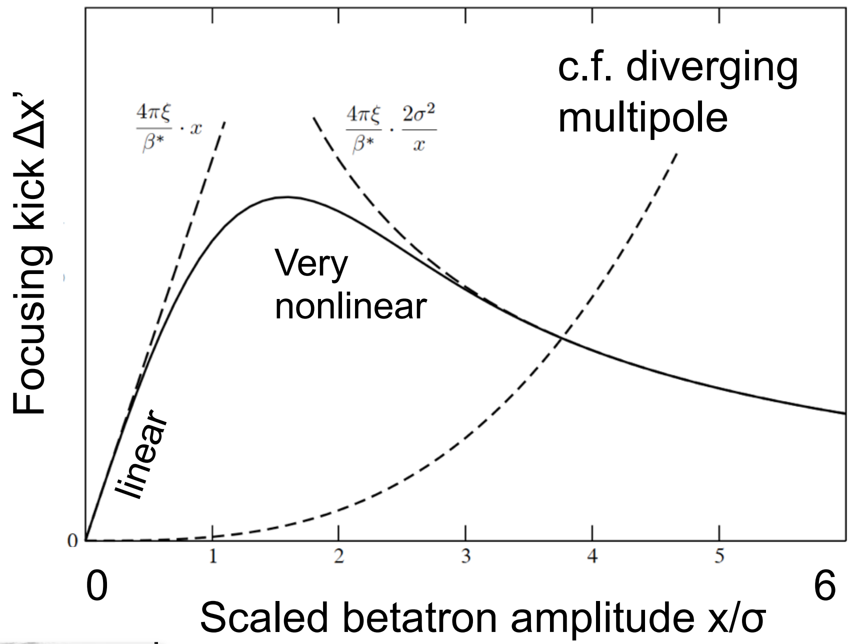
USPAS Accelerator Physics 2024
Hampton VA / Northern Illinois University

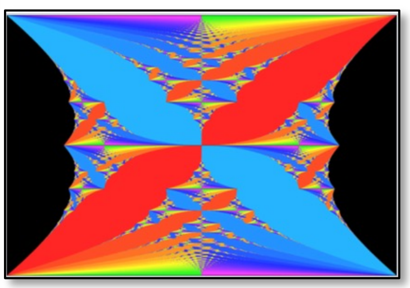
16: Routes to Chaos
(perhaps yet another self-referential lecture)

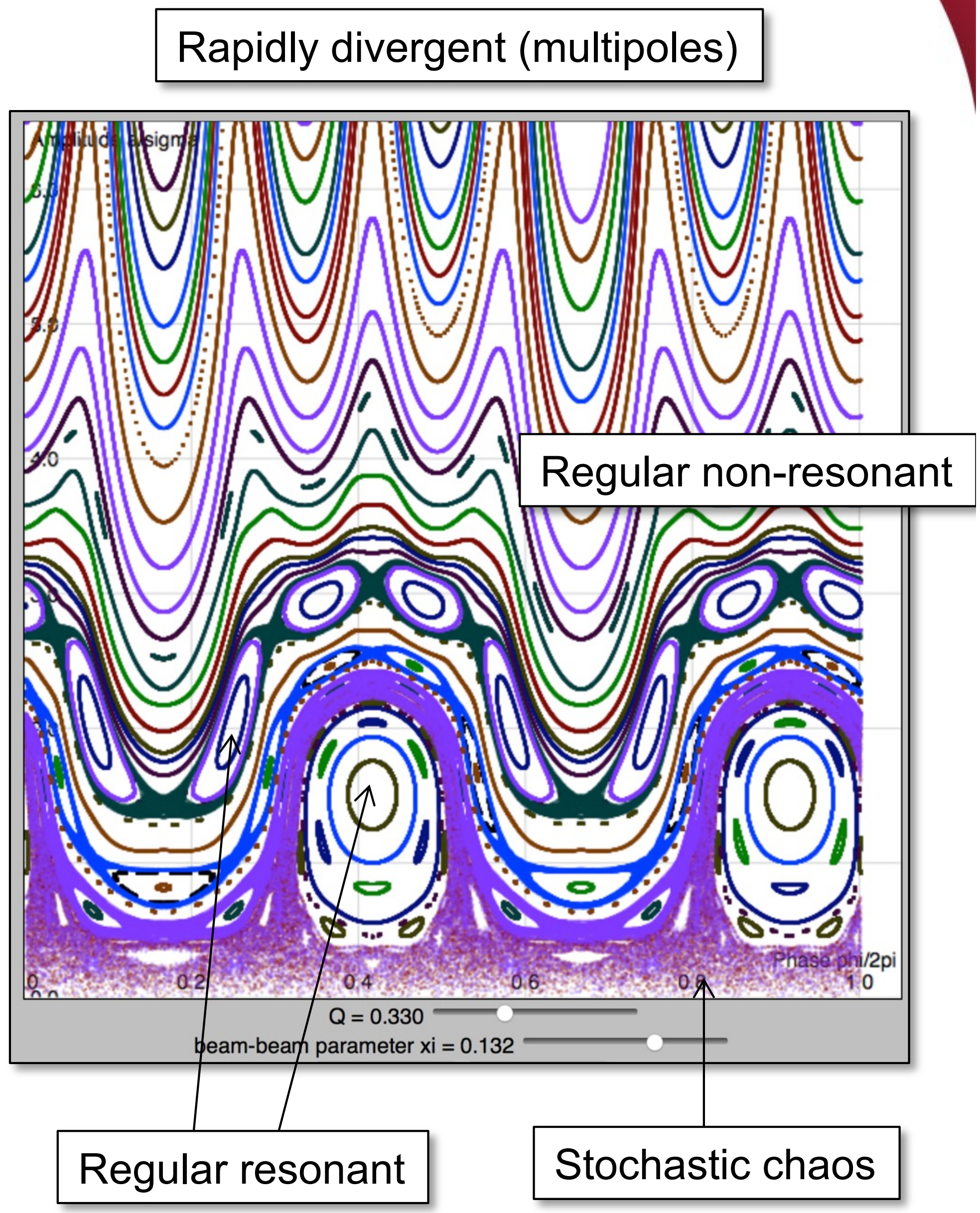
Todd Satogata (Jefferson Lab and ODU) / satogata@jlab.org
Steve Peggs (BNL) / peggs@bnl.gov
Medani Sangroula (BNL) / msangroul@bnl.gov and Alex Coxe / alexcoxe@jlab.org
http://www.toddsatogata.net/2024-USPAS

Happy birthday to Emilio Segre (1959 Nobel), Fritjof Capra, and Harry Styles!
Happy National Baked Alaska Day, National Freedom Day, and Car Insurance Day!

Review: 1D Beam-Beam







D. Hofstadter: "*... an eerie type of chaos ... just behind a facade of order – and yet, deep inside the chaos lurks an even eerier type of order.*"

Review: 1D Beam-Beam

- 1D beam-beam dynamics are surprisingly tractable
  - Can predict where **isolated** resonances are located
    - Steve's lecture and lab 2
  - Can predict **other isolated resonance properties**
    - N-turn Hamiltonians give "island tunes", resonance widths

Fig. 15.7

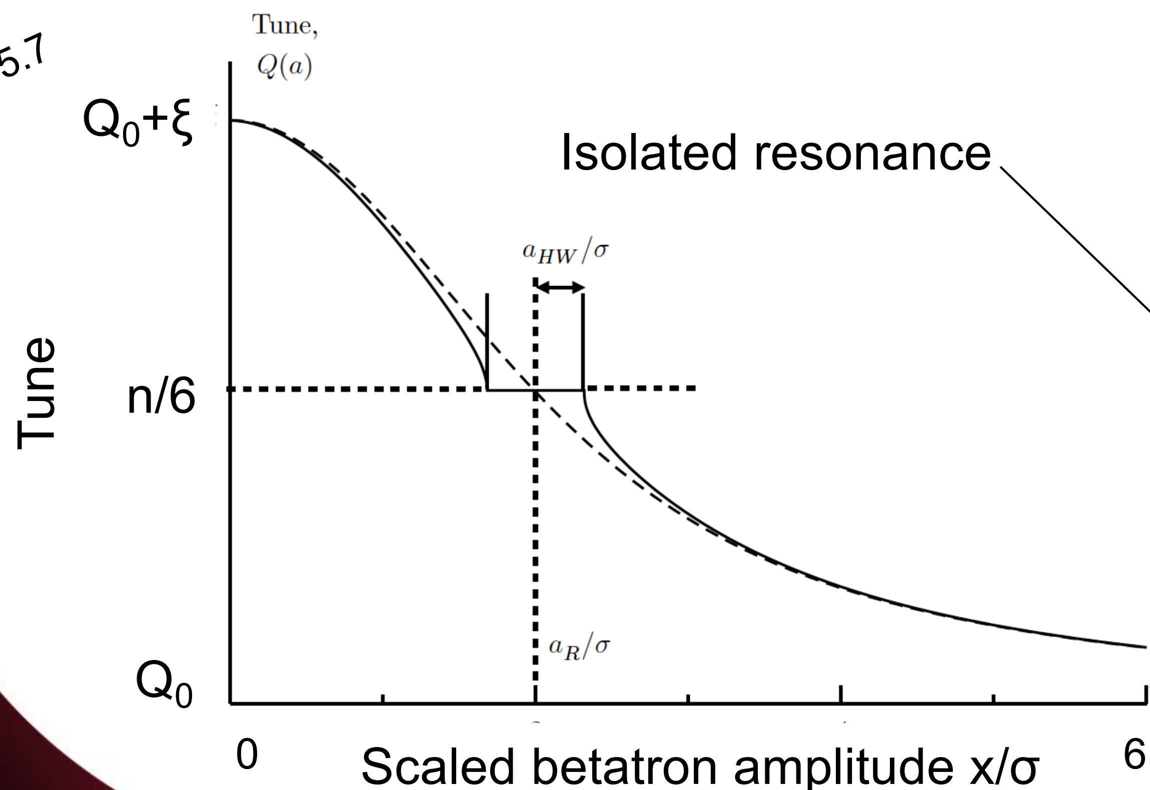
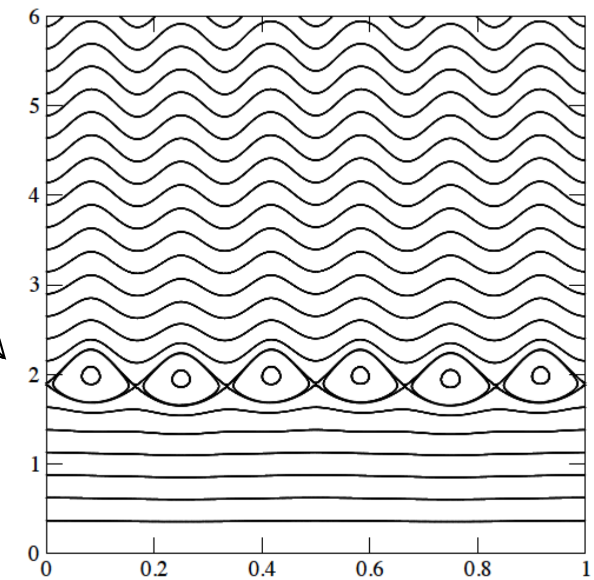


Fig. 15.4



Can we predict the onset of chaos, even roughly?

