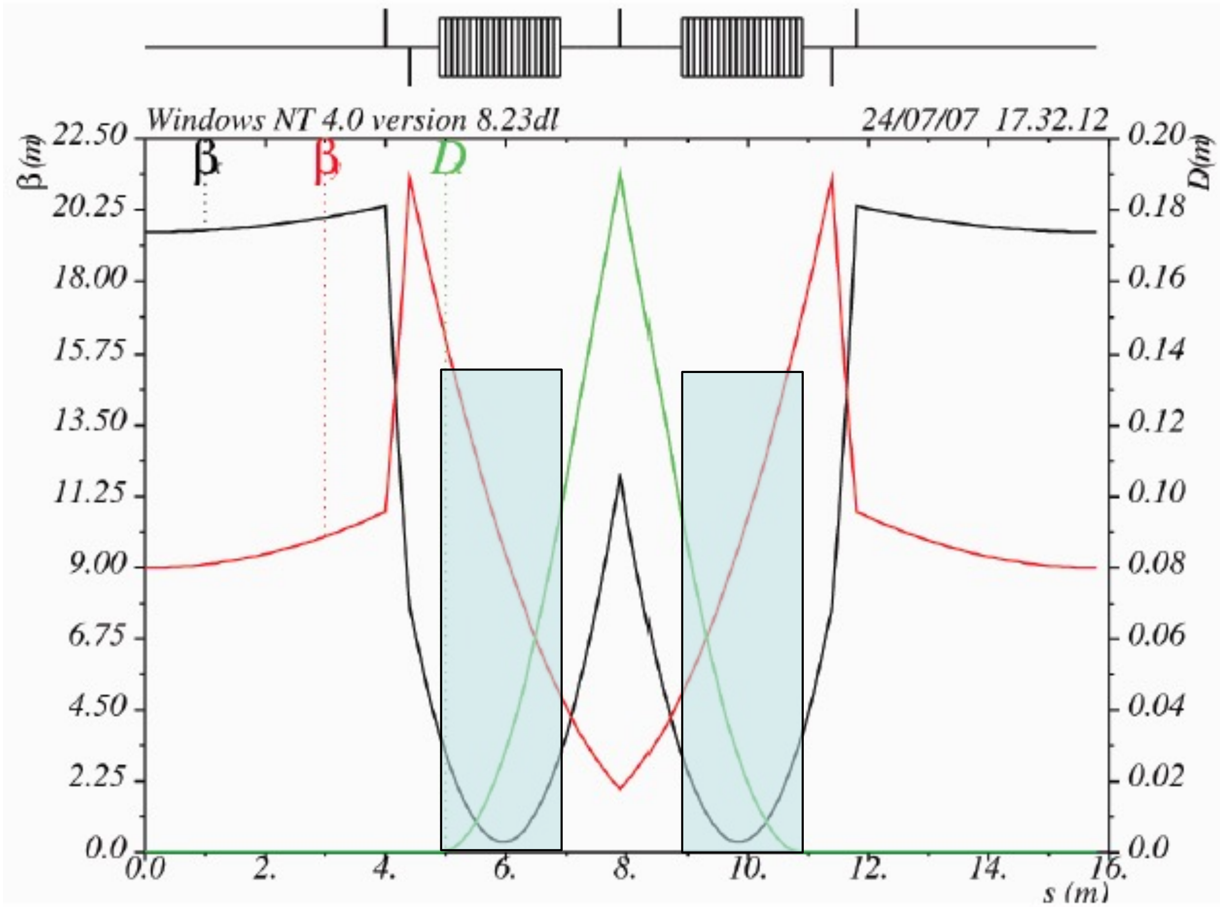
A. Wolski, 2011 CERN Accelerator School Lectures, Greece

FODO Dipole $\mathcal{H}$

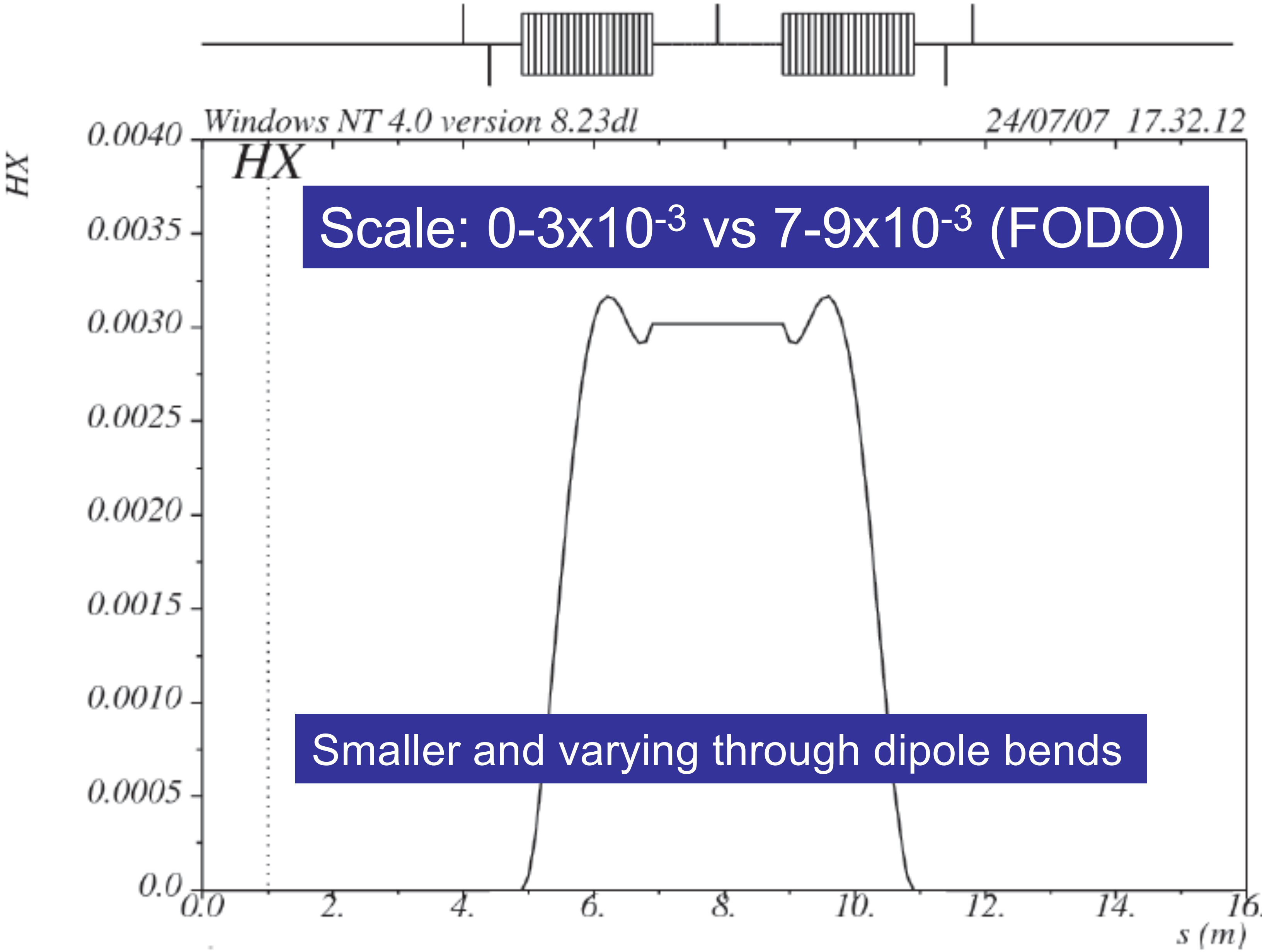


Double Bend Achromats

- If only we had a lattice that had dipoles that had zero η and η' somewhere near their ends
- We do, the double bend achromat!
  - Add extra focusing at ends for periodic matching with $\alpha_{x,y} \approx 0$



