USPAS Accelerator Physics 2024
Hampton VA / Northern Illinois University

Chapter Chao (and Peggs/Satogata 14.5)
(Brief) Intro to (Longitudinal) Instabilities

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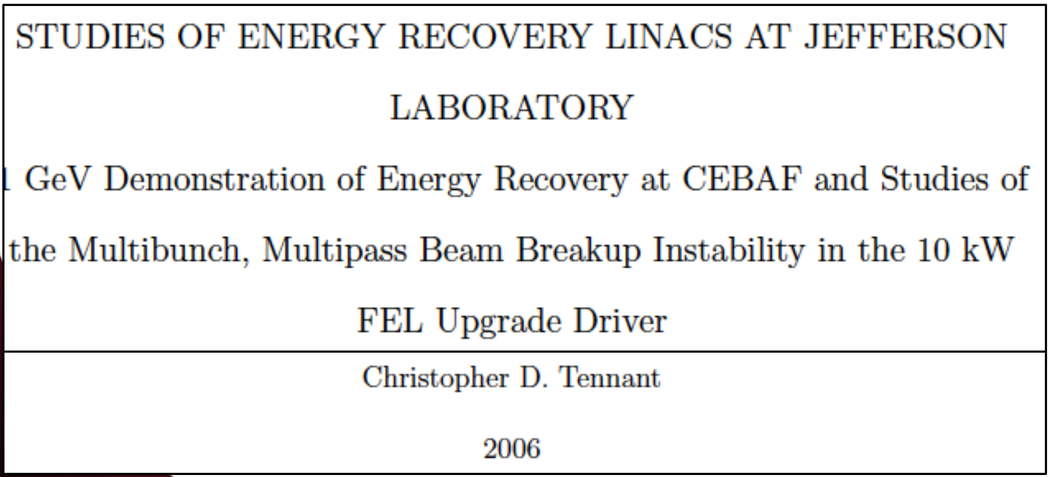
http://www.toddsatogata.net/2024-USPAS

Happy birthday to Irving Langmuir (Nobel 1932) and Rudolf Mossbauer (Nobel 1961)!
Happy National Hot Chocolate Day, sdrawkcaB Day, and Hell Is Freezing Over Day!

Overview

- Thanks to Medani for introduction to impedances!

- Chapter 14.5+: Beam Breakup
  - Introduction: Impedances and instabilities
  - Review: RF higher order modes
  - Linear accelerator beam breakup
    - Amelioration: BNS damping
  - Regenerative (or ERL) beam breakup

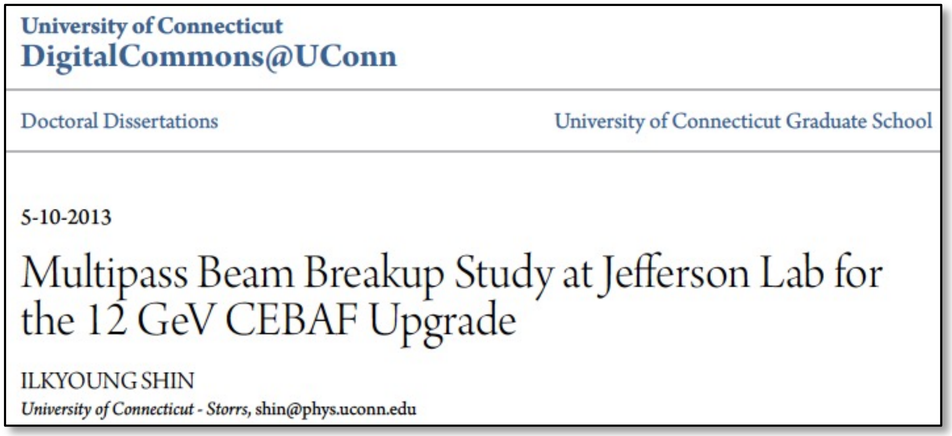


STUDIES OF ENERGY RECOVERY LINACS AT JEFFERSON LABORATORY

GeV Demonstration of Energy Recovery at CEBAF and Studies of the Multibunch, Multipass Beam Breakup Instability in the 10 kW FEL Upgrade Driver

Christopher D. Tennant

2006



University of Connecticut
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Doctoral Dissertations

University of Connecticut Graduate School

5-10-2013

Multipass Beam Breakup Study at Jefferson Lab for the 12 GeV CEBAF Upgrade

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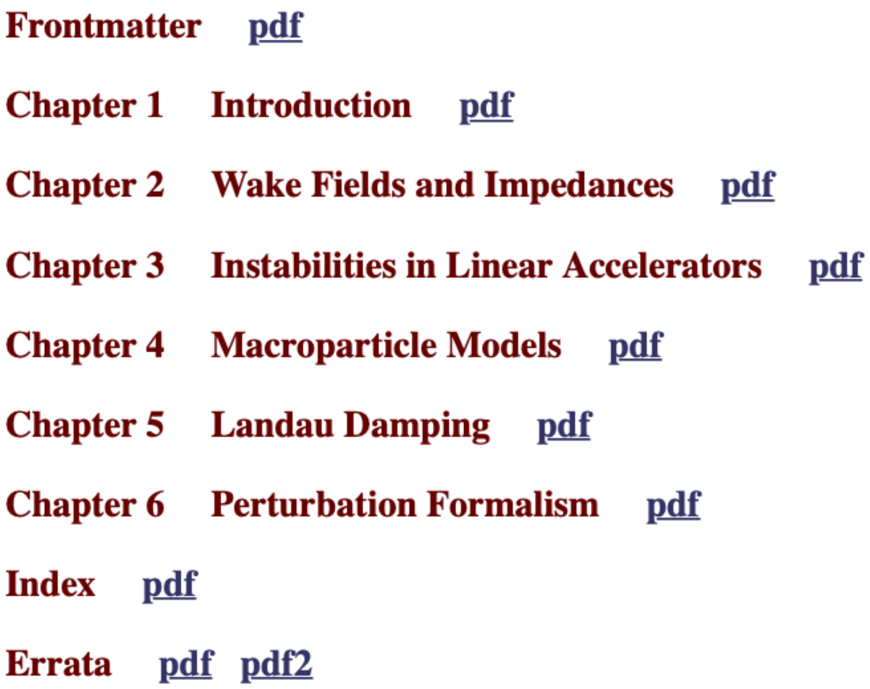
Introduction: Impedances and Instabilities