Review: 1D Beam-Beam

- 1D beam-beam dynamics are surprisingly tractable
  - Can predict where **isolated** resonances are located
    - Steve's lecture and lab 2
  - Can predict **other isolated resonance properties**
    - N-turn Hamiltonians give "island tunes", resonance widths

Fig. 15.7

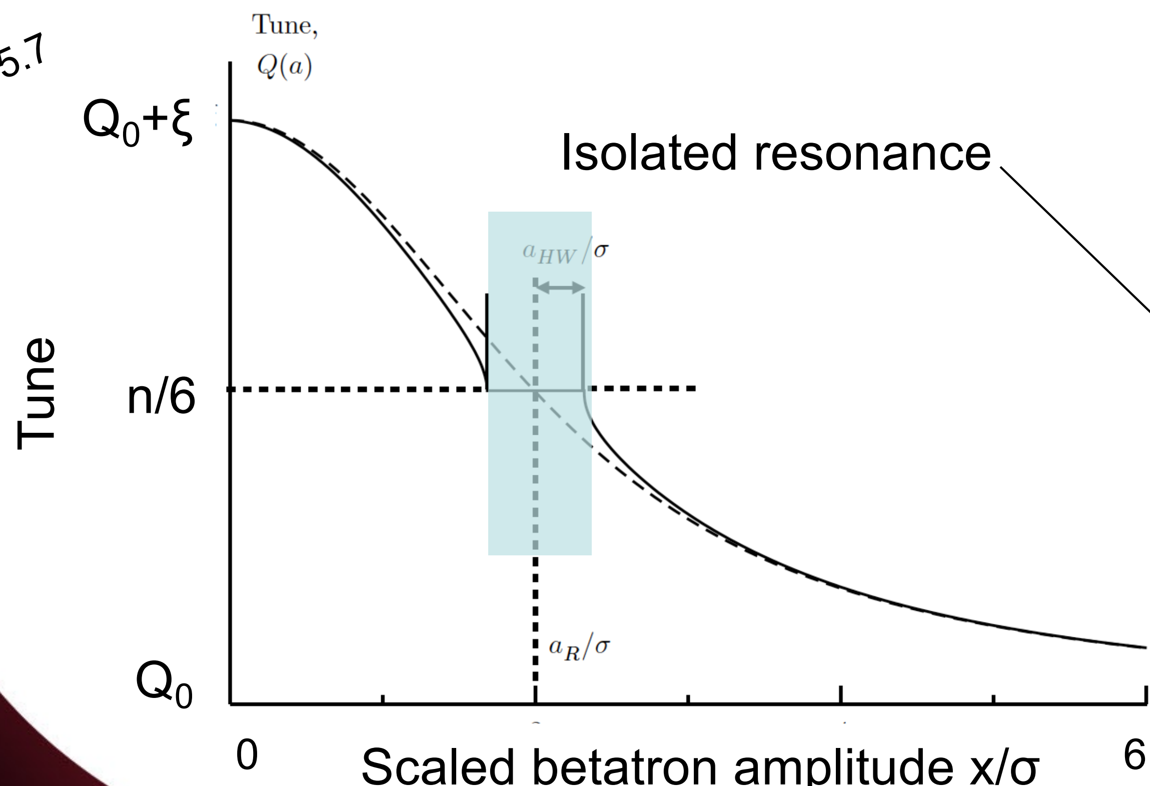
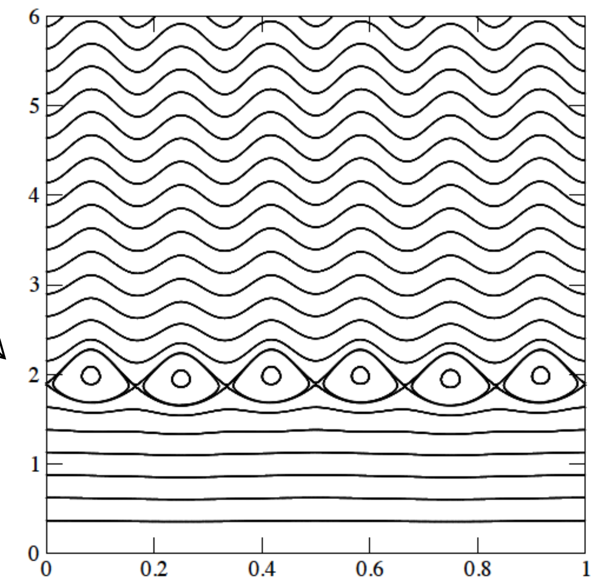


Fig. 15.4

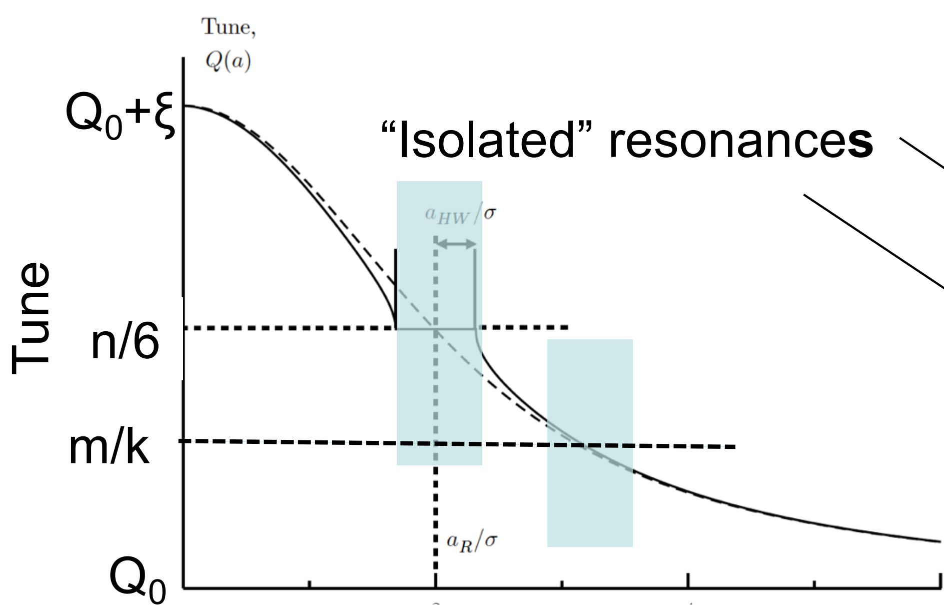


Can we predict the onset of chaos, even roughly?

16.1: Resonance overlap

- We have evaluated isolated resonances using the n-turn action-angle Kobayashi Hamiltonian where $Q_0 - \frac{p}{n} \ll 1$

$$H_n = \underbrace{2\pi\left(Q_0 - \frac{p}{n}\right)J}_{\text{Small-amplitude}} + \underbrace{2\pi\xi\, U(J)}_{\text{U''(J): detuning}} - 2\pi\xi\, V_n(J)\cos(n\phi) - 2\pi\xi\, V_m(J)\cos(m\phi)$$