DBA Dipole$\mathcal{H}$



Scale: 0-3x10^{-3} vs 7-9x10^{-3} (FODO)

Smaller and varying through dipole bends

DBA Radiation Integrals

- We can optimize the beta functions and matching vs dipole length to produce a best (minimum) integral of I_5

$$I_{5,\mathrm{min}} = \frac{1}{4\sqrt{15}} \frac{\theta^4}{\rho} + o(\theta^6)$$

$$I_2 = \int \frac{ds}{\rho^2} = \frac{\theta}{\rho}$$

dipole ends

$$\beta_x \approx L\sqrt{12/5}$$

$$\alpha_x \approx \sqrt{15}$$

$$\epsilon_{0,\mathrm{DBA,min}} = C_q \gamma^2 \frac{I_{5,\mathrm{min}}}{J_x I_2} \approx \frac{1}{4\sqrt{15}} C_q \gamma^2 \theta^3$$

- This is about 13 times smaller (!) than the FODO lattice minimum emittance!

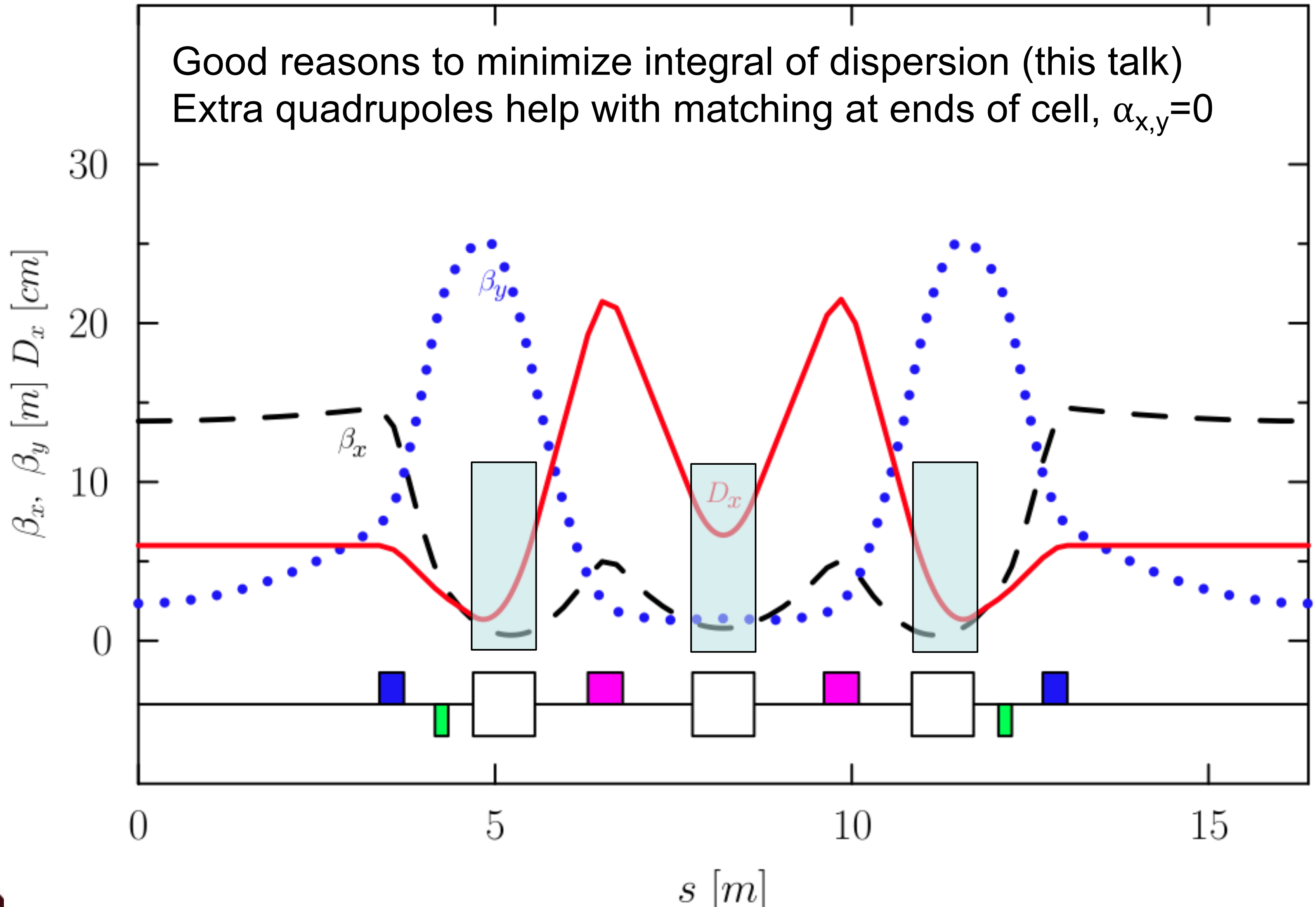
$$\epsilon_0 \approx 1.2\, C_q \gamma^2 \theta^3$$

But We Can Still Do Better

- The double bend achromat was a huge step forward
  - Made NSLS into a very successful light source
  - But we can still further optimize I_5
- One way to do this is the triple bend achromat shown (very briefly) earlier
  - e.g. the ALS, BESSY-II, SLS (Swiss Light Source, PSI)
  - This can place local minima at the dipoles
  - One tradeoff: more complicated lattice, more expensive…
  - More focusing also provides stronger chromatic effects
    - Correction with sextupoles requires nonlinear optimization
- Another solution: minimize I_5 wrt all lattice parameters
  - So-called TME (theoretical minimum emittance) lattices
  - Tend to not be very locally robust solutions
  - But they sure get close to minimizing the natural emittance