- Until now we have mostly only considered beam dynamics as affected by externally-generated EM fields
  - Single-particle dynamics
  - Beam fields dominated by external fields
- But *by definition* the beam has its own EM fields
  - Beam acts like a (time-dependent) current
  - These fields interact with their environment
    - In particular, they interact with resonators in the environment
    - These resonators look very much like impedances in circuits
  - Fields created by the beam can act back upon the beam
  - These fields created in the beam's wake are called "wakefields"
- These interactions create *feedback loops*
  - Like any feedback, this can create unstable feedback loops

Entire Textbooks on Impedances/Instabilities

- My canonical book on this subject is Alex Chao’s textbook
- Free on the web at
  - https://www.slac.stanford.edu/~achao/wileybook.html



Frontmatter pdf

Chapter 1 **Introduction** pdf

Chapter 2 **Wake Fields and Impedances** pdf

Chapter 3 **Instabilities in Linear Accelerators** pdf

Chapter 4 **Macroparticle Models** pdf

Chapter 5 **Landau Damping** pdf

Chapter 6 **Perturbation Formalism** pdf

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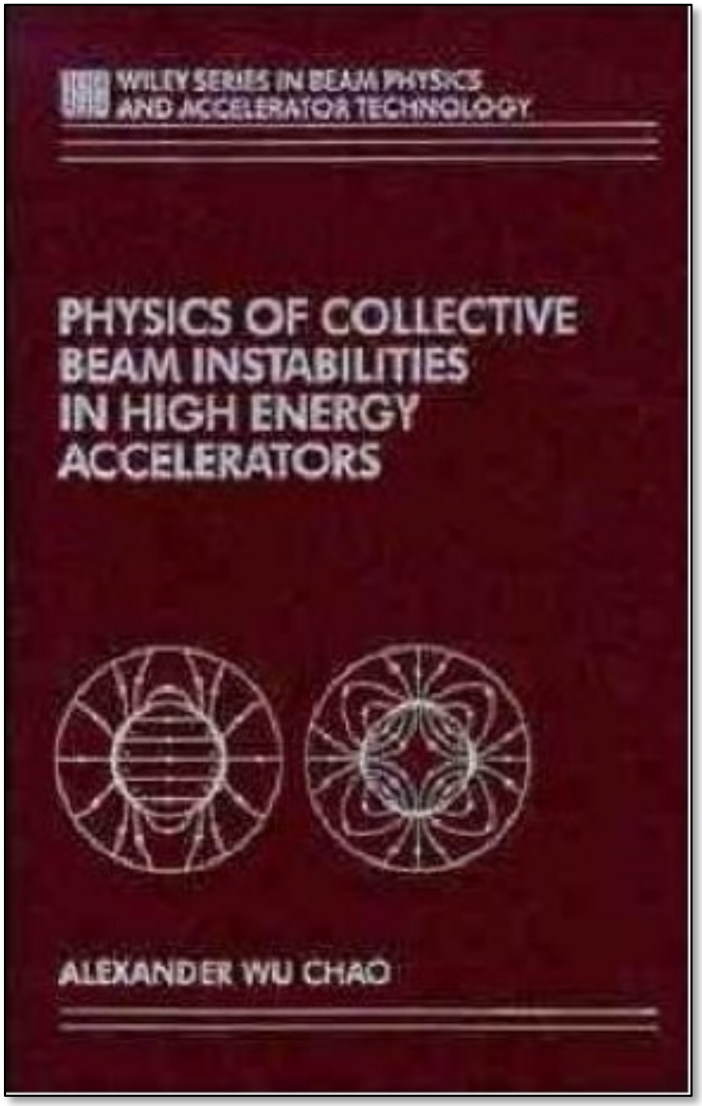

My first USPAS class (1990) with Alex was the basis of this book

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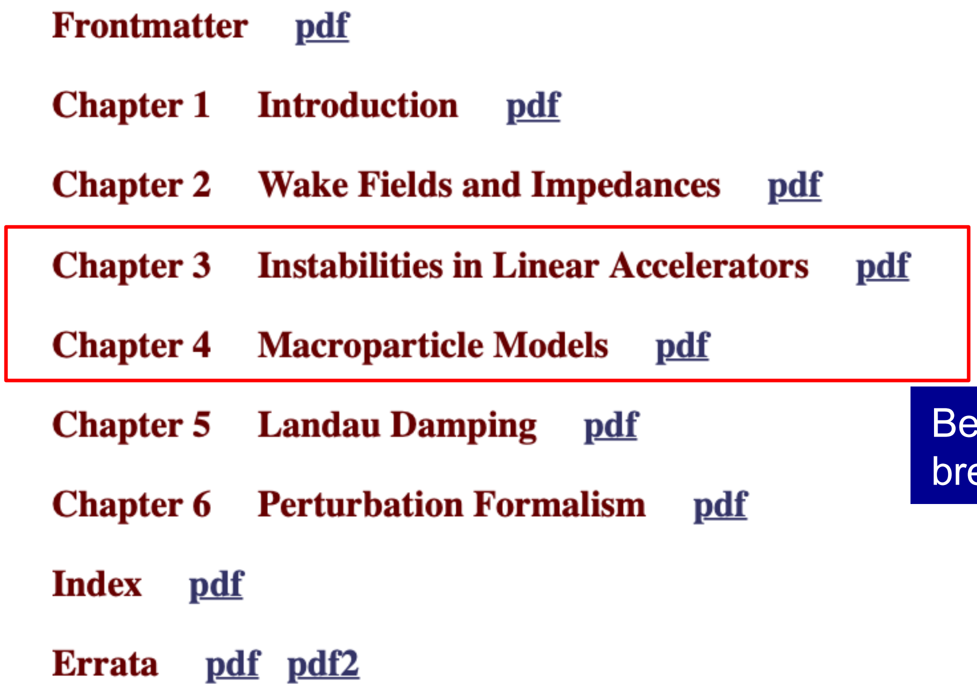