“Isolated” resonances

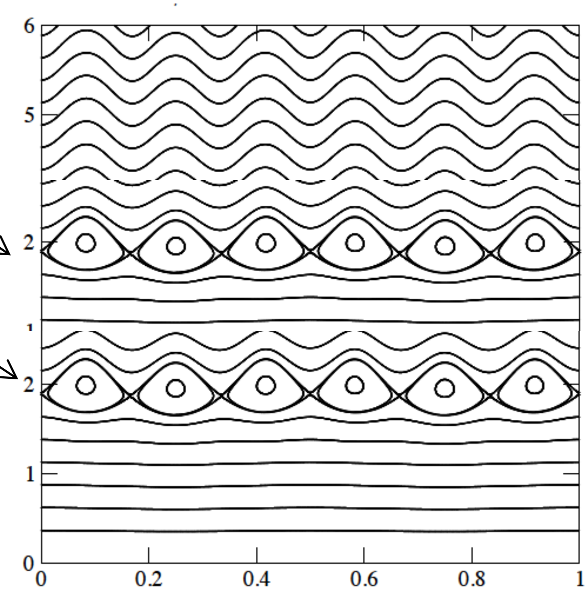
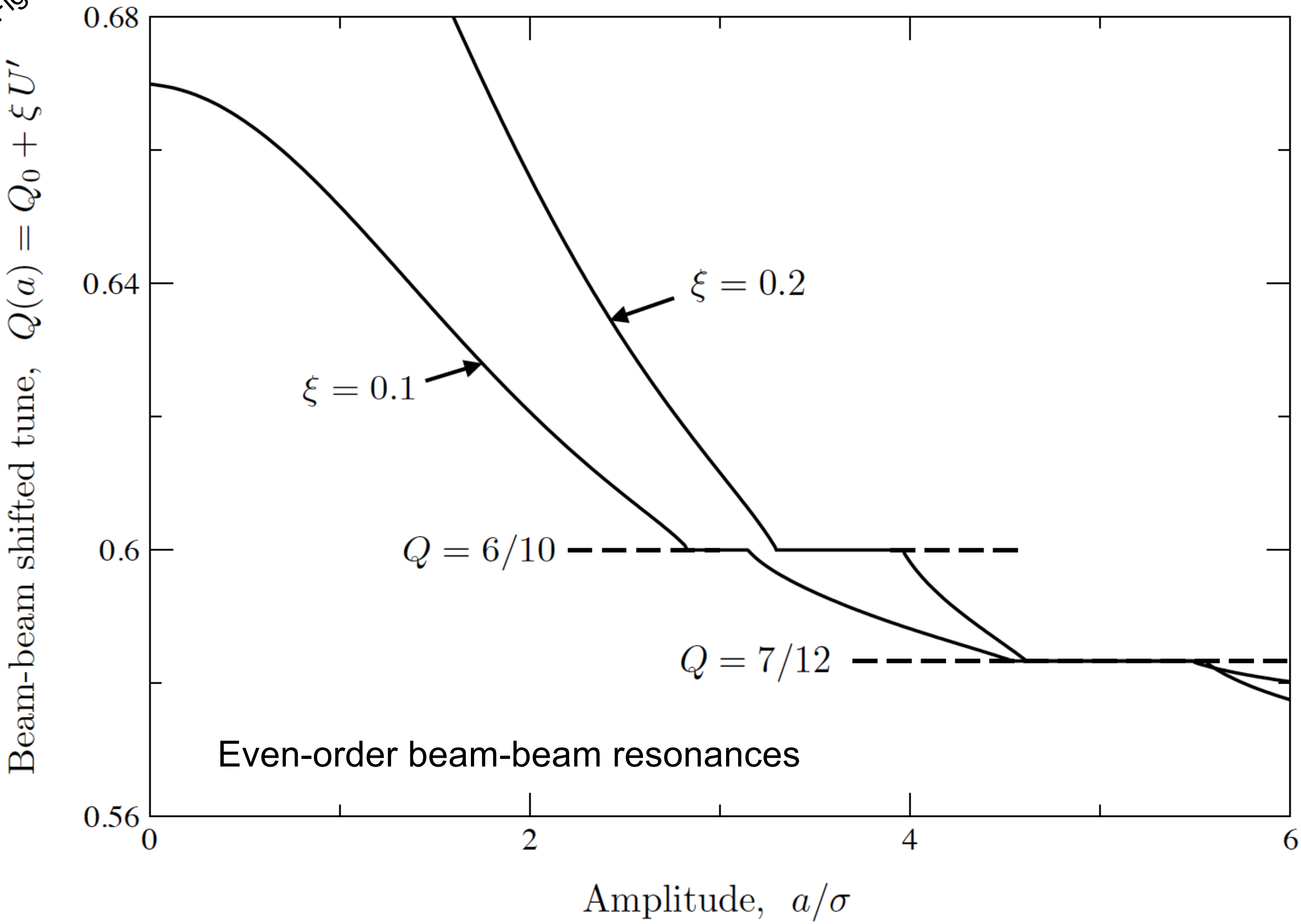
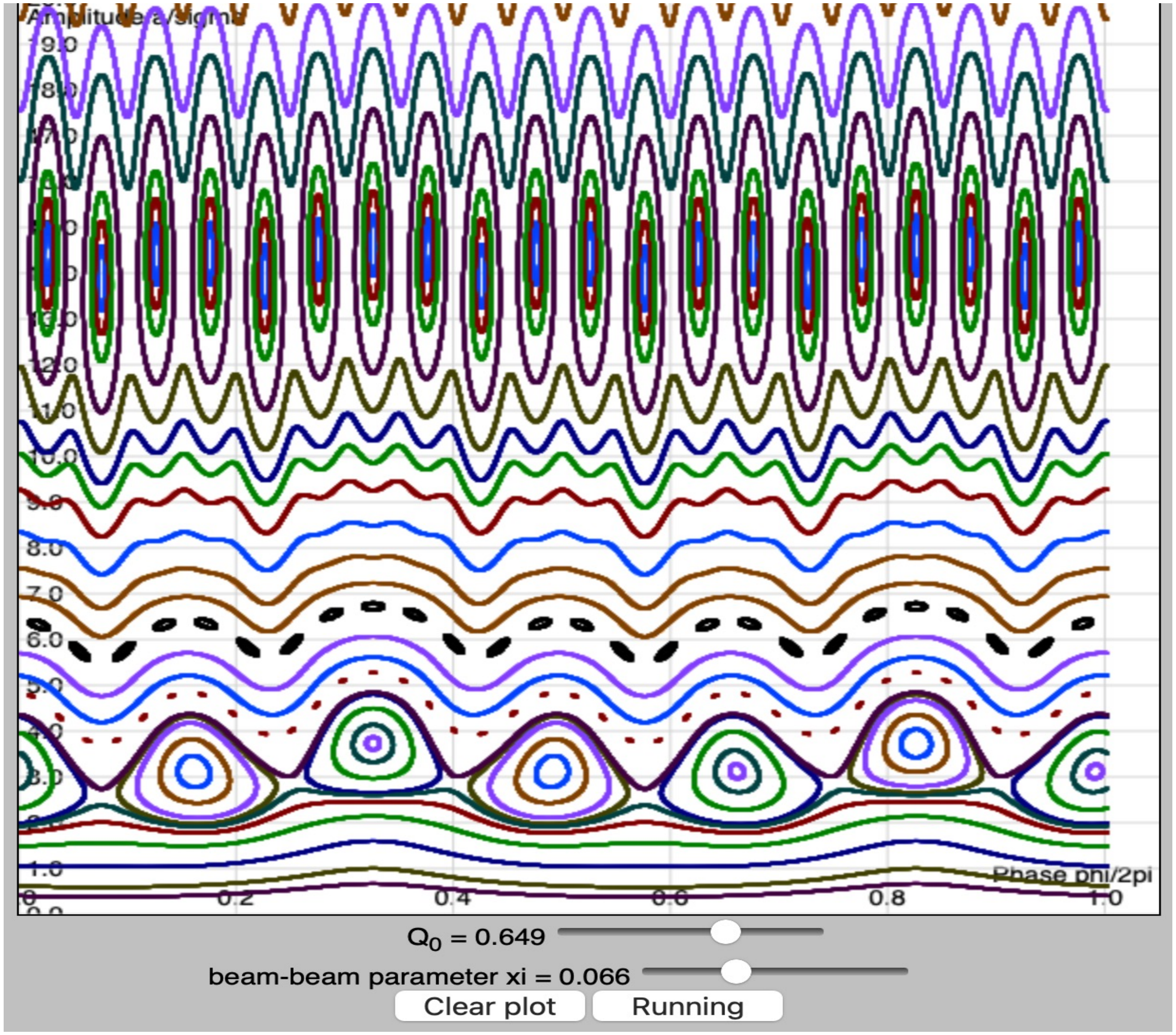


Fig 16.1: Resonance Overlap

Fig. 16.1

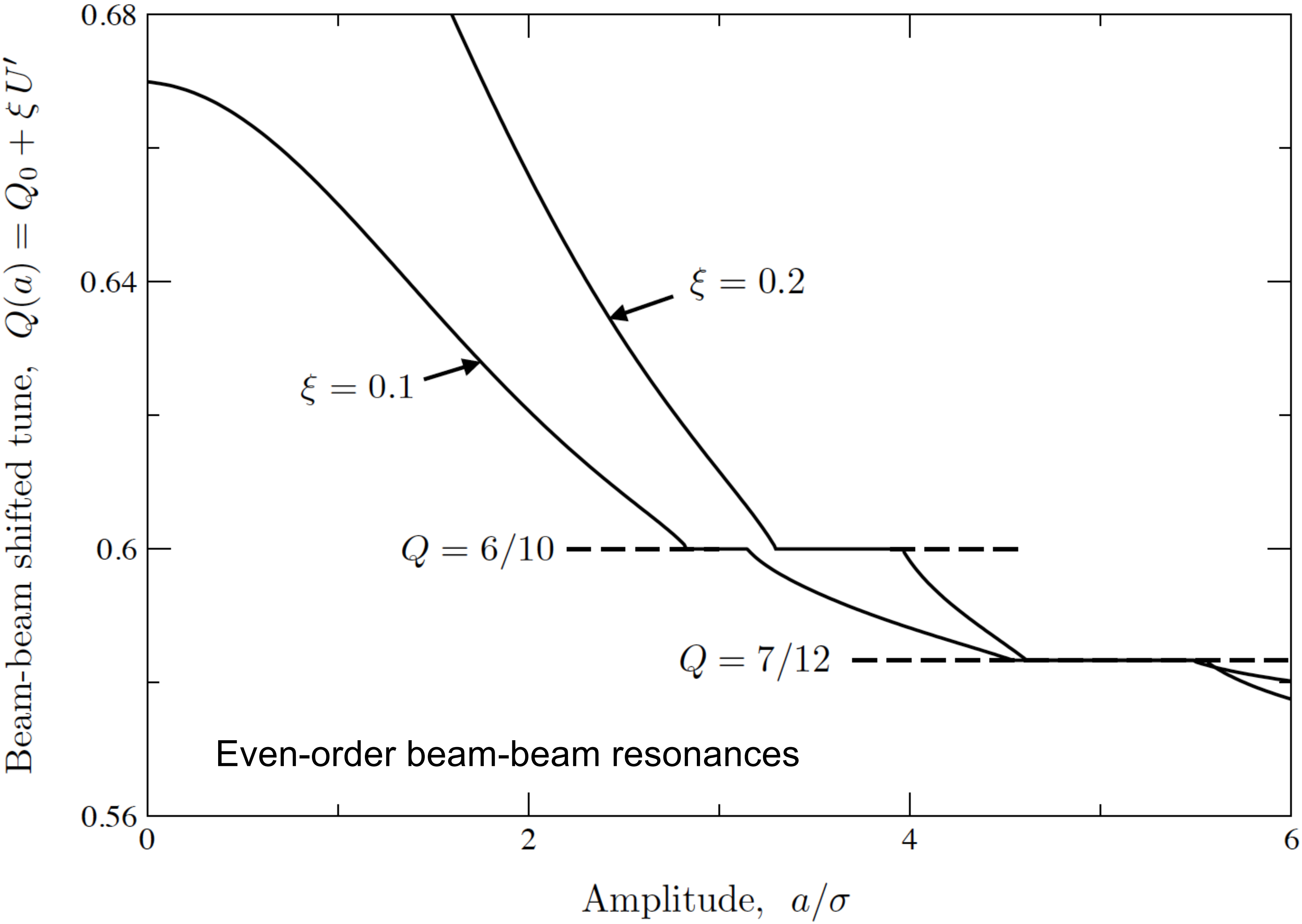


Higher Order "Isolated" Resonances

- 

http://toddsatogata.net/2024-USPAS/homework/BeamBeam2.html

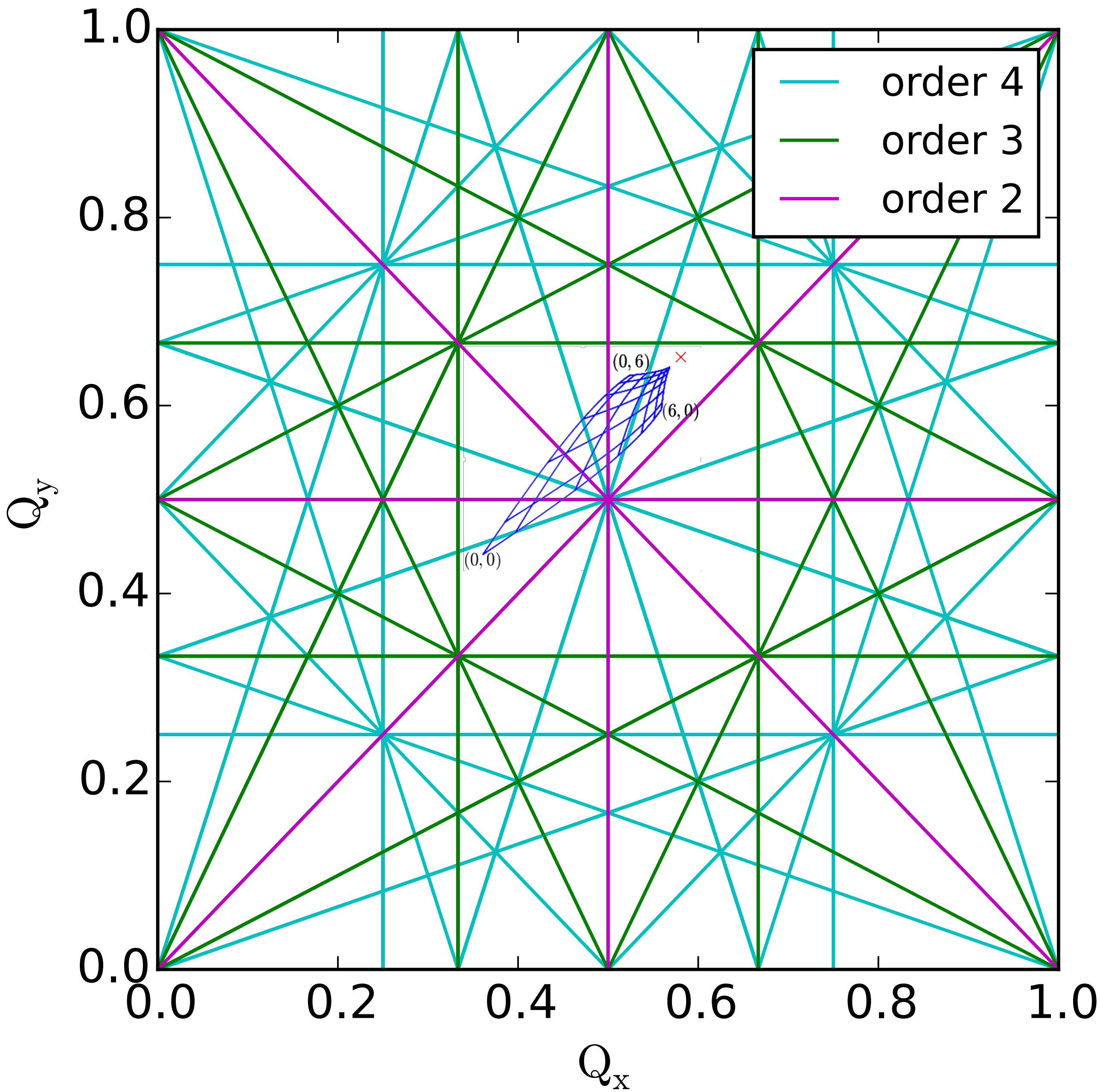
Whoa, Wait a sec: ξ=0.2 could be stable?