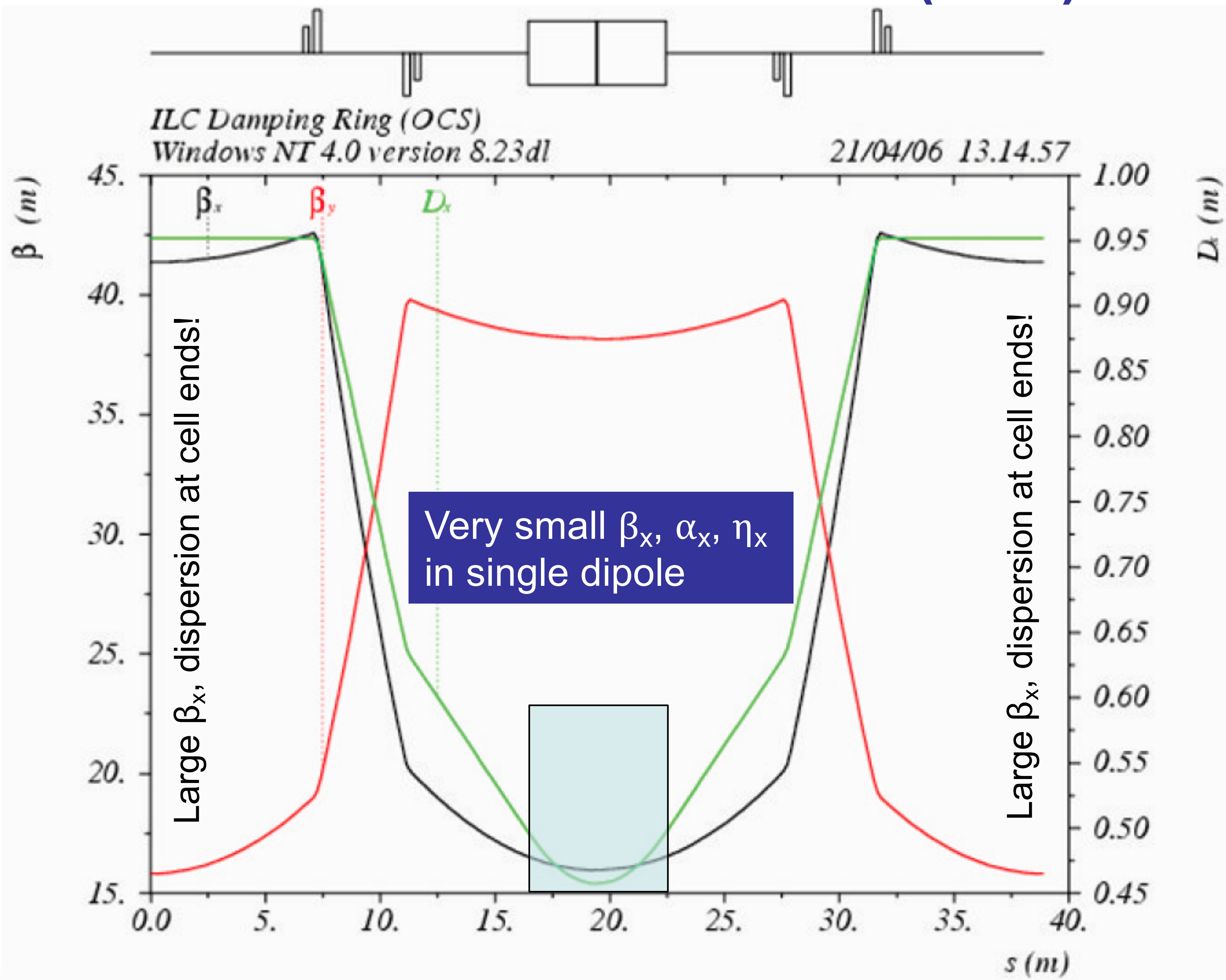
Triple Bend Achromat Cell (ALS at LBL)

Good reasons to minimize integral of dispersion (this talk)
Extra quadrupoles help with matching at ends of cell, $\alpha_{x,y}=0$



L. Yang et al, Global Optimization of an Accelerator Lattice Using Multiobjective Genetic Algorithms, 2009

"Theoretical Minimum Emittance" (TME) Lattice



Summary of Some Minimum Emittance Lattices

Lattice style	Minimum emittance	Conditions/comments
90° FODO	$\varepsilon_0 \approx 2\sqrt{2} C_q \gamma^2 \theta^3$	$\frac{f}{L} = \frac{1}{\sqrt{2}}$
137° FODO	/2.36 $\varepsilon_0 \approx 1.2 C_q \gamma^2 \theta^3$	minimum emittance FODO
DBA	/18.6 $\varepsilon_0 \approx \frac{1}{4\sqrt{15}} C_q \gamma^2 \theta^3$	$\eta_{x,0} = \eta_{px,0} = 0$ $\beta_{x,0} \approx \sqrt{12/5} L \quad \alpha_{x,0} \approx \sqrt{15}$
TME	/3 $\varepsilon_0 \approx \frac{1}{12\sqrt{15}} C_q \gamma^2 \theta^3$	$\eta_{x,\mathrm{min}} \approx \frac{L\theta}{24} \quad \beta_{x,\mathrm{min}} \approx \frac{L}{2\sqrt{15}}$

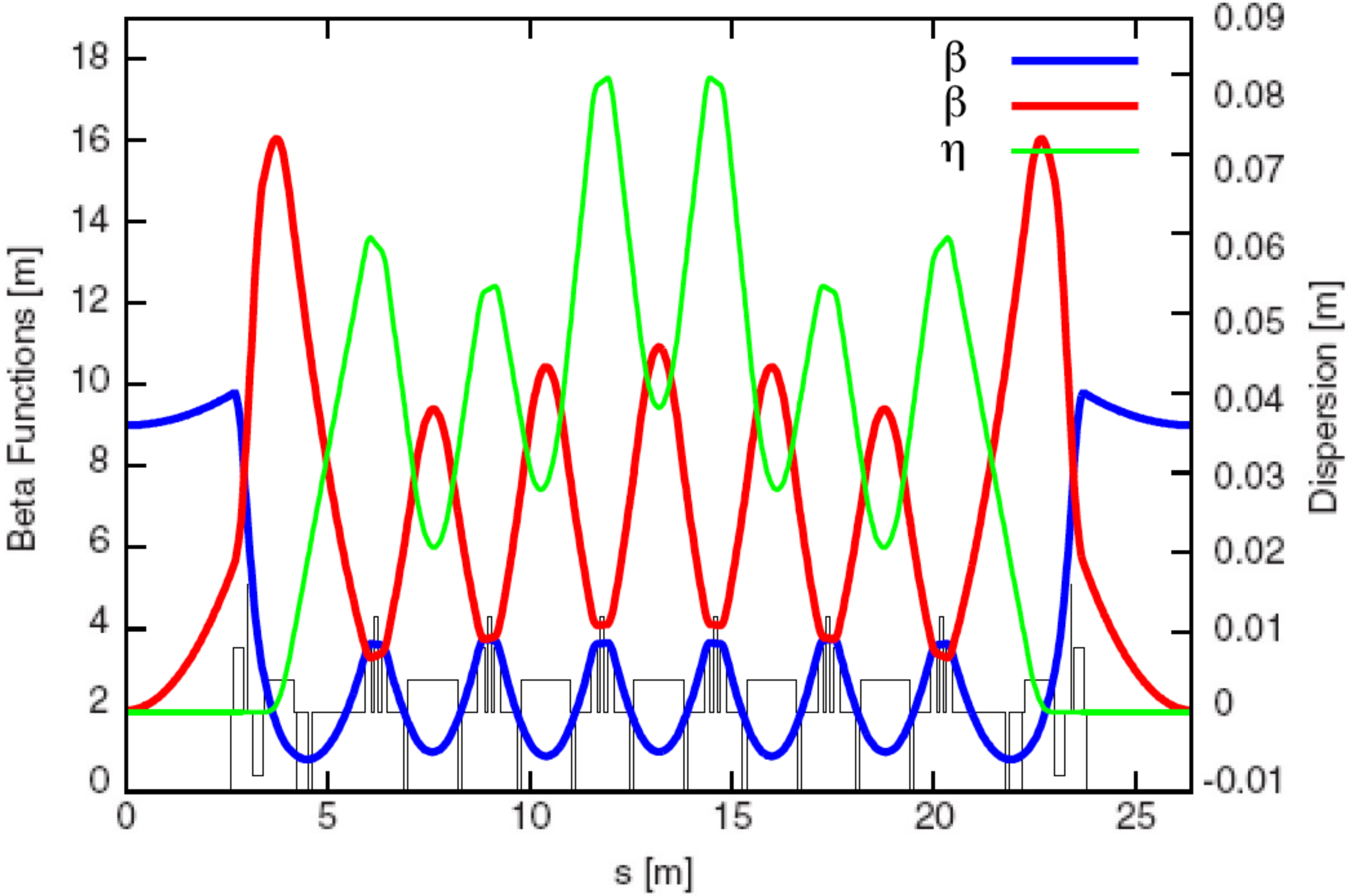
A. Wolski, 2011 CERN Accelerator School Lectures, Greece

Why Not TME All The Time?