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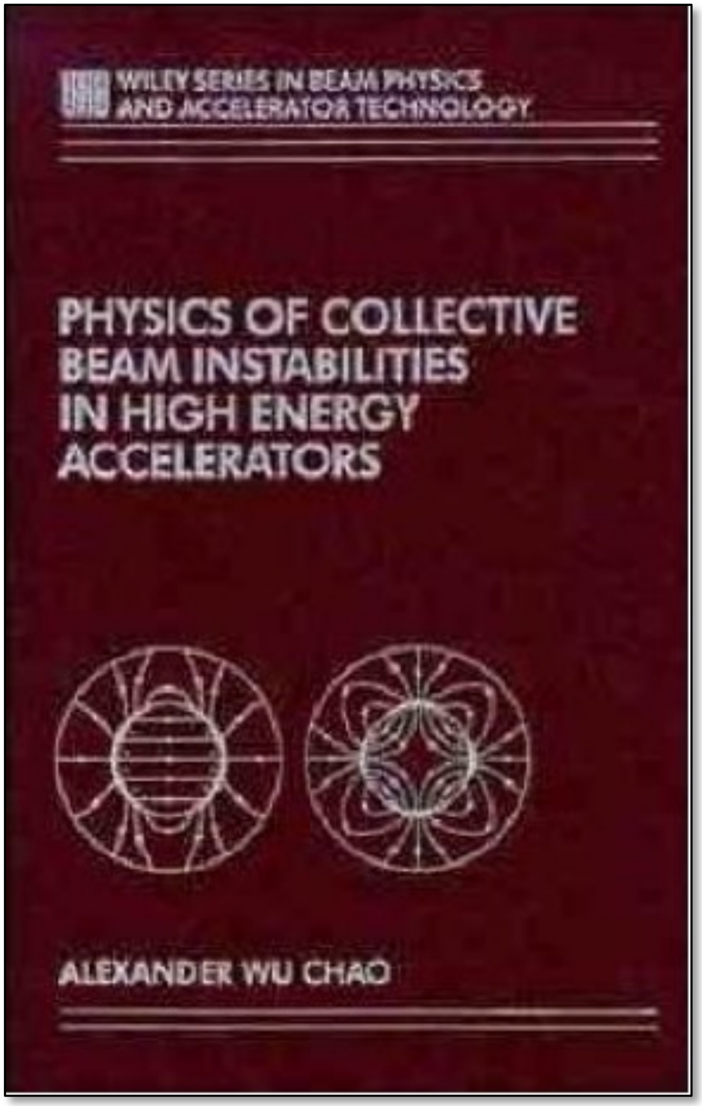
Beam breakup



WILEY SERIES IN BEAM PHYSICS AND ACCELERATOR TECHNOLOGY

PHYSICS OF COLLECTIVE BEAM INSTABILITIES IN HIGH ENERGY ACCELERATORS

ALEXANDER WU CHAO

My first USPAS class (1990) with Alex was the basis of this book

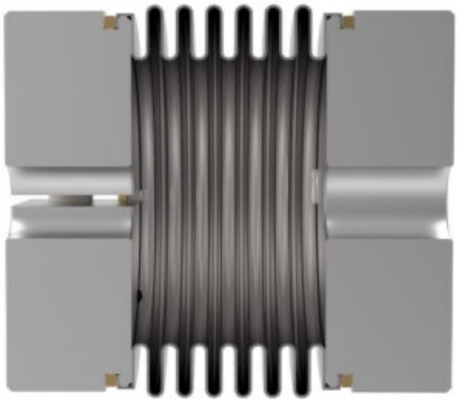
Impedances (Thanks Medani)

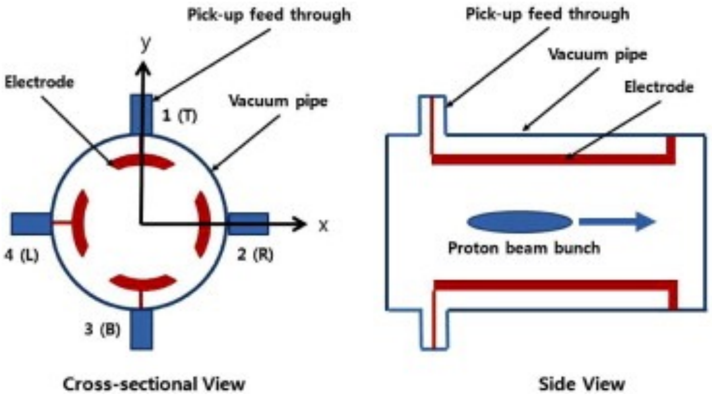
- Any structure that interacts with the beam fields can be treated as an impedance
  - Fortunately many of them are bad high frequency resonators (low Q)
    - Wakefields do not persist a long time compared to time between bunches, or even bunch length
    - Damp quickly in residual resistance of conductors
- Examples of impedances that can contain wakefields
  - Beam pipe
  - Bellows

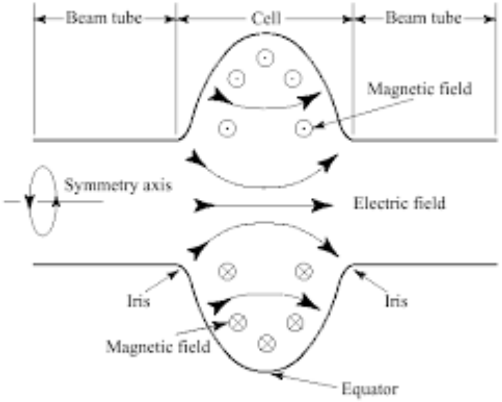
Diagnostics

RF cavities

high Q by design!