Tune Diagram and Beam-Beam Tune Spread

-

ξ=0.2



(Some) EIC Parameters

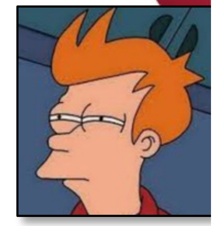


Table 3.3: EIC beam parameters for different center-of-mass energies $\sqrt{s}$, with strong hadron cooling. High divergence configuration.

Species	proton	electron	proton	electron	proton	electron	proton	electron	proton	electron
Energy [GeV]	275	18	275	10	100	10	100	5	41	5
CM energy [GeV]	140.7		104.9		63.2		44.7		28.6	
Bunch intensity [10^{10}]	19.1	6.2	6.9	17.2	6.9	17.2	4.8	17.2	2.6	13.3
No. of bunches	290		1160		1160		1160		1160	
Beam current [A]	0.69	0.227	1	2.5	1	2.5	0.69	2.5	0.38	1.93
RMS norm. emit., h/v [μm]	5.2/0.47	845/71	3.3/0.3	391/26	3.2/0.29	391/26	2.7/0.25	196/18	1.9/0.45	196/34
RMS emittance, h/v [nm]	18/1.6	24/2.0	11.3/1.0	20/1.3	30/2.7	20/1.3	26/2.3	20/1.8	44/10	20/3.5
β^*, h/v [cm]]	80/7.1	59/5.7	80/7.2	45/5.6	63/5.7	96/12	61/5.5	78/7.1	90/7.1	196/21.0
IP RMS beam size, h/v [μm]	119/11		95/8.5		138/12		125/11		198/27	
K_x	11.1		11.1		11.1		11.1		7.3	
RMS $\Delta\theta$, h/v [μrad]	150/150	202/187	119/119	211/152	220/220	145/105	206/206	160/160	220/380	101/129
BB parameter, h/v [10^{-3}]	3/3	93/100	12/12	72/100	12/12	72/100	14/14	100/100	15/9	53/42
RMS long. emittance [10^{-3}, eV·s]	36		36		21		21		11	
RMS bunch length [cm]	6	0.9	6	0.7	7	0.7	7	0.7	7.5	0.7
RMS $\Delta p/p$ [10^{-4}]	6.8	10.9	6.8	5.8	9.7	5.8	9.7	6.8	10.3	6.8
Max. space charge	0.007	neglig.	0.004	neglig.	0.026	neglig.	0.021	neglig.	0.05	neglig.
Piwinski angle [rad]	6.3	2.1	7.9	2.4	6.3	1.8	7.0	2.0	4.2	1.1
Long. IBS time [h]	2.0		2.9		2.5		3.1		3.8	
Transv. IBS time [h]	2.0		2		2.0/4.0		2.0/4.0		3.4/2.1	
Hourglass factor H	0.91		0.94		0.90		0.88		0.93	
Luminosity [10^{33}cm^{-2}s^{-1}]	1.54		10.00		4.48		3.68		0.44	

EIC Max Beam-Beam Parameters

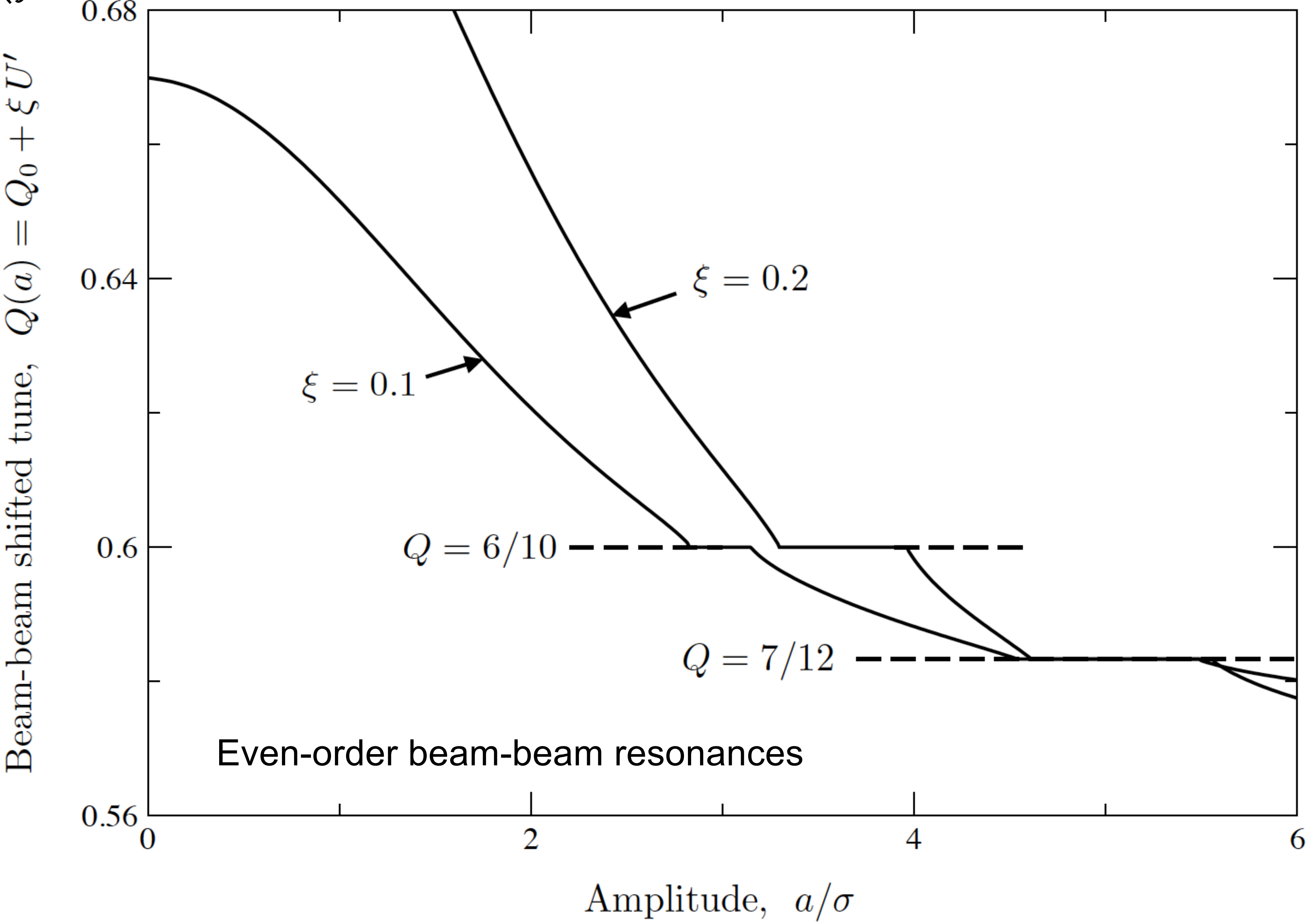
Species	proton	electron
Energy [GeV]	100	5
CM energy [GeV]	44.7	
BB parameter, h/v $[10^{-3}]$	14/14	100/100

- Proton beam-beam: ξ=0.014
  - Comparable to Tevatron p/pbar experience (ξ=0.012)
- Electron beam-beam: ξ=0.1
  - Comparable to SuperKEK e- experience (e+ up to ξ=0.12)
- Both are aggressive for an aggressive collider
- HERA achieved (0.019,0.045) e-, (0.0012,0.0009) p

Useful updated exhaustive table of collider parameters (2022):
https://pdg.lbl.gov/2022/reviews/rpp2022-rev-hep-collider-params.pdf

Fig 16.1: Resonance Overlap

Fig. 16.1



Even-order beam-beam resonances

16.1: Resonance overlap

- We evaluated isolated resonances using the n-turn action-angle Kobayashi Hamiltonian where $Q_0 - \frac{p}{n} \ll 1$

$$H_n = \underbrace{2\pi\left(Q_0 - \frac{p}{n}\right) J}_{\text{Small-amplitude frequency}} + \underbrace{2\pi\xi\, U(J)}_{U''(J)\text{: detuning}} - \underbrace{2\pi\xi\, V_n(J)\cos(n\phi)}_{V_n(J)\text{: resonance driving}}$$

Small-amplitude frequency

U''(J): detuning

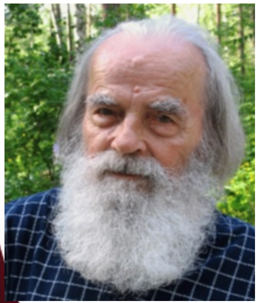
V_n(J): resonance driving

- Convenient way to write equations of motion (tracking)
  - Conserved quantity: phase space "KAM tori"
- Here we had assumed that all other resonances either
  - have small V_n (so their widths are small enough to ignore) or
  - their Hamiltonian terms phase average to near zero over many iterations of this map (over many turns)

16.1: Resonance overlap: Chirikov

- What happens when this assumption breaks down?
  - Resonances approach the point of overlapping
  - Separatrices are the first to interact
    - But separatrices have infinite period and therefore are infinitely vulnerable to perturbation

- Chirikov hypothesized that chaos emerges when nearby resonance widths are large enough that they overlap