- Optimizing one parameter (beam emittance) does not necessarily optimize the facility performance!
  - TME lattices are considered by many to be over-optimized
  - High chromaticities give very sensitive sextupole distributions
    - These in turn give very sensitive nonlinear beam dynamics
    - Momentum aperture, dynamic aperture, …
    - More tomorrow and Thursday

- Usually best to back off TME to work on other optimization
  - Another alternative is to move towards machines with many dipoles
    - Reduces bending angle per dipole and brings emittance down
    - MAX-IV: 7-bend achromat; SPRING-8 6- and 10-bend achromats

MAX-IV 7-Bend Achromat



MAX-IV Nonlinear Optics

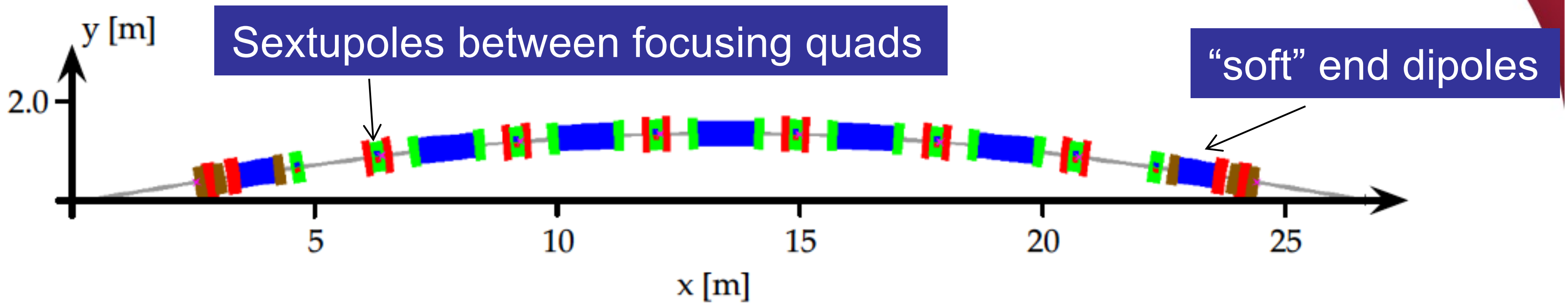


Figure 1: Schematic of one of the 20 achromats of the MAX IV 3 GeV storage ring. Magnets indicated are gradient dipoles (blue), focusing quadrupoles (red), sextupoles (green), and octupoles (brown).

- MAX-IV represents an interesting case in optics design
  - Soft end dipoles minimize synchrotron radiation on SC IDs
  - All dipoles have vertical gradient
  - Strong focusing -> large chromaticities
  - Low dispersion -> very strong chromaticity sextupoles
  - Three sextupole families optimize higher-order chromaticity and driving terms
  - Additional octupoles also correct tune change vs amplitude

S. Leeman, ICFA Beam Dynamics Newsletter 57 (2012)

MAX-IV Parameters

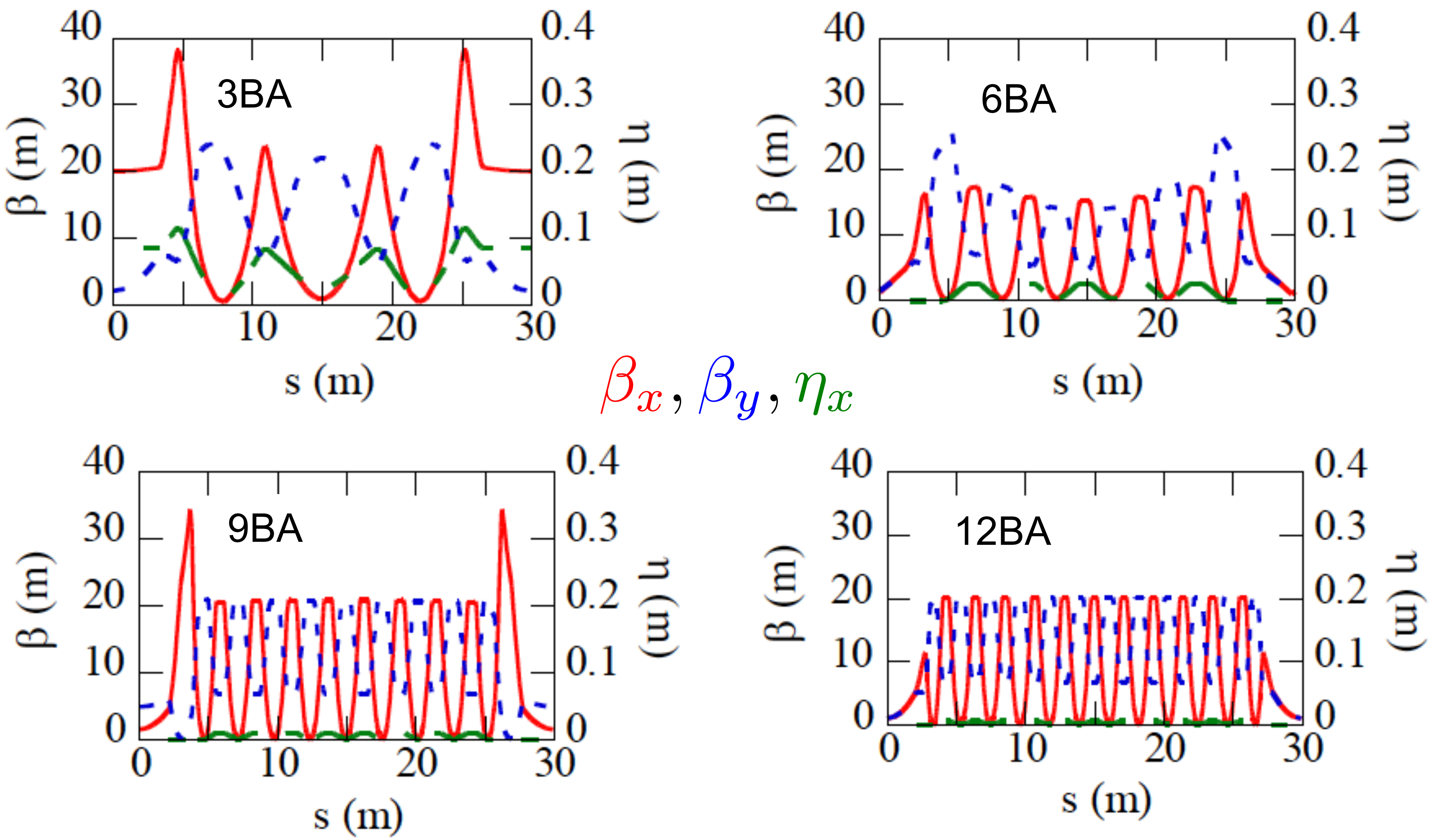
Parameter	***Unit***	***Value***
Energy	GeV	3.0
Main radio frequency	MHz	99.931
Circulating current	mA	$\boxed{500}$
Circumference	m	528
Number of achromats	…	20
Number of long straights available for IDs	…	19
Betatron tunes (H/V)	…	$\boxed{42.20 / 16.28}$
Natural chromaticities (H/V)	…	-50.0 / -50.2
Corrected chromaticities (H/V)	…	+1.0 / +1.0
Momentum compaction factor	…	3.07×10^{-4}
Horizontal damping partition	…	1.85
Horizontal emittance (bare lattice)	nm·rad	0.326
Radiation losses per turn (bare lattice)	keV	360.0
Natural energy spread	…	0.077%
Required momentum acceptance	…	$\boxed{4.5\%}$