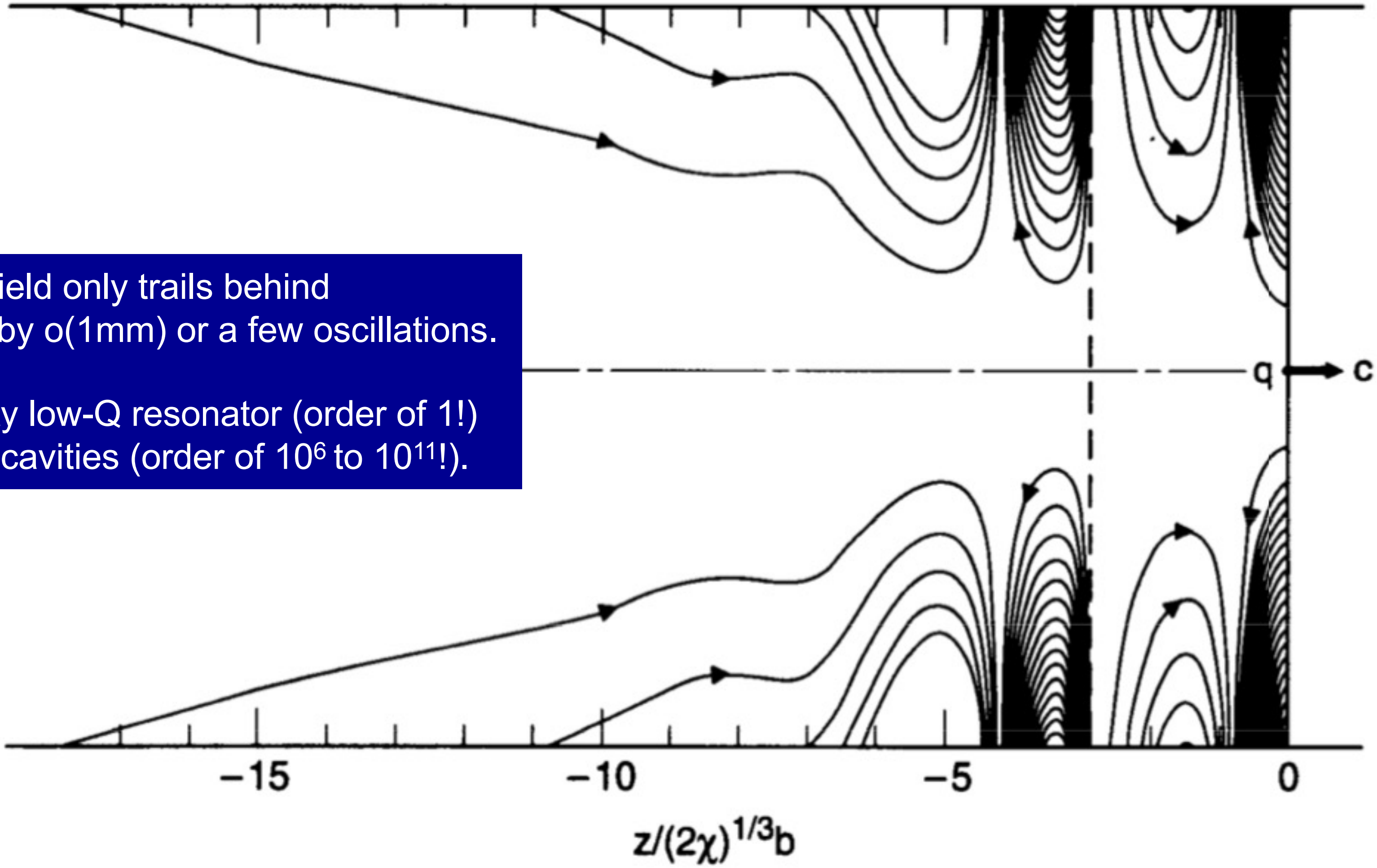






Digression on Q: http://toddsatogata.net/2017-USPAS/ResonantDrivenOscillator.pdf

Beam Pipe Wakefield



Wakefield only trails behind beam by o(1mm) or a few oscillations.

A pretty low-Q resonator (order of 1!) vs RF cavities (order of 10^6 to 10^{11}!).

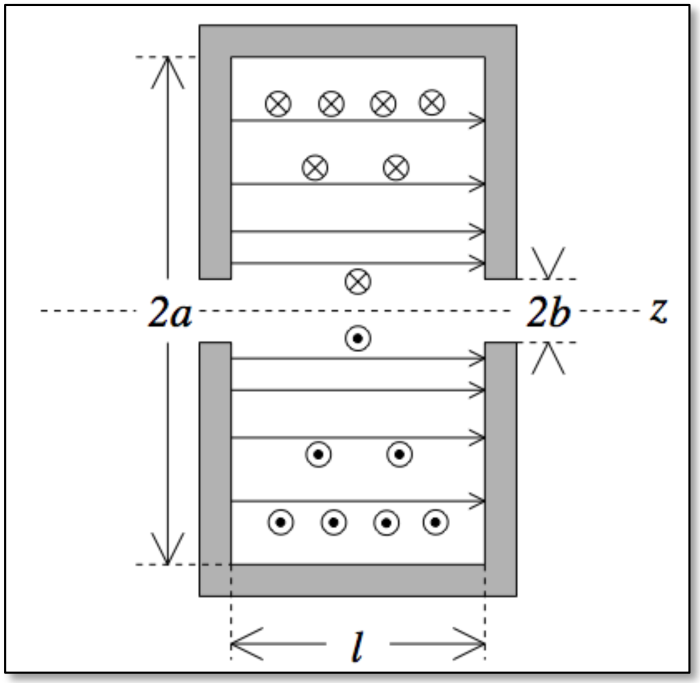
Figure 2.3. Wake electric field lines in a resistive wall pipe generated by a point charge q. The field pattern shows oscillatory behavior in the region $|z| \lesssim 5(2\chi)^{1/3}b$ (or $|z| \lesssim 0.35$ mm for an aluminum pipe with $b = 5$ cm). The field line density to the left of the dashed line has been magnified by a factor of 40. (Courtesy Karl Bane, 1991.)