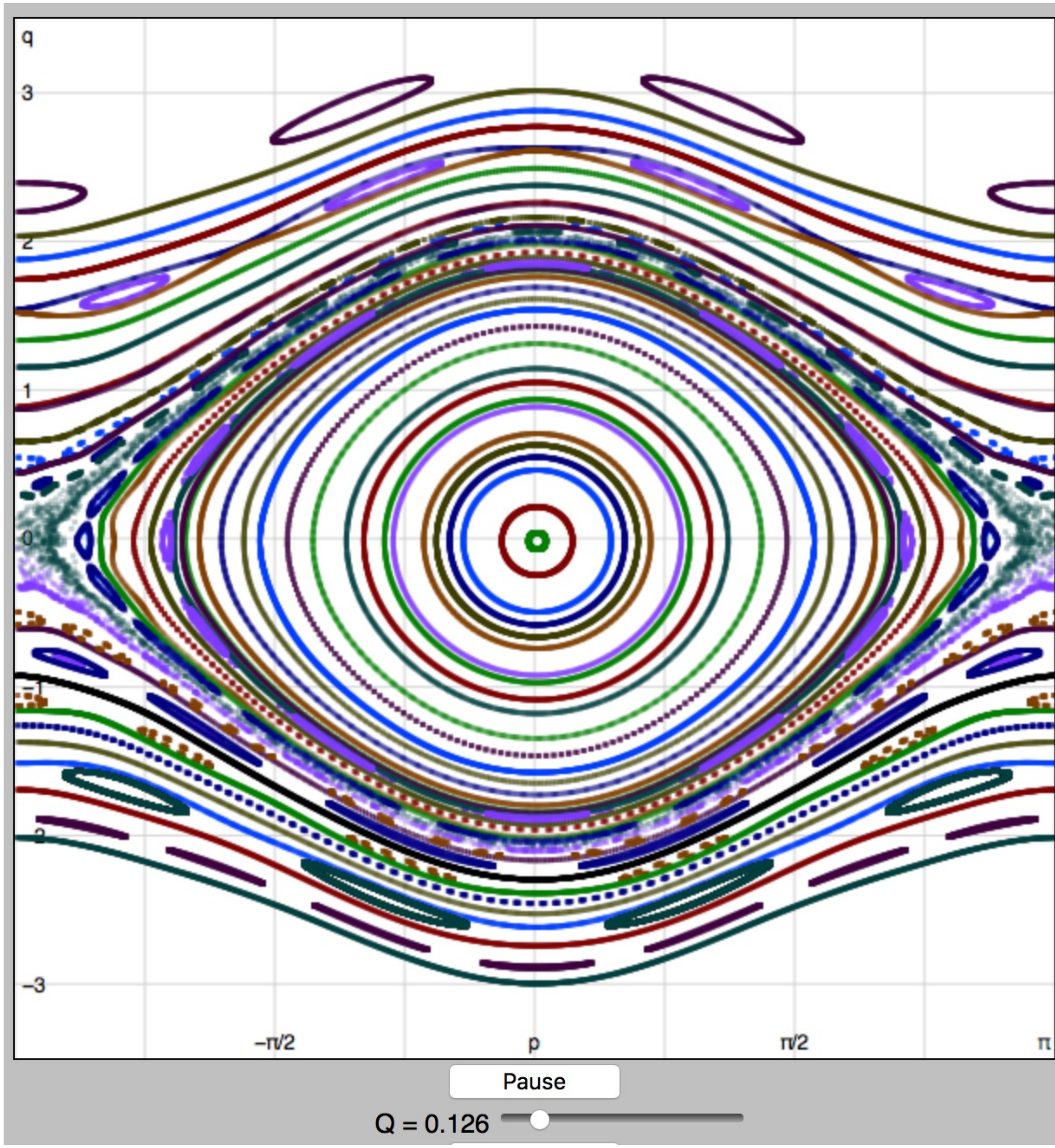
  - This is calculable and verifiable
  - "A Universal Instability of Many-Dimensional Oscillator Systems" – Physics Reports **52** 5, May 1979, pp. 263-379.
  - 5000+ citations: a "famous" paper and surprisingly readable

16.1: Chirikov Overlap and the Standard Map

Chaotic resonance overlap: separatrix becomes chaotic

RF motion with large Qs!

Difference map



"Isolated" resonances

16.2: 6D Motion and Tune Modulation

- We have been (understandably) rather naive
  - This is a perfect 1D uncoupled nonlinear model

$$\Delta x' = \frac{B'L}{(B\rho)} x$$

- Reality (aside from noise)
  - Dispersion couples longitudinal and transverse motion
    - Almost always have to bend the beam somewhere
    - Off-center motion in quadrupoles also gives dipole “feed-down”
  - Coupling couples transverse motion
    - Quadrupoles have random rotations relative to design plane
  - Sextupoles are necessary (mostly)
    - Chromaticity correction in large accelerators
- Any coupling adds different frequencies to our system
  - Tune modulation, e.g. $Q_0 = Q_{00} + q \, \sin(2\pi Q_s t)$

16.2: Tune modulation

- The isolated resonance Kobayashi Hamiltonian was

$$H_n = 2\pi\left(Q_0 - \frac{p}{n}\right)J + 2\pi\xi\, U(J) - 2\pi\xi\, V_n(J)\cos(n\phi)$$
$$Q_0 = Q_{00} + q\ \sin(2\pi Q_s t)$$

- Modulation of the tune looks like a time-dependent driving term
  - Poincare and periodicity asides, large-N-turn maps
- To first order, the phase modulation also appears in the resonance driving term!
  - Phase-modulated pendulum: Mathieu equation
  - Todd's dissertation: http://www.toddsatogata.net/Thesis/
  - Sidebands and sideband overlap leading to chaotic motion

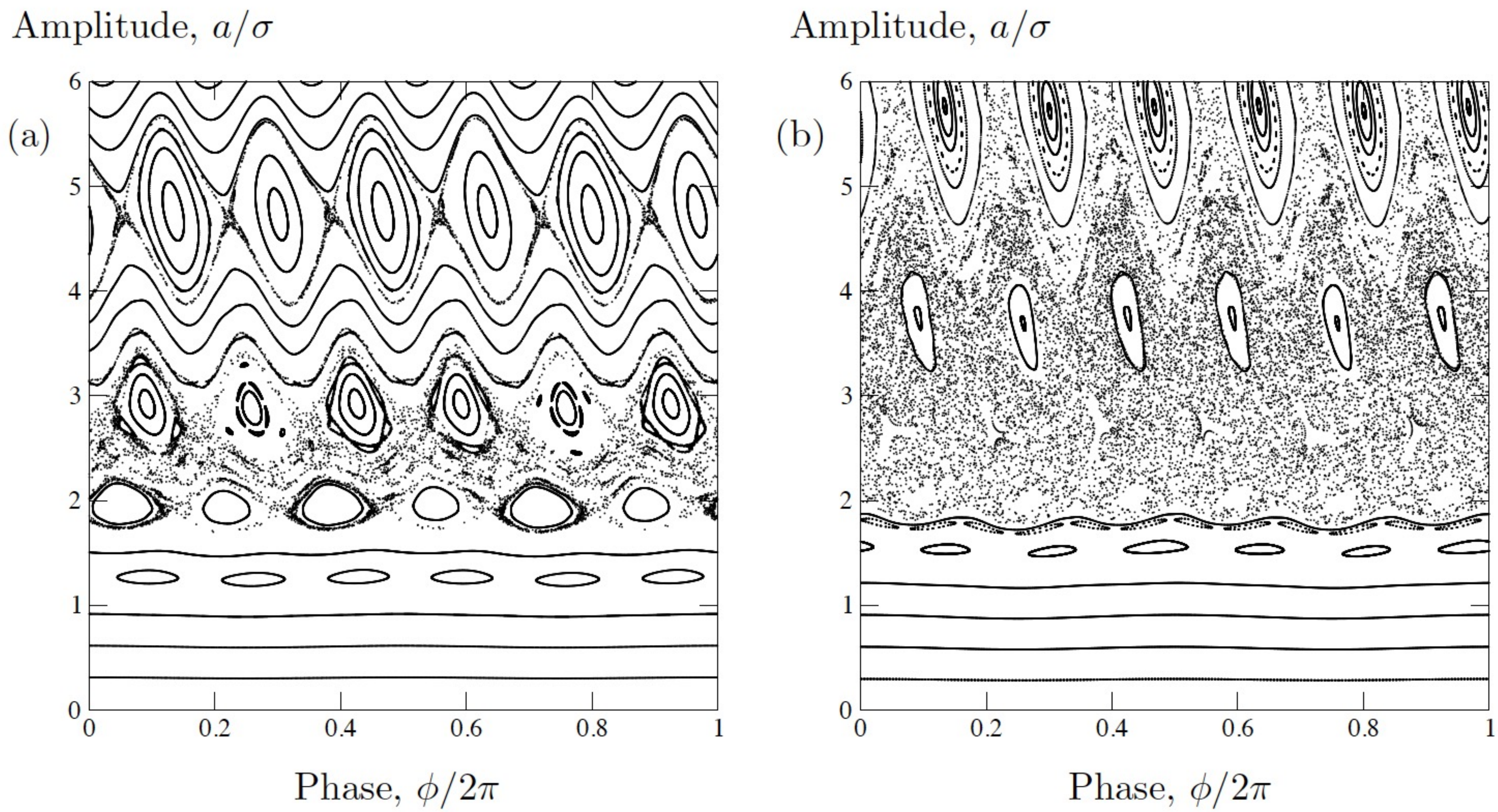


Figure 16.2 Simulated phase space structure due to one round beam-beam kick of strength $\xi = 0.0042$ (a), and $\xi = 0.006$ (b), with the parameters of Equation 16.19. The modest increase in ξ moves the tune modulation side-bands closer together, and dramatically broadens the chaotic sea, allowing