S. Leeman, ICFA Beam Dynamics Newsletter 57 (2012)

Lattice	Type	E [GeV]	ε_x [nm·rad]	ε^*_x [nm·rad]	J_x	$<\mathcal{H}_x>$ [$\times 10^{-3}$]	F_{rel}	ξ_x/ν_x	S
SPring-8	11×DB-4	8	3.4	3.7	1.0	1.42	4.6	2.2	58
ESRF	DB-32	6	3.8		1.0	1.68	3.5	3.6	89
APS	DB-40	7	2.5	3.1	1.0	1.35	3.3	2.5	69
PETRA III	Mod. FODO	6	1		1.0	3.62	39.8	1.2	20
SPEAR3	DB-18	3	11.2		1.0	5.73	7.4	5.5	73
ALS	TB-12	1.9	6.3	6.4	1.0	4.99	10.4	1.7	24
BESSY II	TBA-10	1.9	6.1		1.0	4.83	2.9	2.8	40
SLS	TBA-12	2.4	5		1.0	3.38	2.6	3.2	56
DIAMOND	DB-24	3	2.7		1.0	1.46	4.2	2.9	76
ASP	DB-14	3	7		1.4	5.60	3.0	2.1	28
ALBA	DB-16	3	4.3		1.3	2.96	2.6	2.1	39
SOLEIL	DB-16	2.75	3.7	5.5	1.0	1.79	2.0	2.8	67
CLS	DBA-12	2.9	18.3		1.6	16.79	2.0	1.3	10
ELETTRA	DBA-12	2	7.4		1.3	9.12	1.4	3.0	31
TPS	DB-24	3	1.7		1.0	1.08	2.7	2.9	87
NSLS-II	DBA-30	3	2		1.0	3.78	2.0	3.1	50
MAX-IV	7BA-20	3	0.33		1.9	0.40	18.1	1.2	59
PEP-X (TME)	4×8TME-6	4.5	0.095		1.0	0.34	3.3	1.7	90
PEP-X (USR)	8×7BA-6	4.5	0.029		1.0	0.10	5.3	1.4	145
TeVUSR	30×7BA-6	11	0.0031		2.4	0.02	12.0	1.4	360
TeVUSR	30×7BA-6	9	0.0029		2.7	0.02	18.4	1.4	281

J. Bengtsson, 2012, Nonlinear Dynamics Optimization in Low Emittance Rings, ICFA Beam Dynamics Newsletter 57, April 2012

Multiple Bend Achromats (BAs)



Y. Shimosaki, 2012, Nonlinear Dynamics Optimization in Low Emittance Rings, ICFA Beam Dynamics Newsletter 57, April 2012

Ultimate Storage Ring Concept: PEP-X

E = 4.5 GeV

I = 1.5 A

ε_x = 150 pm-rad

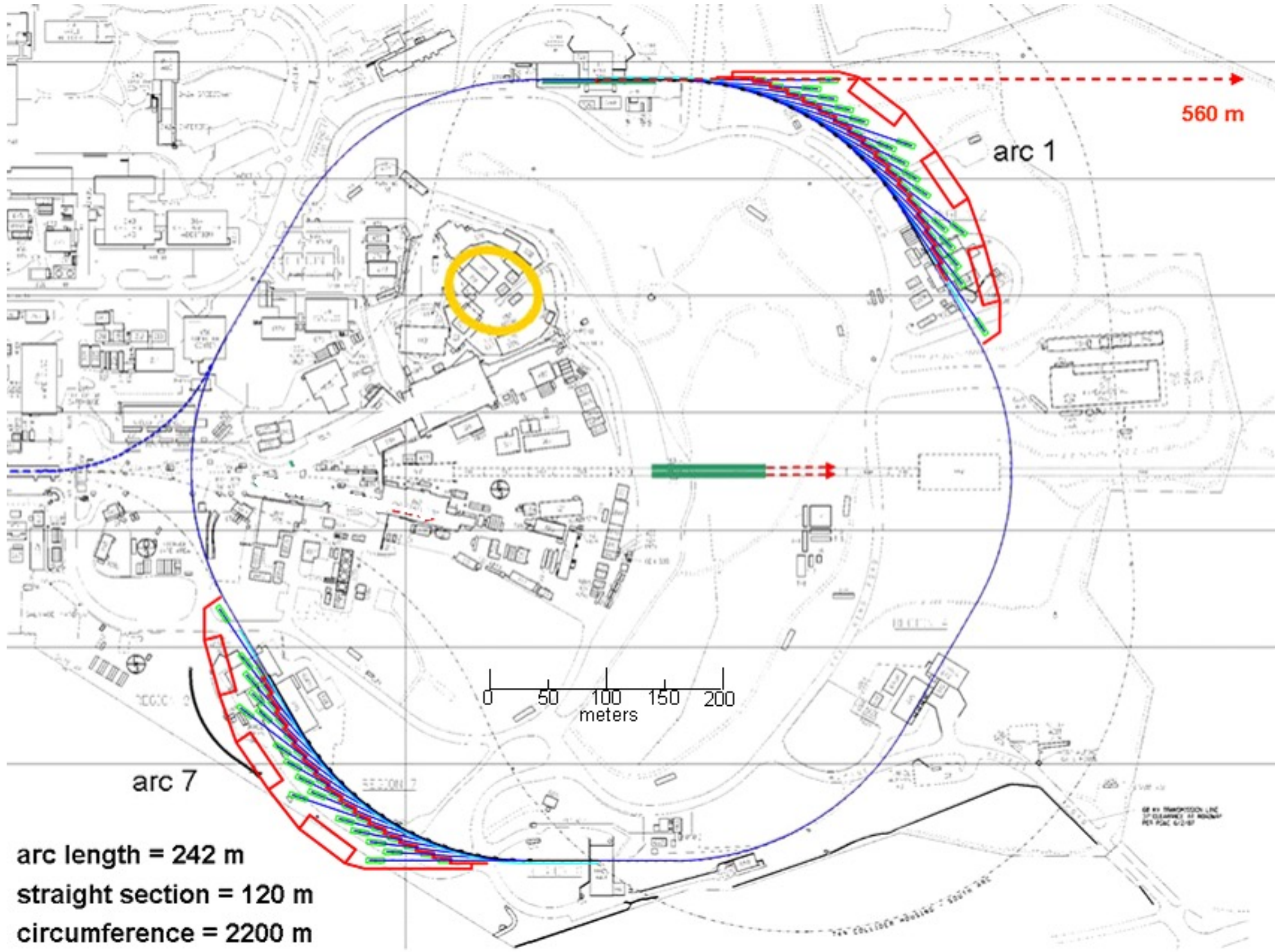
(~0.06 nm-rad w/o IBS)

ε_y = 8 pm-rad

σ_s = 3/6 mm
(without/with 3rd harm rf)

τ = ~1 h

top-up injection every few seconds
(~7 nC, multiple bunches)



- 2 arcs of DBA cells with 32 ID beam lines (4.3-m straights)
- 4 arcs of TME cells

- ~90 m damping wigglers
- 6 ea 120-m straights for injection, RF, damping wigglers long IDs, etc.