From Chao textbook: https://www.slac.stanford.edu/~achao/wileybook.html

Review: RF Cavity Modes

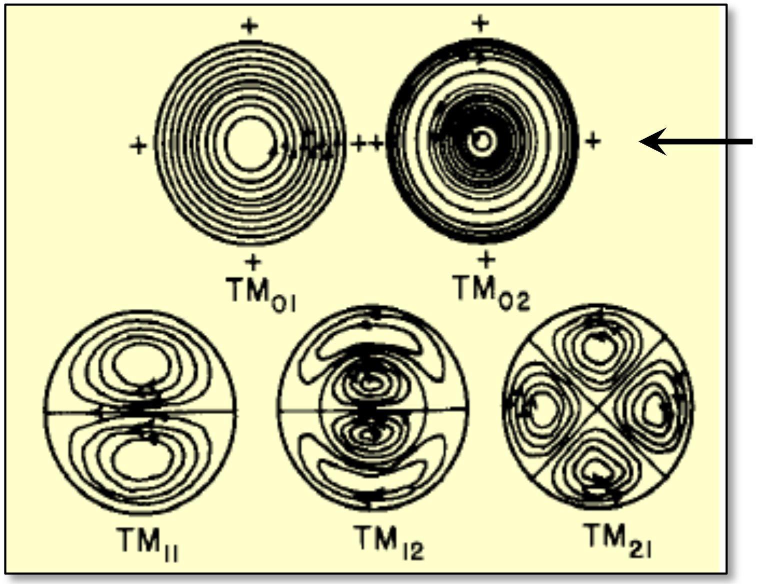
- Currents couple to RF cavity modes through electric field
  - We therefore restrict our discussion to TM$_{njl}$ modes
    - These are the RF modes that have longitudinal electric field

$$E_z(r,\theta,z) \sim J_n(k_c r)(C_1 \cos n\theta + C_2 \sin n\theta) \cos\left(\frac{m\pi z}{l}\right)$$



$$H_z = 0 \text{ everywhere}$$

$$J_n(k_c a) = 0 \qquad X_{nj} \text{ is the j}^{\text{th}} \text{ root of } J_n$$



TM$_{0j}$ are only modes with E field on axis. Radial magnetic forces.