$$Q_0 = Q_{00} + q\ \sin(2\pi Q_s t)$$ q = 0.001 Qs=0.00515

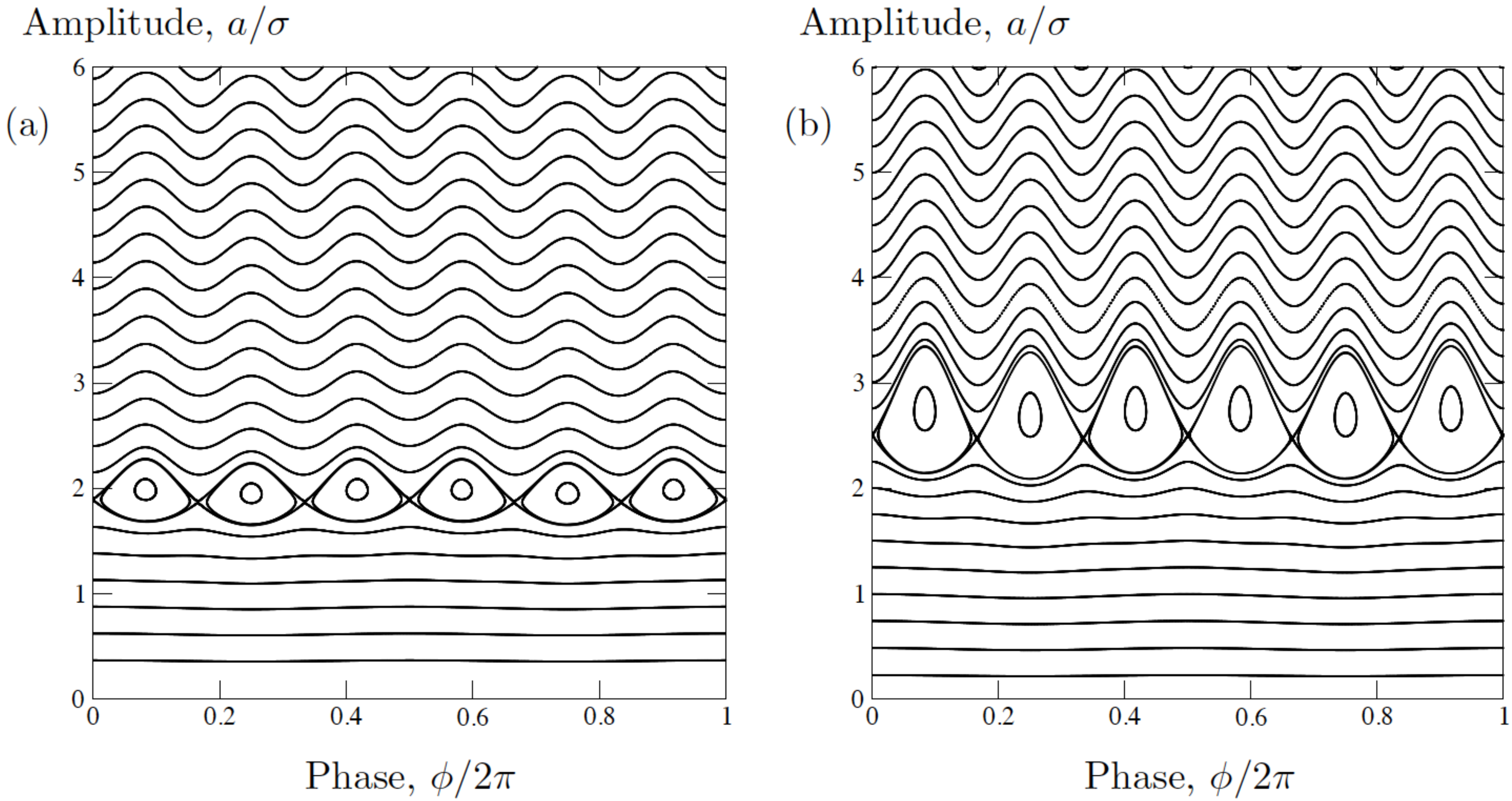
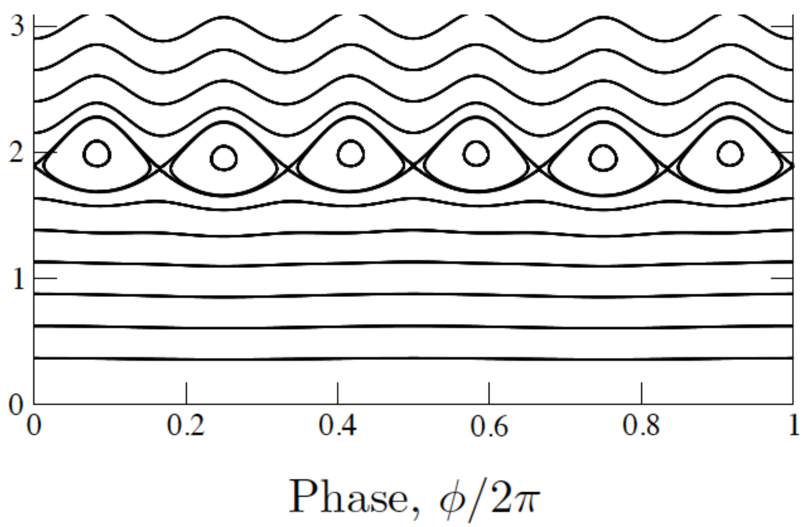
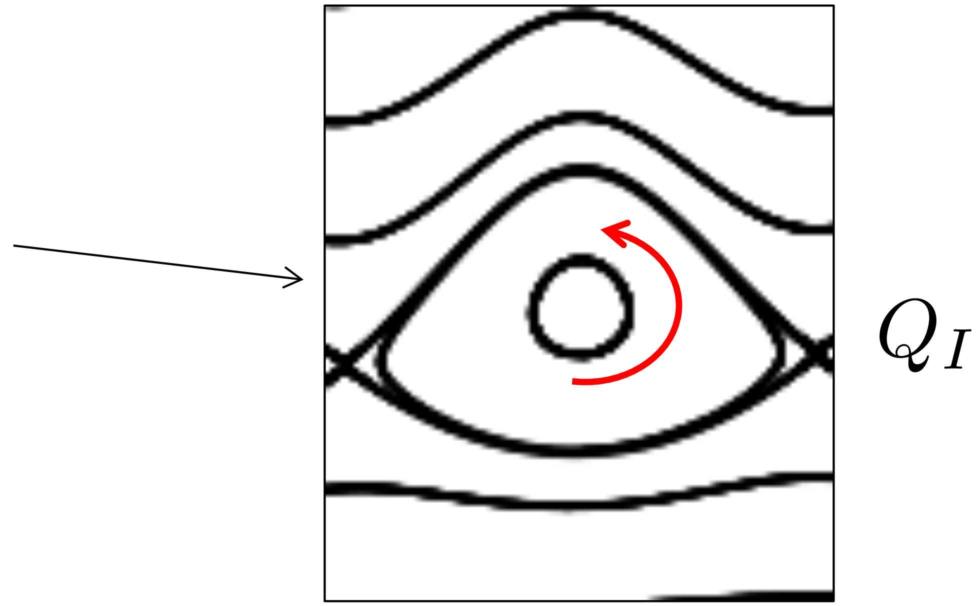


Figure 15.4 Six island chains from the simulation of a single round beam-beam interaction of strength $\xi = 0.0042$ (a), and $\xi = 0.006$ (b), with a base tune of $Q_0 = 0.331$ [40]. The amplitude width of the islands increases as the chain moves to a larger resonance amplitude when ξ is increased. (See also Figure 16.2.)

Tune Modulation Frequency Scale

- Remember, tune modulation is really "just" modulating a pendulum
  - We know kicking around a pendulum near its natural frequency produces excitement
  - What is the natural frequency of resonant motion?
  - Remember these are topologically equivalent to pendula
  - "Island" tune: $Q_I \ll Q_{00}$





Slow Tune Modulation: Amplitude Modulation

$$H_n = 2\pi\left(Q_0 - \frac{p}{n}\right)J + 2\pi\xi\, U(J) - 2\pi\xi\, V_n(J)\cos(n\phi)$$

$$Q_0 = Q_{00} + q\,\sin(2\pi Q_s t)$$

- If modulation frequency is much lower than island tune

$$Q_s \ll Q_I$$

- then the modulation really looks like a slow variation of Q_0
  - Nearly adiabatic with respect to the resonance “island” motion
  - Amplitudes of resonances “breathe” up and down
  - Island widths also vary because their amplitudes are changing

- Conversely, modulation frequency >> island tune...

Tune Modulation Diagram

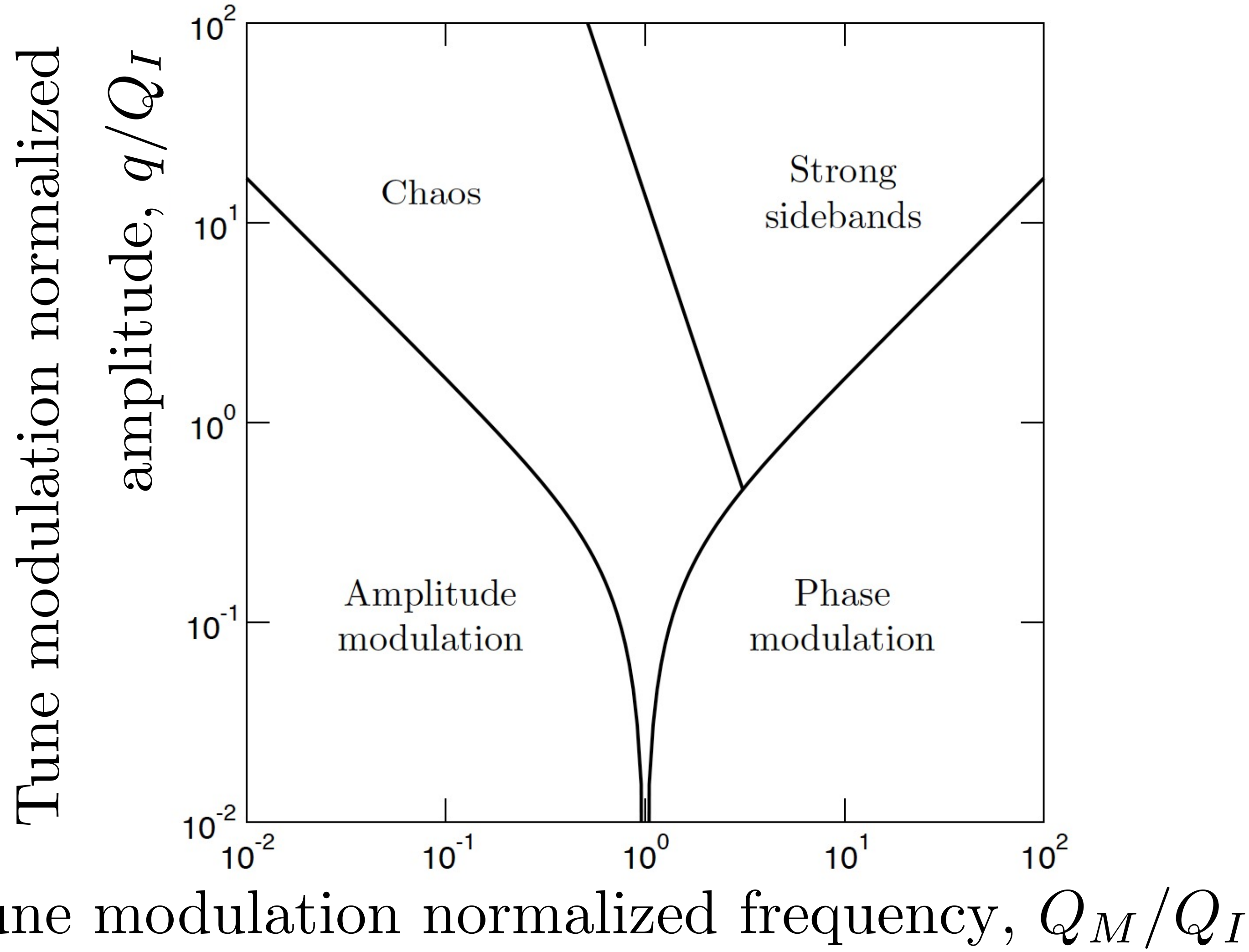
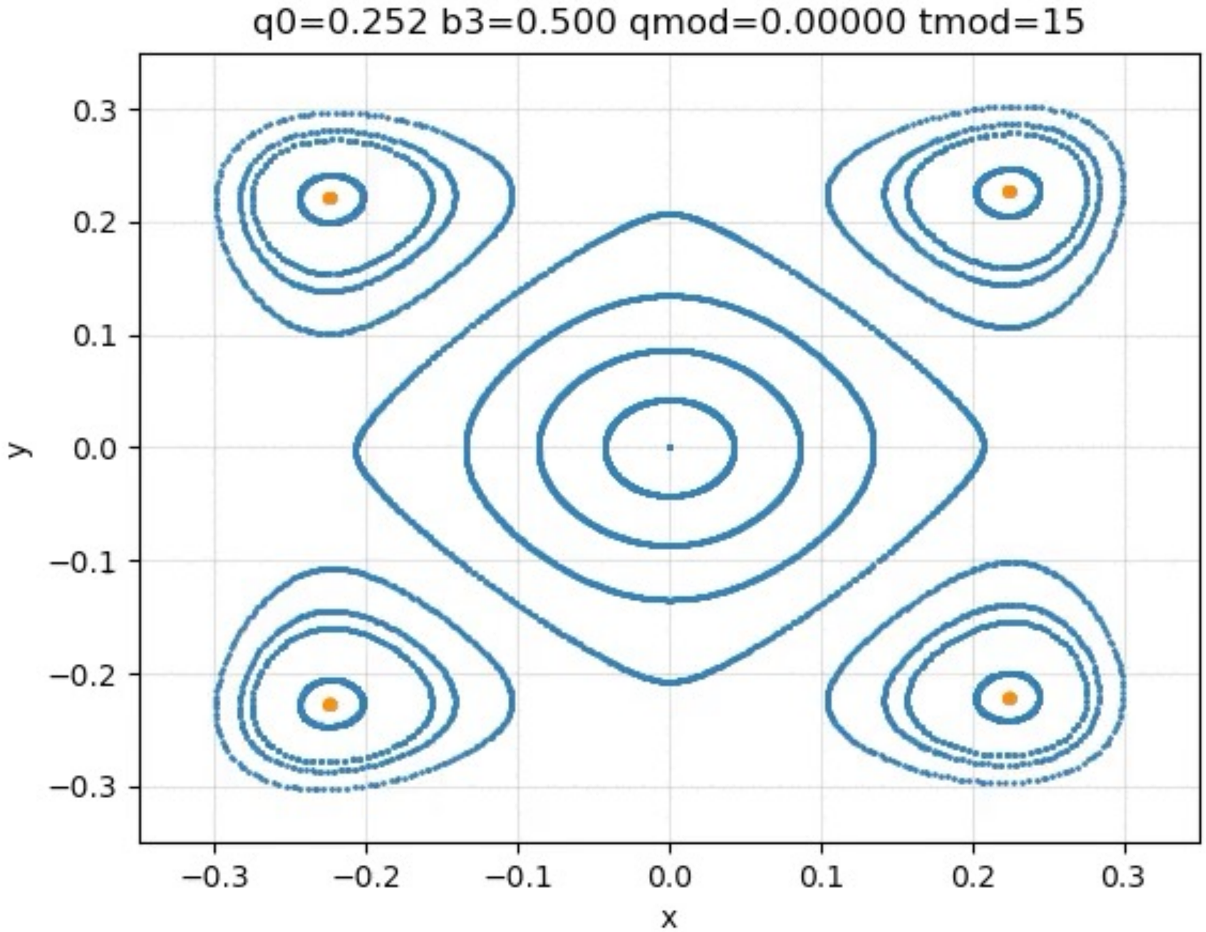
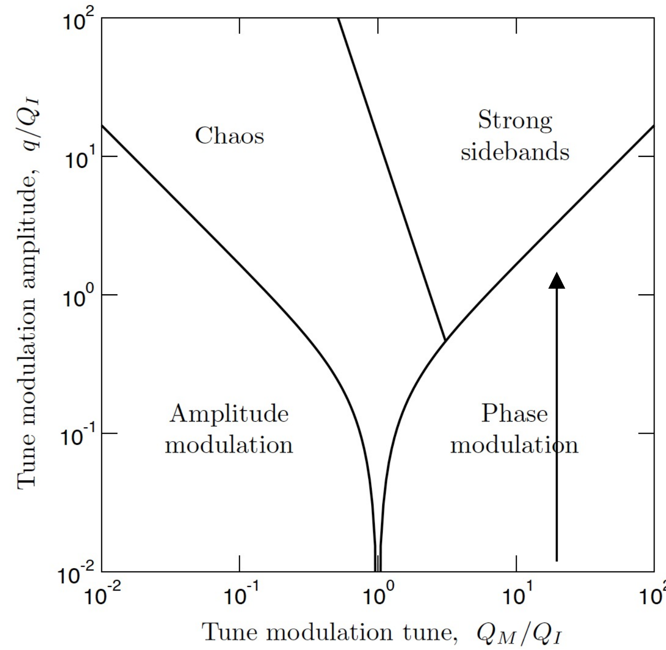