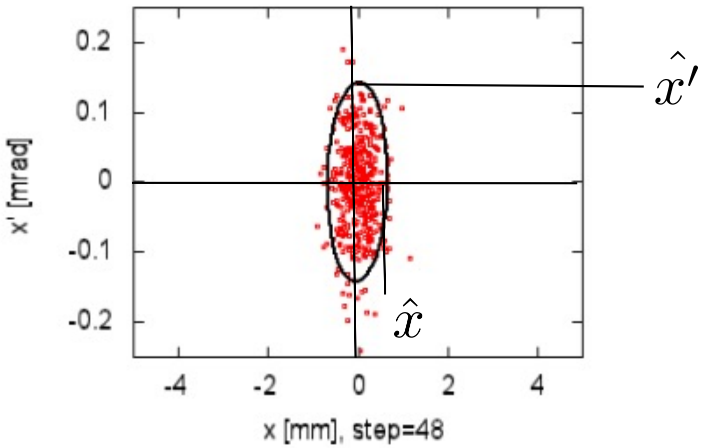
B. Hettel (SLAC) Future Light Sources 2012 Workshop

Small Emittance Drawbacks: Touschek Scattering

- Electrons within the electron bunches in a synchrotron light storage ring do sometimes interact with each other
  - They're all charged particles, after all

- Fortunately most of these interactions are negligible for high energy, ultrarelativistic electron beams
  - $\gamma \gg 1$ so, e.g., time dilation reduces effect of space charge $\propto \gamma^{-2}$
  - But these are long-distance Coulomb repulsions
  - High angle scattering can lead to sudden large momentum changes for individual electrons
  - Low emittance and high brilliance enhances this effect
    - Tighter distributions of particles => more likelihood of interactions
  - Large momentum changes can move electrons out of the stable RF bucket => particle loss

Rough Order of Magnitude



$$\hat{x}' \qquad \hat{x}$$

$$\hat{x} = \sqrt{\mathcal{W}\beta(s)}$$

$$\hat{x}' = \sqrt{\mathcal{W}/\beta(s)}$$

$$\sigma_{\mathrm{RMS}} = \sqrt{\epsilon\beta}$$

$$\sigma'_{\mathrm{RMS}} = \sqrt{\epsilon/\beta}$$

$$\epsilon = \pi\mathcal{W}$$

- For a given particle, $\hat{x} = \beta\hat{x}' = \frac{\beta\hat{p_x}}{p_0} \qquad \hat{p_x} = \frac{p_0\hat{x}}{\beta}$
- If **all** transverse momentum is transferred into δ then

$$\Delta p = \gamma p_x = \gamma\frac{p_0\hat{x}}{\beta}$$

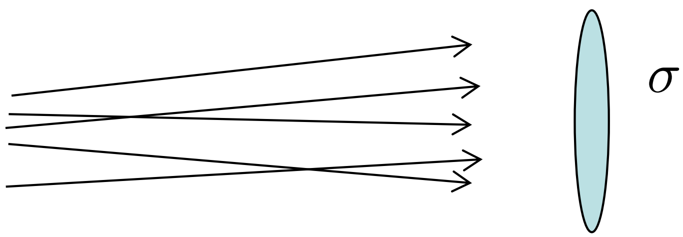
- For realistic numbers of 2 GeV beam (γ~4000), β_x=10m, and $100\mu m$ beam displacement, we find

$$\Delta p \approx 80 \text{ MeV/c} \approx 0.04\, p_0$$

- This scattering mechanism can create electron loss
  - Even worse for particles out in Gaussian tails

Cross Section

- **Cross section** is used in high energy physics to express the probability of scattering processes: units of area



$$1 \text{ barn} \equiv 10^{-28} \text{ m}^2 = 10^{-24} \text{ cm}^2$$

- Often expressed as a **differential cross section**, probably of interaction in a given set of conditions (like interaction angle or momentum transfer): $d\sigma/d\Omega$
- In particle colliders, **luminosity** is defined as the rate of observed interactions of a particular type divided by the cross section $\mathcal{L} \equiv \frac{\text{event rate}}{\sigma}$ units $[\text{s}^{-1}\text{ cm}^{-2}]$

Integrating this over time gives an expected number of events in a given time period to calculate experiment statistics

Touschek Scattering Calculations

- Touschek Scattering calculations use the Moller electron elastic interaction cross section in the rest frame of the electrons
  - Then relativistically boost back into the lab frame
  - This is all too involved for this lecture!
    - Really 2nd year graduate level scattering theory calculation
  - See Carlo Bocchetta's talk at CERN Accelerator School
    - http://cas.web.cern.ch/cas/BRUNNEN/Presentations/PDF/Bocchetta/Touschek.pdf
  - As usual we'll just quote the result
  - Touschek loss exponential decay lifetime

$$V_{\text{bunch}} = 8\pi\sigma_x\sigma_y\sigma_z$$

$$\tau = \frac{\gamma^3 V_{\text{bunch}} \sigma'_{\text{x,RMS}} \delta^2_{\text{acceptance}}}{c r_0^2 N_{\text{bunch}} (\ln(2)\sqrt{\pi})} \frac{1}{C(\epsilon)}$$

$$C(\epsilon) \approx -[\ln(1.732\epsilon) + 1.5]$$

$$\epsilon \equiv \left(\frac{\delta_{\text{acceptance}}}{\gamma\sigma'_{\text{x,RMS}}}\right)^2$$

$$\delta_{\text{acceptance}}: \quad \frac{\Delta p}{p_0} \text{ at which particles are lost}$$

$$r_0 \approx 2.818 \times 10^{-13} \text{ cm}$$

Touschek Scaling

$$\boxed{\tau = \frac{\gamma^3 V_{\text{bunch}} \sigma'_{\text{x,RMS}} \delta^2_{\text{acceptance}}}{c r_0^2 N_{\text{bunch}} (\ln(2)\sqrt{\pi})} \frac{1}{C(\epsilon)}}$$

$$V_{\text{bunch}} = 8\pi\sigma_x\sigma_y\sigma_z$$

$$C(\epsilon) \approx -[\ln(1.732\epsilon) + 1.5]$$

$$\epsilon \equiv \left(\frac{\delta_{\text{acceptance}}}{\gamma\sigma'_{\text{x,RMS}}}\right)^2$$

$\delta_{\text{acceptance}}$: $\dfrac{\Delta p}{p_0}$ at which particles are lost

$$r_0 \approx 2.818 \times 10^{-13}\ \text{cm}$$

- High lifetime is good, low lifetime is bad
  - Higher particle phase space density $N_{\text{bunch}}/V_{\text{bunch}}$ makes loss faster
    - But we want this for higher brilliance!
  - Smaller momentum acceptance makes loss faster
    - But tighter focusing requires sextupoles to correct chromaticity
    - Sextupoles and other nonlinearities reduce $\delta_{\text{acceptance}}$
  - Higher beam energy γ_r makes loss slower
    - Well at least we win somewhere!