BBU Mechanism: TM110 mode

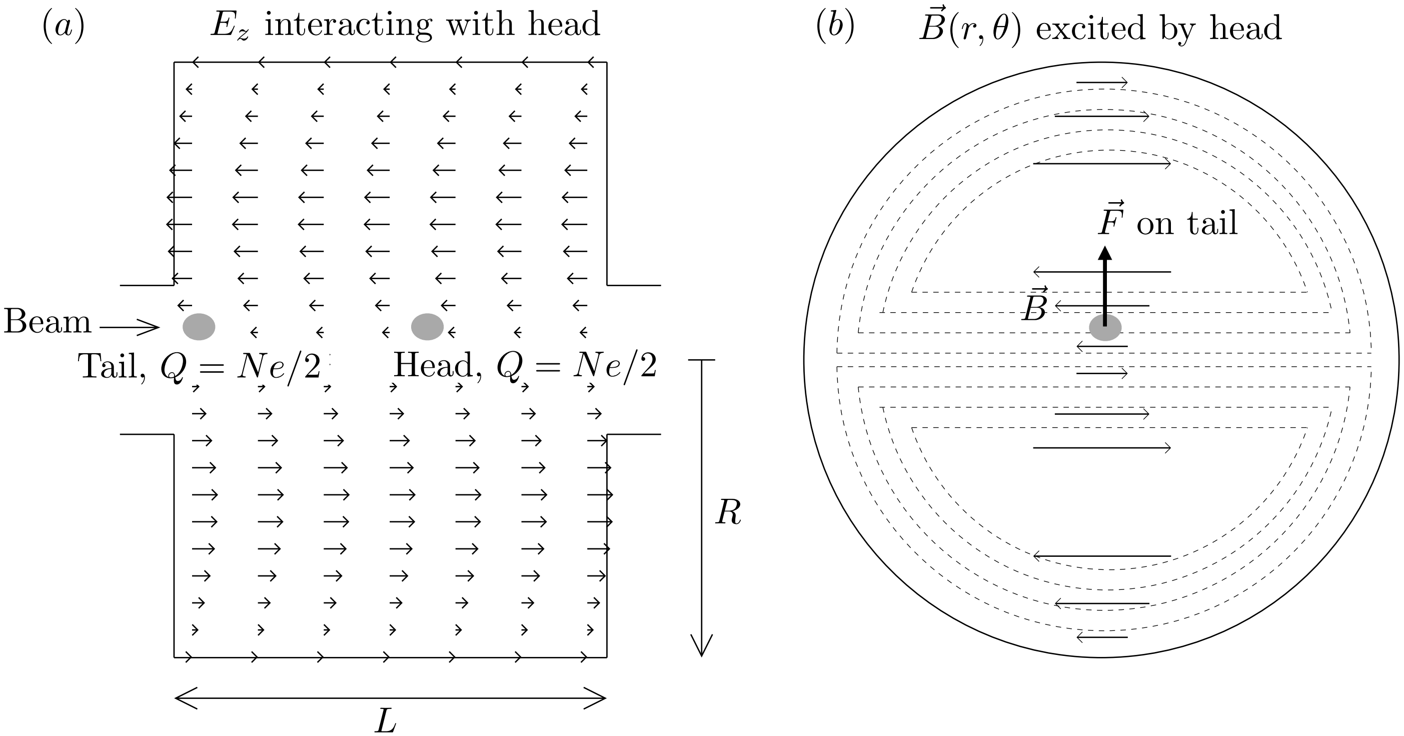
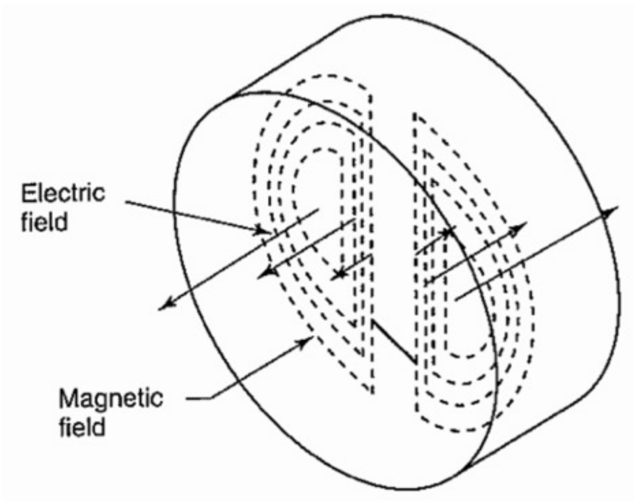
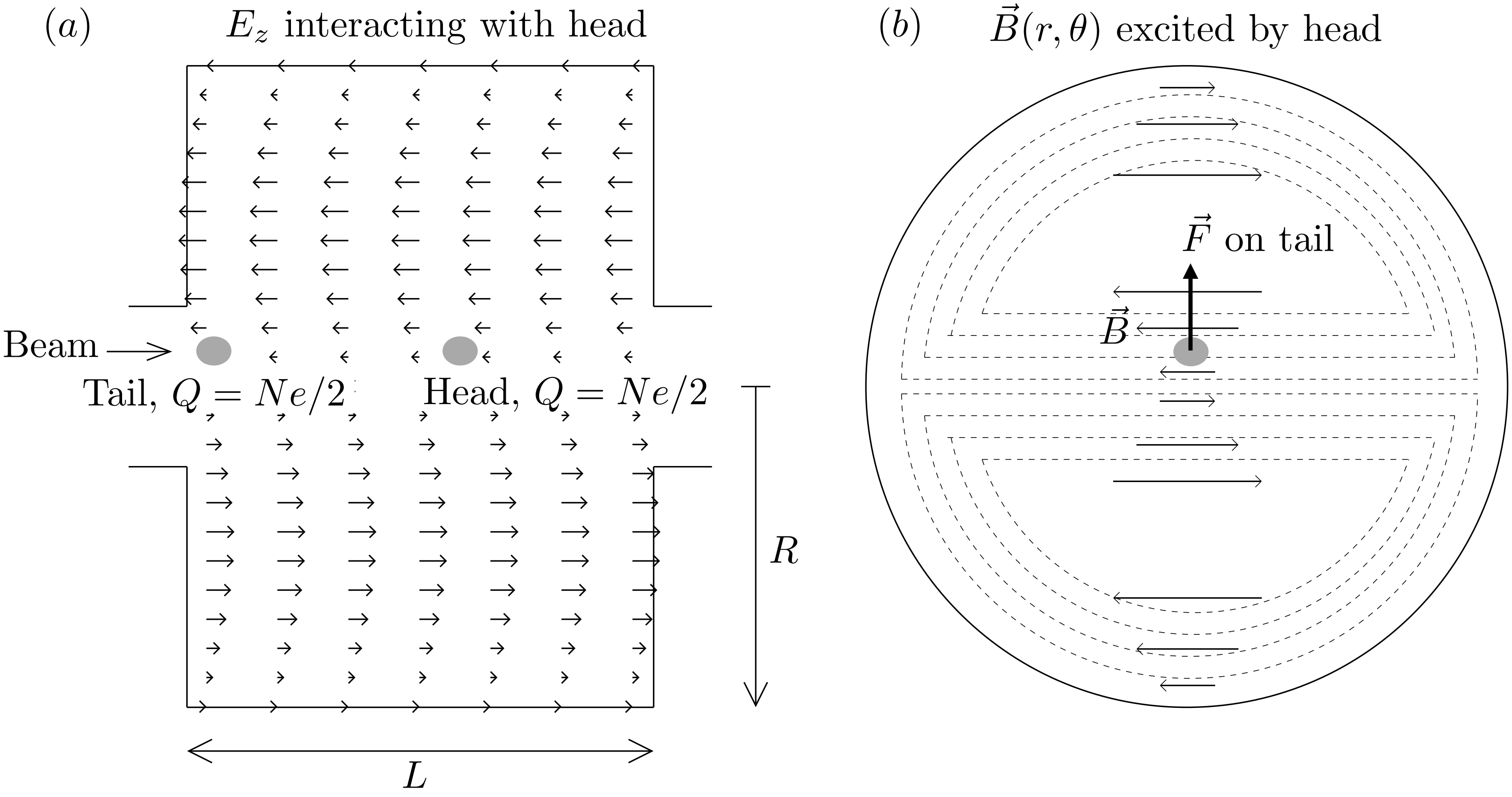


Figure 14.6



- Beam breakup RF cavity HOM
    - TM110 mode shown here: illustrates mechanism
- High Q HOM modes are most dangerous
    - Deposited power rings for longer time
    - More chance for unstable feedback loop with later beam

14.5: HOMs, BBU and BNS Damping



- Passing electron bunches can have transverse displacement
  - Interact with HOMs and deposit energy in cavity
  - Later beam at right phases can add energy constructively
  - Eventually field gets large enough to trip beam or trip RF control

Wakefield Formalism

- From Chao, the impulses (integrated forces) from a general order *m* wakefield are given by

$$\int_{-L/2}^{L/2} ds\, \vec{F}_\perp = -eI_m W_m(z) m r^{m-1} (\hat{r} \cos m\theta - \hat{\theta} \sin m\theta),$$

$$\int_{-L/2}^{L/2} ds\, F_\parallel = -eI_m W_m'(z) r^m \cos m\theta, \qquad (2.50)$$

- Here we only consider the first nontrivial transverse force wakefield with *m*=1:

$$\int_{-L/2}^{L/2} ds\, \vec{F}_\perp = -e\, I\, W_1(z)\, \hat{x} = -N\, e^2\, W_1(z)\, \hat{x}$$