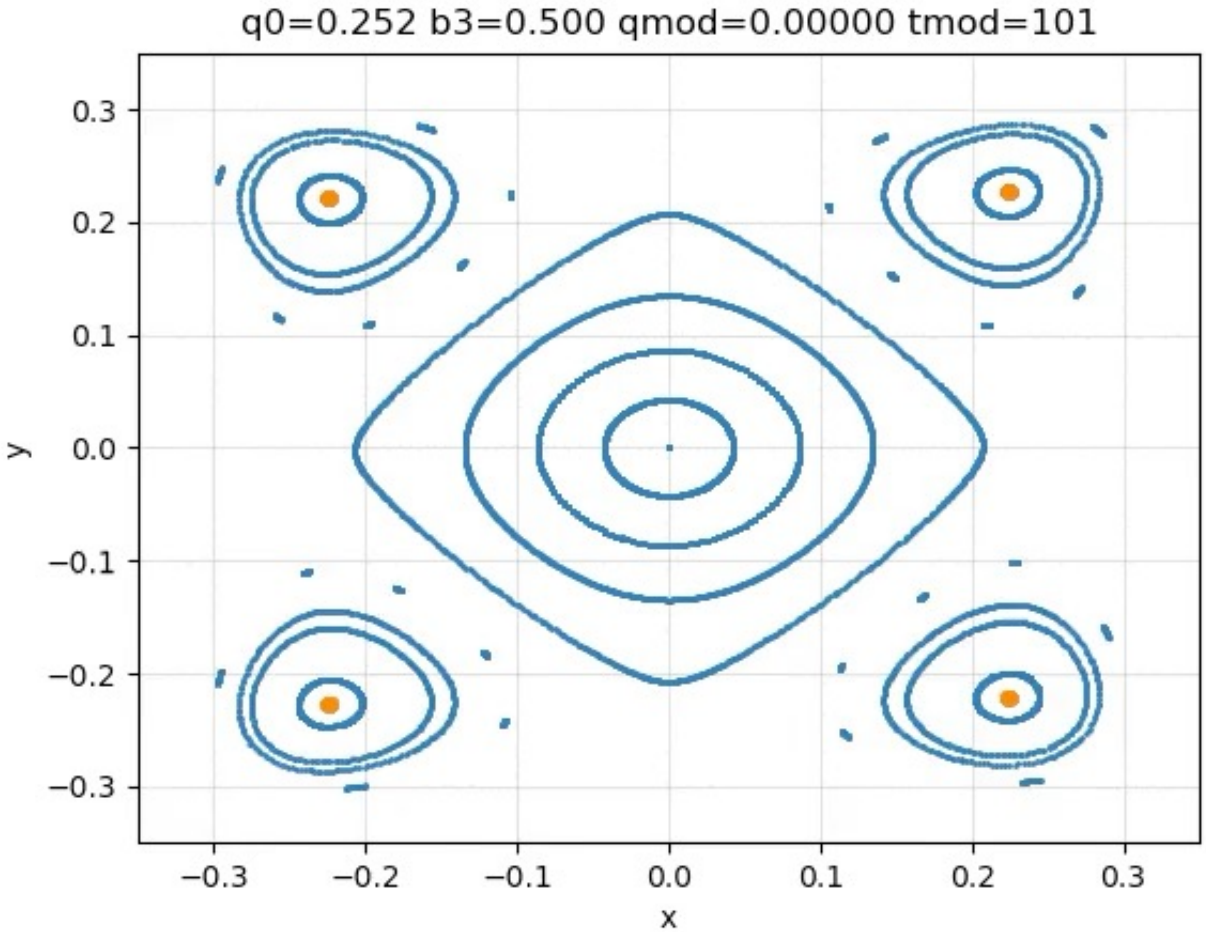
Figure 16.3 Dynamical zones universally predicted in normalised tune modulation space $(q/Q_I, Q_M/Q_I)$ for n = 6, with the boundaries defined in Equation 16.23. The island tune Q_I, a scale factor on both axes, is a parameter of central importance.

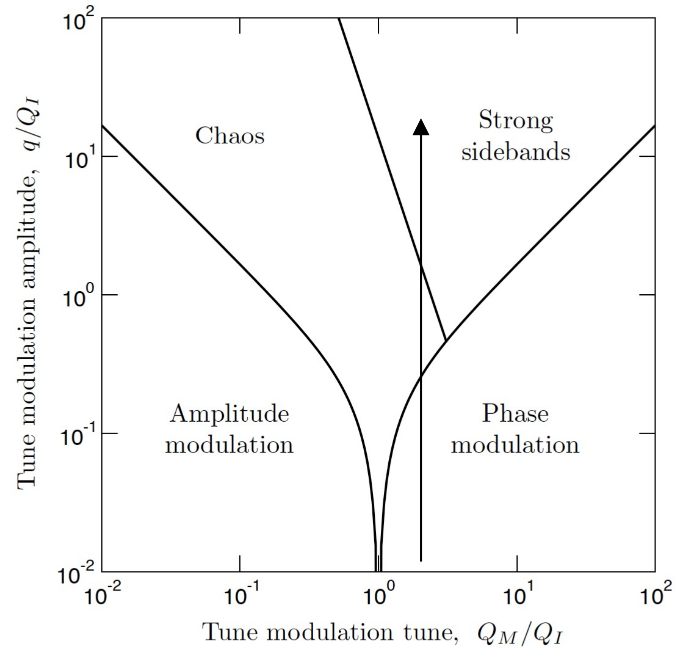
Fun Phase Space Movies: Phase Modulation