Touschek Lifetime Calculations

- Generally one must do some simulation of Touschek losses

Touschek Lifetime Calculations for NSLS-II

B. Nash, S. Kramer, Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

The Touschek effect limits the lifetime for NSLS-II. The basic mechanism is Coulomb scattering resulting in a longitudinal momentum outside the momentum aperture. The momentum aperture results from a combination of the initial betatron oscillations after the scatter and the non-linear properties determining the resultant stability. We find that higher order multipole errors may reduce the momentum aperture, particularly for scattered particles with energy loss. The resultant drop in Touschek lifetime is minimized, however, due to less scattering in the dispersive regions. We describe these mechanisms, and present calculations for NSLS-II using a realistic lattice model including damping wigglers and engineering tolerances.[1]

INTRODUCTION

LINEAR AND NON-LINEAR DYNAMICS MODELING

NSLS-II has a 15-fold periodic DBA lattice. The lattice functions for NSLS-II are shown in Figure 1. The linear lattice results in the equilibrium beam sizes around the ring that enter into Eqn. (1). Non-linear dynamics enter through the parameter $\delta_{\rm acc}(s)$. This is the maximum momentum change that a scattered particle can endure before it is lost. There are two elements to this stability question. The first is the amplitude of the initial orbit which comes from the off-momentum closed orbit (dispersion) and beta functions. These are shown in Figures 2 and 3. The amplitude of the induced betatron oscillation following a scatter with relative energy change $\delta = \frac{\Delta E}{E_0}$ is given by

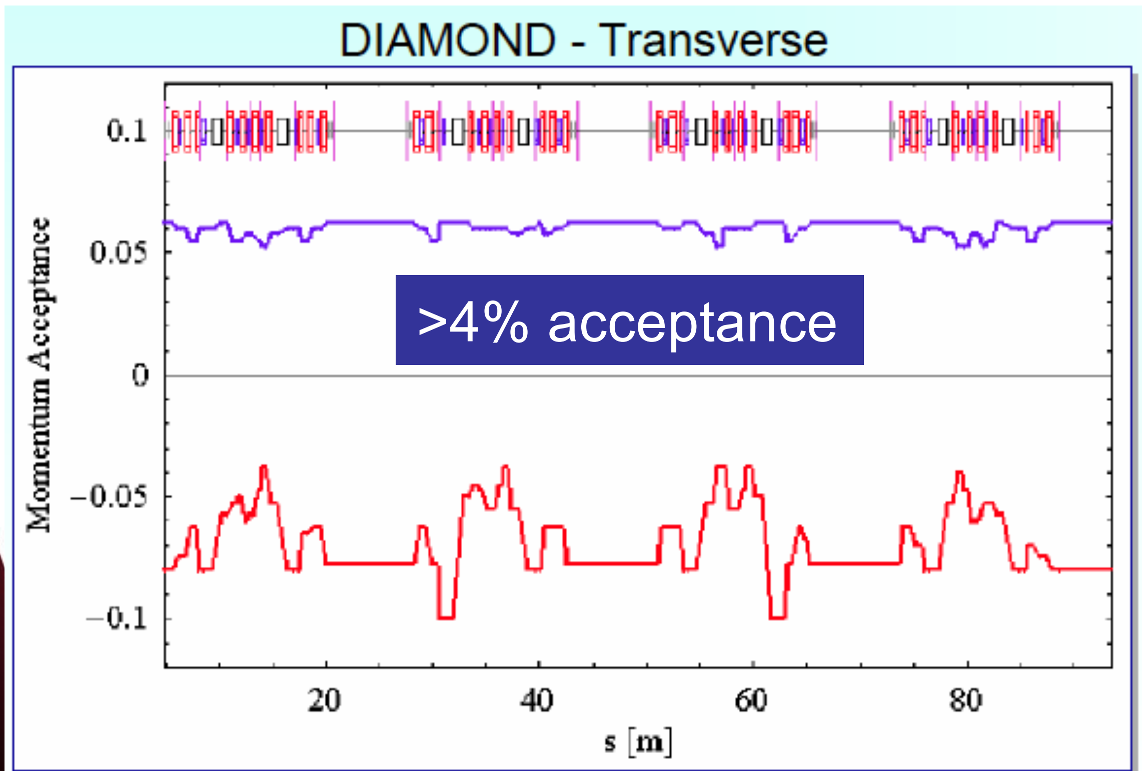
$$x_2 = (\eta^{(1)}(s_2) + \sqrt{\mathcal{H}(s_1)\beta_x(s_2)})\delta + \eta^{(2)}(s_2)\delta^2 \quad (3)$$

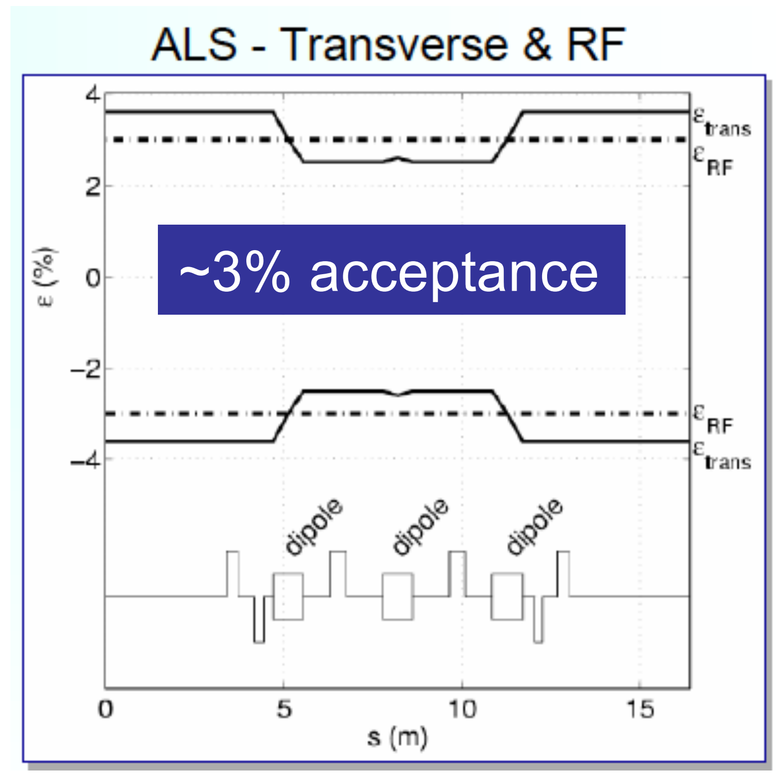
where $\mathcal{H} = \gamma_x\eta_x^2 + 2\alpha_x\eta_x\eta_x' + \beta_x\eta_x'^2$ is the dispersion in-