- Averaging the integral over one cavity RF period gives

$$\vec{F}_{\perp,\mathrm{ave}} = -\frac{Ne^2}{2L}\, W_1(z)\, \hat{x}$$

$$F = \frac{dp_x}{dt} = \frac{dp_x}{ds}\frac{ds}{dt} = \frac{dp_x}{ds}\beta c \quad \Rightarrow \quad \frac{dp_x}{ds} = \frac{F}{\beta c}$$

$$x'' = \frac{dx'}{ds} = \frac{d}{ds}\frac{p_x}{p_0} = \frac{dp_x}{ds}\frac{1}{p_0} = \frac{1}{p_0}\frac{F}{\beta c} = \frac{F}{\beta^2 \gamma m c^2} \approx \frac{F}{E}$$

14.5: BBU Formalism

$$x_{\text{head}}(s) = \hat{x}\cos(k_\beta s)$$

Regular betatron motion

Tail betatron motion is a driven harmonic oscillator:

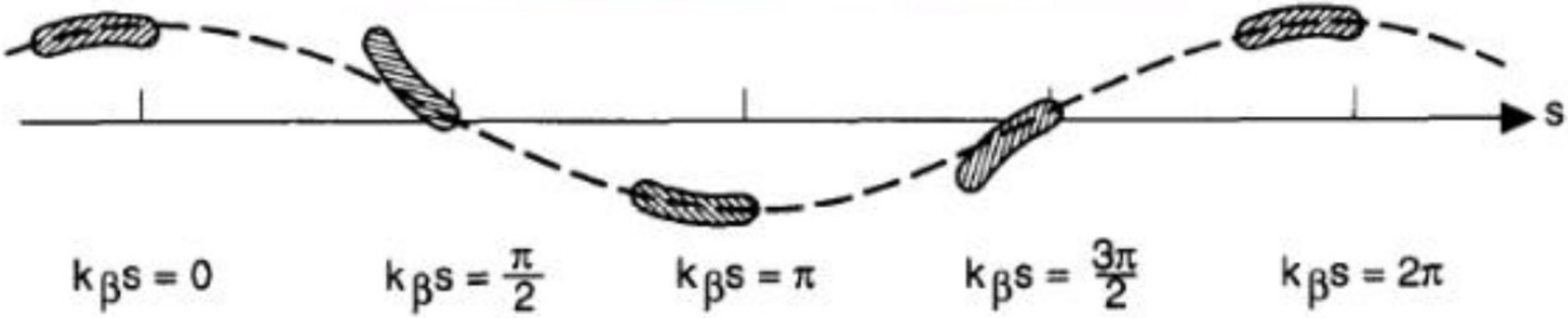
$$x''_{\text{tail}}(s) + k_\beta^2\, x_{\text{tail}}(s) = -\left(\frac{Ne^2 W_1(z)}{2EL}\right)\hat{x}\cos(k_\beta s) \qquad (14.23)$$

Solution of driven harmonic oscillator:

$$x_{\text{tail}}(s) = \hat{x}\cos(k_\beta s) - \left(\frac{Ne^2 W_1(z)}{4k_\beta EL}\right) s\,\hat{x}\sin(k_\beta s) \qquad (14.24)$$

- The tail sees an additional force from the RF wakefields driven by particles in the front (“head”) of the bunch
    - Solution grows linearly with s coordinate = distance along linac
    - A big problem for long linacs like the SLC
    - Establishes tight tolerance on beam alignment in RF cavities

BBU Illustrated



Tail amplitude grows over traversal of many cavities

From Chao textbook: https://www.slac.stanford.edu/~achao/wileybook.html

14.5: BBU Amelioration via BNS

- Plugging in realistic numbers (e.g. for SLAC linac)
  - Beam centering tolerance in RF cavities smaller than beam size! A tough sell!
- Instead, accelerate beam a little off crest
  - Tail has less energy than head, gets more betatron focusing

$$x''_{\text{tail}}(s) + (k_\beta + \delta k_\beta)^2 \, x_{\text{tail}}(s) \;=\; -\left(\frac{Ne^2 W_1(z)}{2EL}\right) \hat{x} \cos(k_\beta s) \qquad (14.25)$$

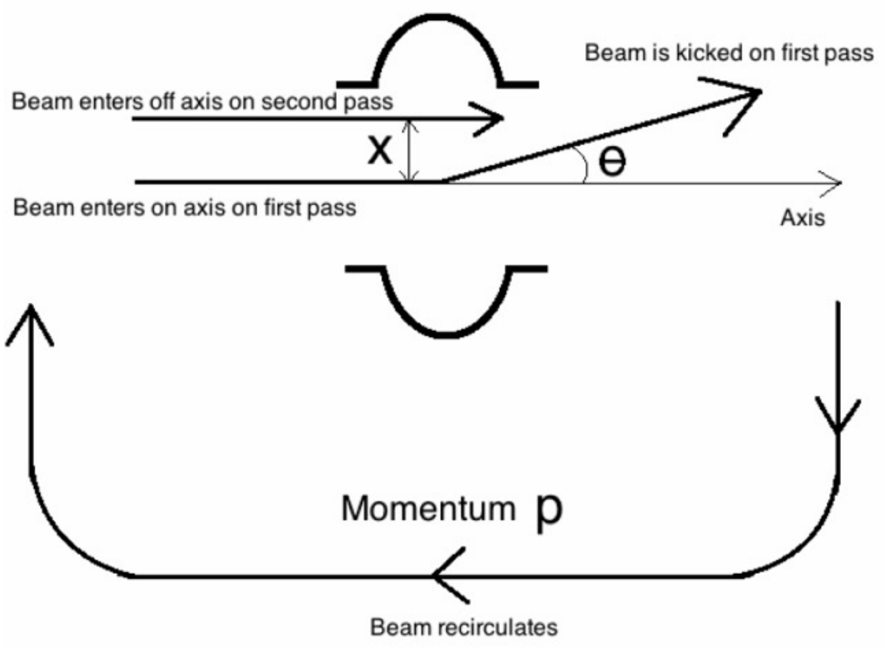
  - This can be safely kept stable if the additional betatron focusing is enough to satisfy the condition

$$\delta k_\beta \;=\; -\frac{Ne^2 W_1(z)}{4\, k_\beta\, EL}$$

  - This is called BNS damping, and it “rescued” the SLC http://accelconf.web.cern.ch/AccelConf/p85/PDF/PAC1985_2389.PDF
  - https://cds.cern.ch/record/2751715/files/10.23732_CYRCP-2020-009.68.pdf
    - BNS: Balakin, Novokhatski and Smirnov (Novosibirsk LC study)