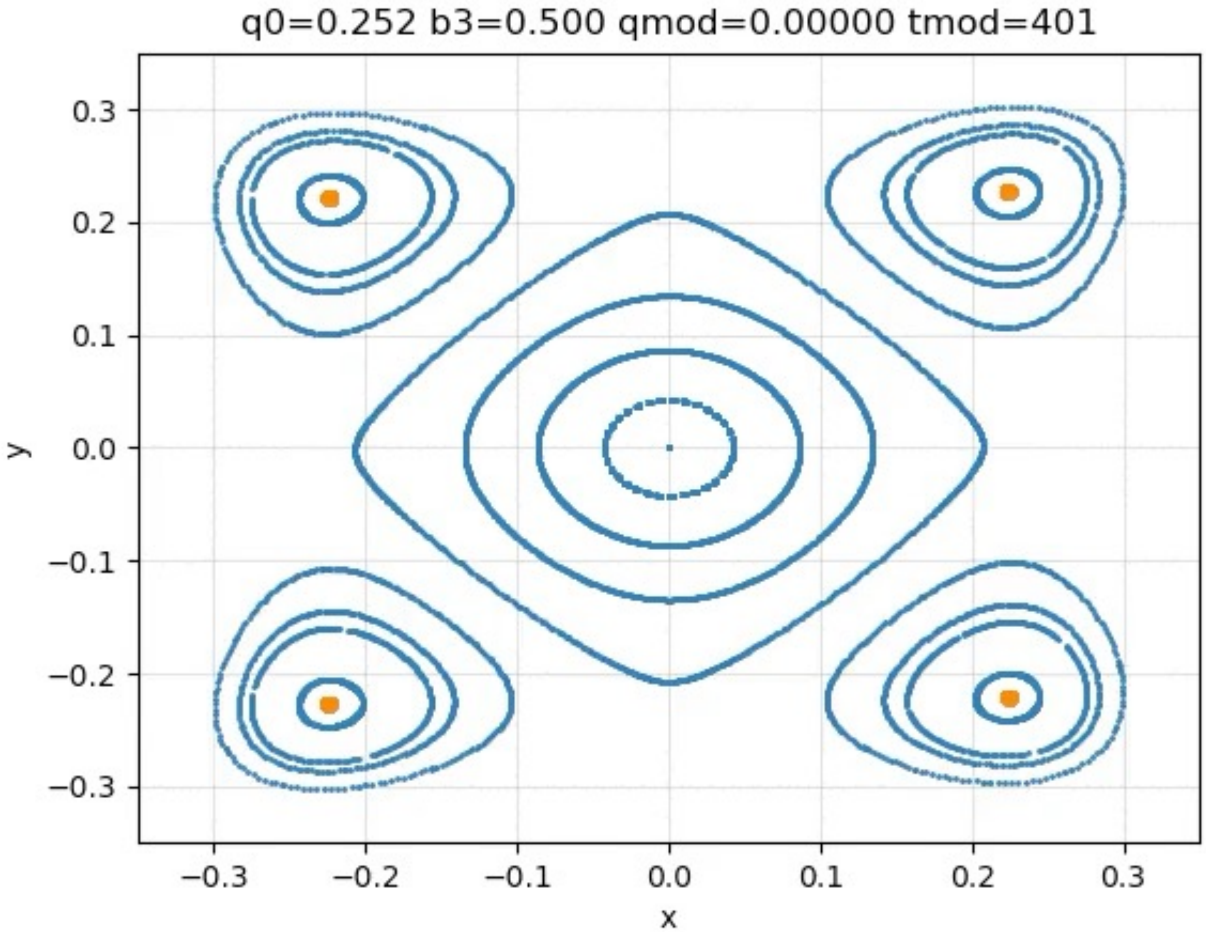


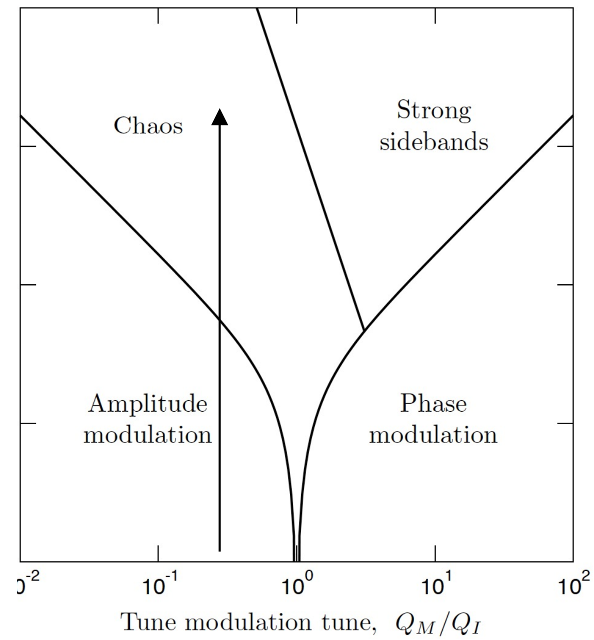
Fun Phase Space Movies: Above Resonance





Fun Phase Space Movies: Below Resonance





E778 Persistant Signal

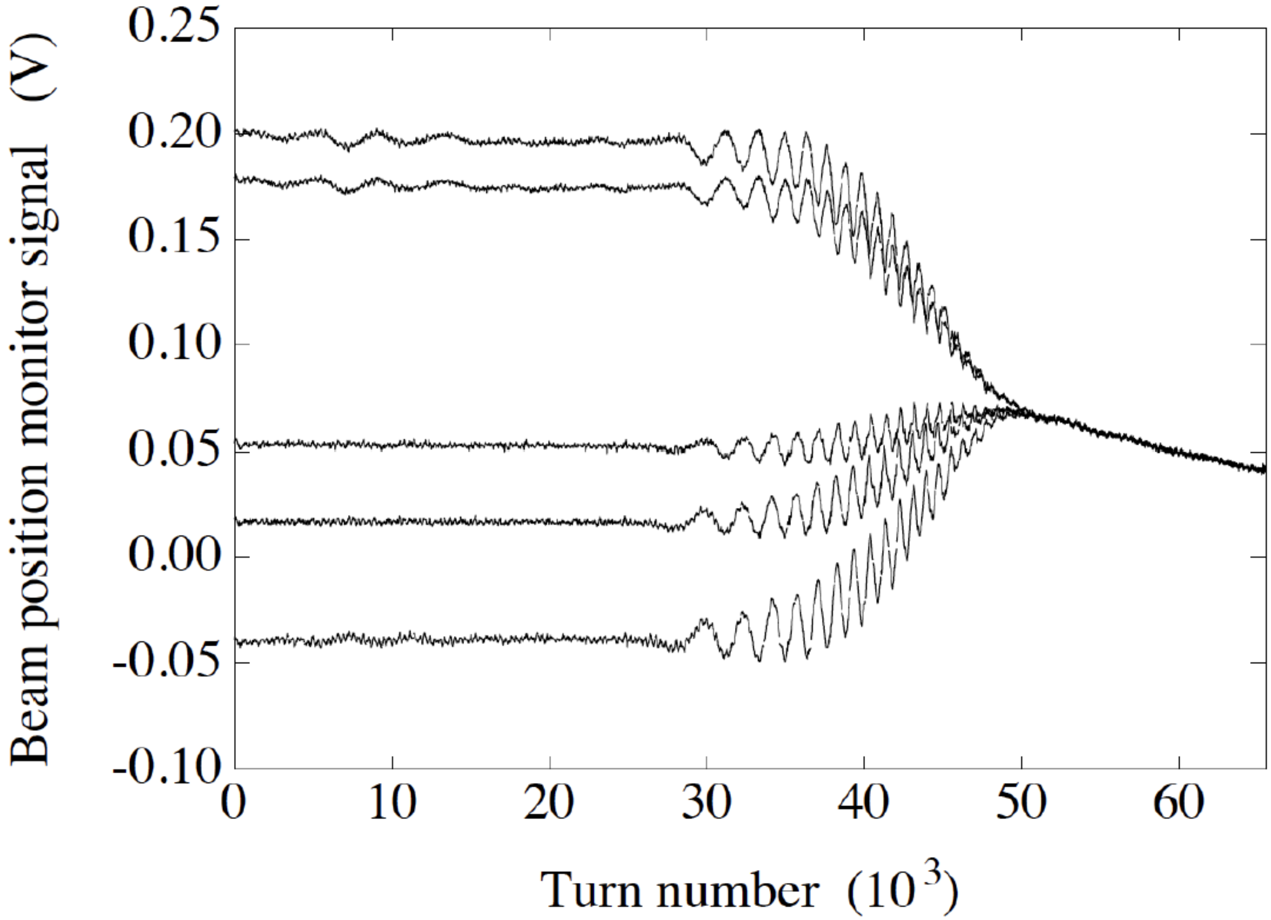
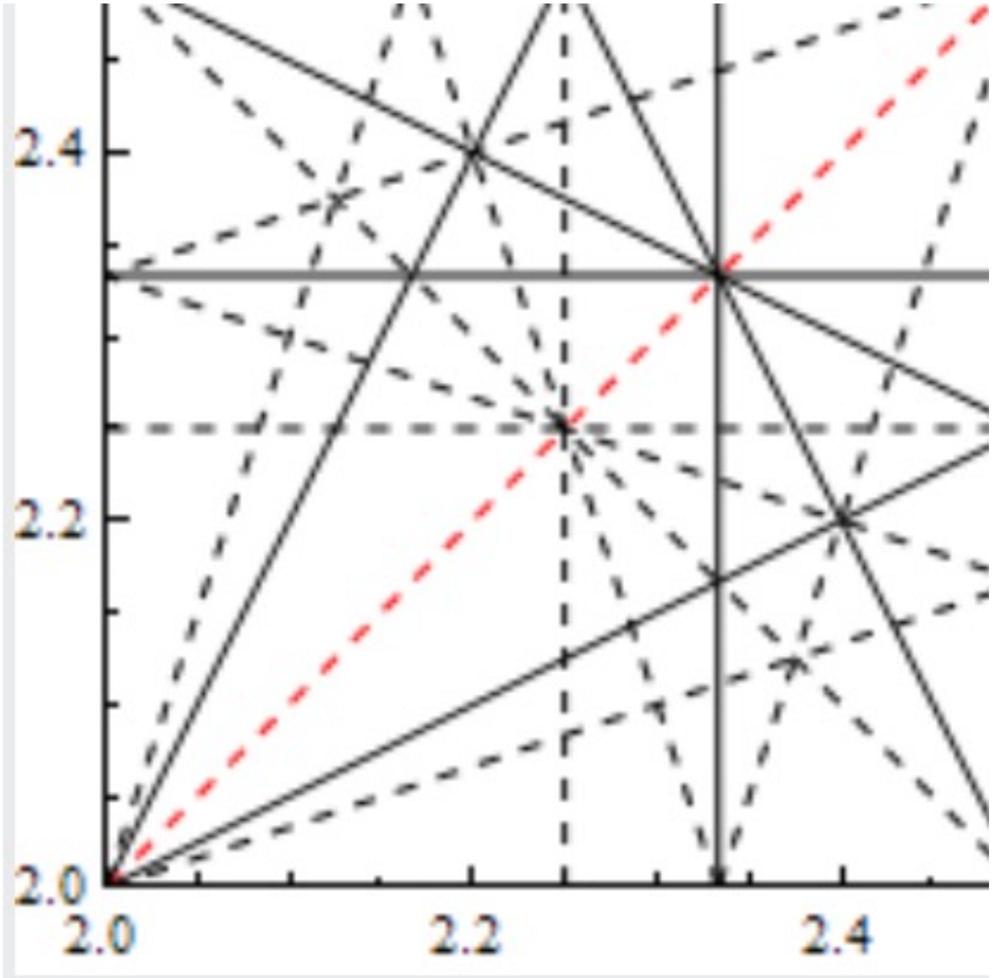


Figure 16.4 Turn-by-turn persistent signal data due to beam trapped in an $n = 5$ resonance island in the nonlinear dynamics Tevatron experiment E778 [47, 48]. The resonance is driven by sextupoles. Chirping the operating point from the *amplitude modulation* zone into the *chaos* zone in Figure 16.3 destroys the resonance, and the persistent signal.

Arnold Diffusion and Integrability

- This is still just 1.5 dimensions!!
  - One dimension plus time
- In more dimensions, particles can move along resonance lines and diffuse in tune space
- A mechanism for long-term amplitude growth in tracking and perhaps even reality
- Visualization makes my brain hurt
- Integrability class next door



Lichtenberg and Lieberman, Jose and Saletan