PAC'09 Conference: http://www.bnl.gov/isd/documents/70446.pdf

Momentum Aperture and Touschek

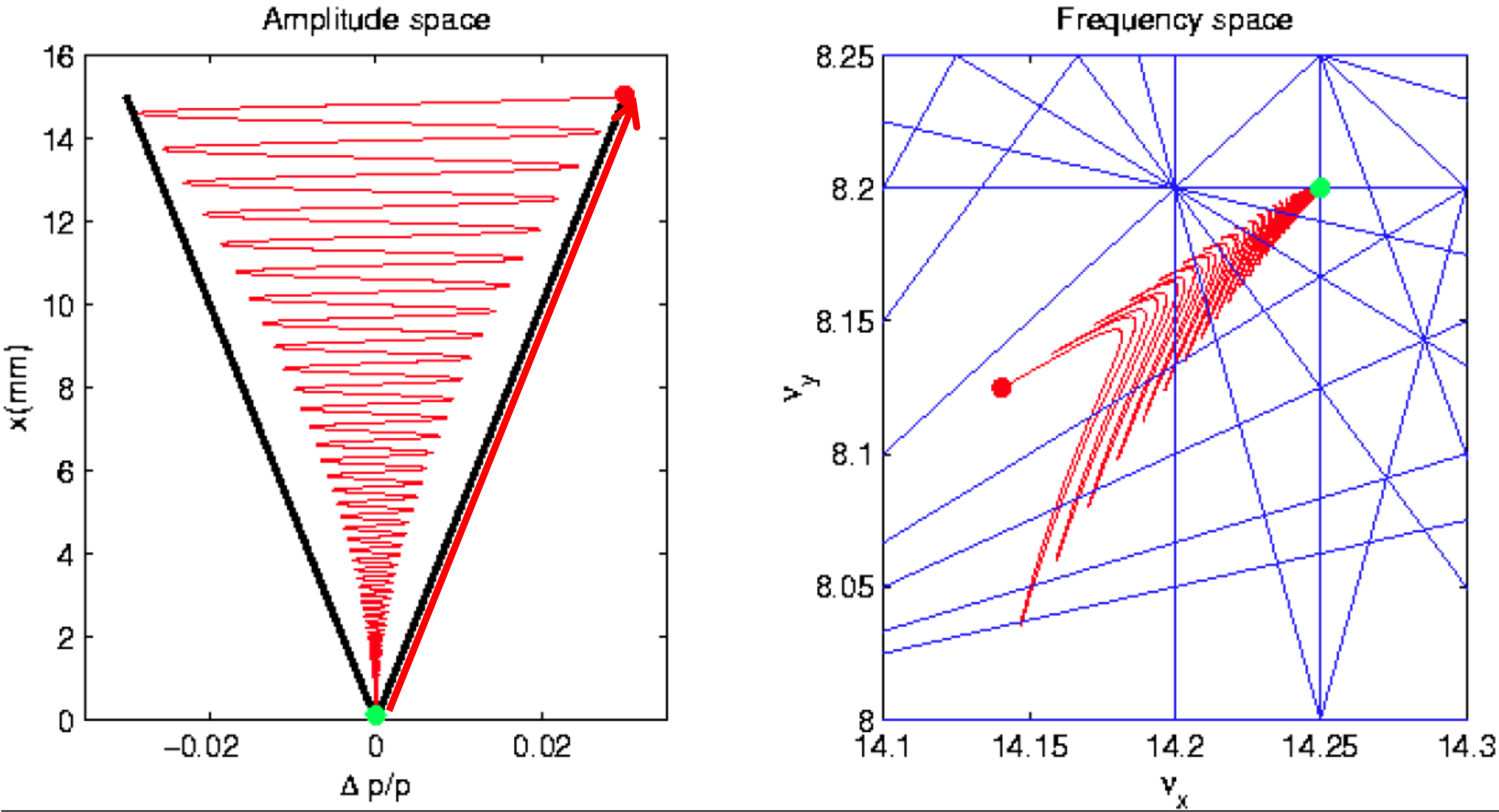
- Most third generation storage rings have limiting transverse acceptance
  - Much work to optimize transverse momentum aperture
  - Particularly modern machines (e.g. DIAMOND, SOLEIL)
  - Detailed nonlinear dynamics measurements required





Carlo Bocchetta's talk at CERN Accelerator School

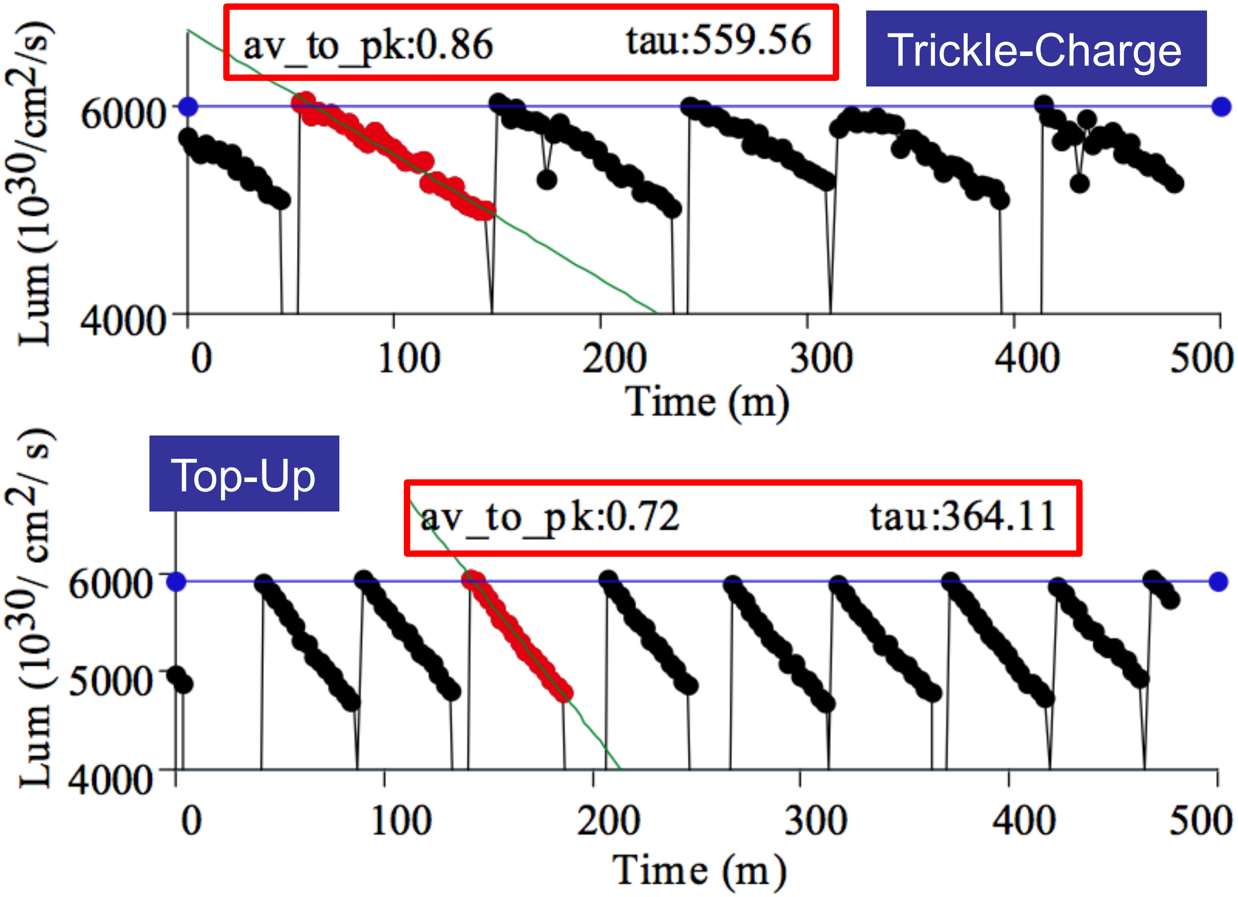
Kicked Electron Damping



- After a Touschek kick, electrons damp again
  - But they move through tunes and amplitudes in complicated way
  - Will see more of "tune space" and resonances tomorrow

D. Robin, ALS

Top-Up and Trickle-Charge



- Top-up: add beam at discrete times to "top-up" beam current
  - Turn off detectors during top-up, dominated by beam lifetime
- Trickle-charge: add small trickle of beam continuously
  - Dominated by injection jitter detector trips, other injector stability

J. L. Turner et al, "Trickle-Charge: A New Operational Mode for PEP-II", SLAC-PUB-11175