Regenerative Beam Breakup (BBU)

- Regenerative beam breakup
  - Positive feedback loop between beam power and higher order mode RF power
  - Couples through beam transport
  - Many RF higher order modes communicate with beam, each other in near-exponential complexity
  - Limits total beam current



Ilkyoung Shin dissertation

14.5: Regenerative BBU

The threshold current for horizontal regenerative beam breakup is

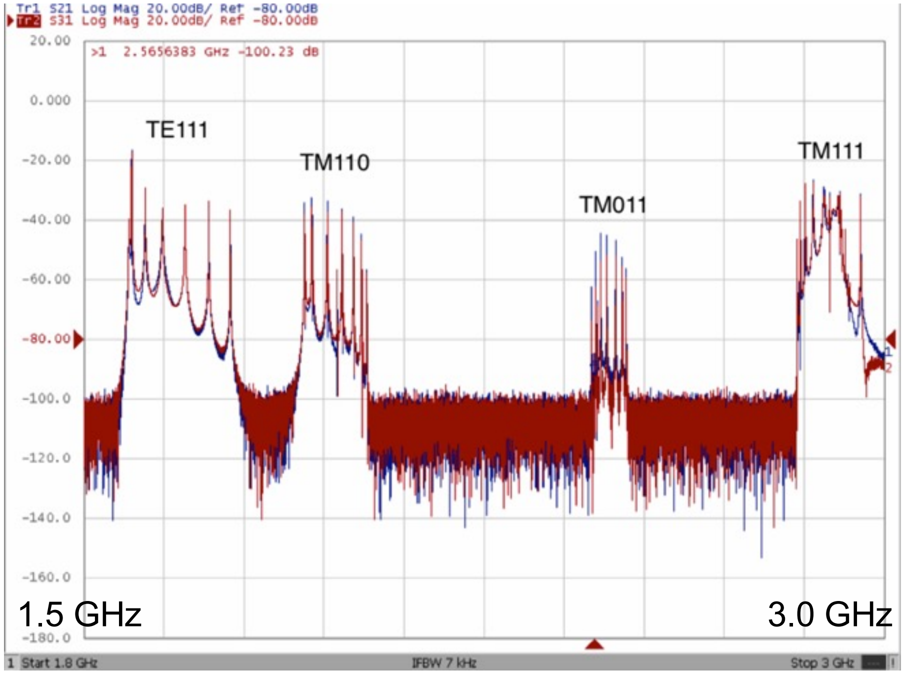
$$I_{\text{th}} = -\frac{2Ec}{e\omega\,(R/Q)Q\,M_{12}\,\sin(\omega T)} \qquad (14.28)$$

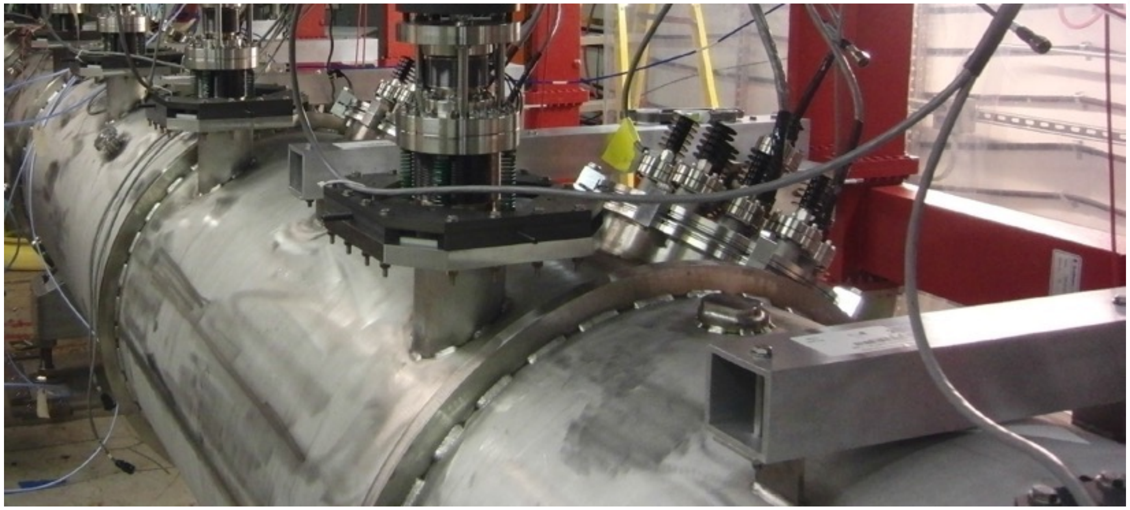
where E is the total beam energy, ω is the HOM frequency, Q and (R/Q) are properties of the HOM, M_{12} is the recirculation beam transport matrix element, and T is the recirculation time [45]. Instability is only possible if

$$M_{12}\,\sin(\omega T) < 0 \qquad (14.29)$$

Otherwise the feedback loop damps the HOM, and the beam is predicted to remain stable at all currents. Regenerative BBU can be avoided for some HOMs by careful control of M_{12}. However, $\omega T \gg 1$ in realistic systems so stability cannot be assured for all HOMs.

BBU Measurements: CEBAF C100 Warm HOM Loads





- C100 HOM, BBU experiment: Ilkyoung Shin’s JLab PhD thesis
- Surveyed HOMs using warm coupler ports in CMTF, tunnel
  - With and without beam loading, varying recirculation optics
- Based on techniques described in Chris Tennant’s JLab thesis
- HOM power and BBU measurements are accessible